Three-phase Circuits

Mohammad Hadi

mohammad.hadi@sharif.edu

@MohammadHadiDastgerdi

Fall 2021

Overview

Electricity Delivery System

Electricity Delivery System

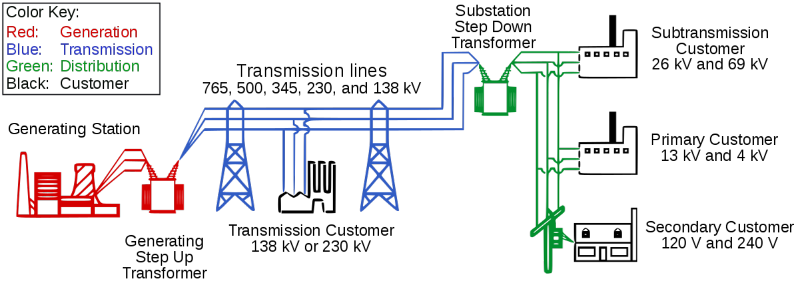


Figure: Electricity delivery system.

- Generation, transmission, and distribution subsystems
- Three-phase and high-voltage transmission

High-voltage Transmission

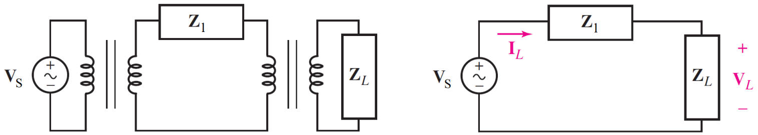


Figure: Simplified power transmission model with $Z_L = R_L + jX_L$ and $Z_l = R_l + jX_l$.

- Load power: $P_L = |V_L||I_L|\cos(\angle V_L - \angle I_L)$
- Transmission loss: $P_l = R_l|I_l|^2 = \frac{R_l P_L^2}{V_L^2\cos^2(\angle V_L - \angle I_L)}$
- High transmission voltage to reduce transmission loss
- Unit power factor to reduce transmission loss

Three-phase Transmission

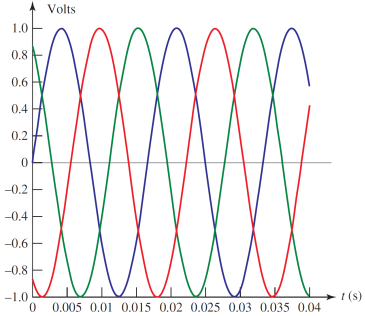


Figure: An example set of three voltages, each of which is 120° out of phase with the other two.

- Three-phase synchronous generators
- Three-phase loads and motors
- Three-wire transmission line
- Constant instantaneous power delivery
- Efficient rectification for DC generation

Three-phase Voltage Source

Three-phase Voltage Source

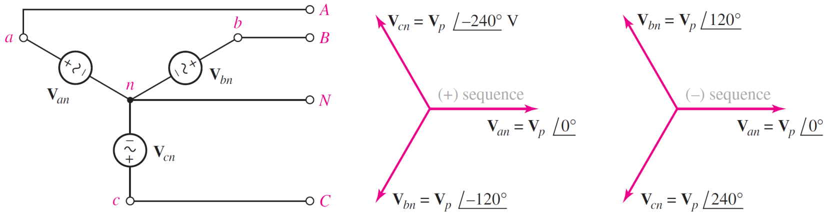


Figure: Positive (abc) and negative (acb) phase sequence in three-phase voltage source.

- Positive sequence:

$$\begin{cases} v_{an}(t) = V\cos(\omega t) & \equiv V_{an} = V\angle 0^\circ \\ v_{bn}(t) = V\cos(\omega t - 120^\circ) & \equiv V_{bn} = V\angle -120^\circ \\ v_{cn}(t) = V\cos(\omega t + 120^\circ) & \equiv V_{cn} = V\angle 120^\circ \end{cases}$$

- Negative sequence:

$$\begin{cases} v_{an}(t) = V\cos(\omega t) & \equiv V_{an} = V\angle 0^\circ \\ v_{bn}(t) = V\cos(\omega t + 120^\circ) & \equiv V_{bn} = V\angle 120^\circ \\ v_{cn}(t) = V\cos(\omega t - 120^\circ) & \equiv V_{cn} = V\angle -120^\circ \end{cases}$$

$$v_{an}(t) + v_{bn}(t) + v_{cn}(t) = 0 \Rightarrow V_{an} + V_{bn} + V_{cn} = 0$$

Three-phase Balanced Circuits

Three-phase Balanced Circuits

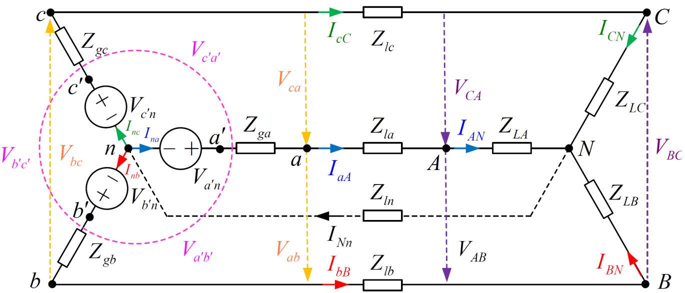


Figure: A three-phase system, connected Y-Y and including a neutral.

- KCL: $\frac{V_{Nn}}{Z_{ln}} + \frac{V_{Nn}-V_{a'n}}{Z_{ga}+Z_{la}+Z_{LA}} + \frac{V_{Nn}-V_{b'n}}{Z_{gb}+Z_{lb}+Z_{LB}} + + \frac{V_{Nn}-V_{c'n}}{Z_{gc}+Z_{lc}+Z_{LC}} = 0$
- Balanced condition: $\begin{cases} V_{a'n} + V_{b'n} + V_{c'n} = 0 \\ Z_{ga} = Z_{gb} = Z_{gc} = Z_g \\ Z_{la} = Z_{lg} = Z_{lc} = Z_l \\ Z_{LA} = Z_{LB} = Z_{LC} = Z_L \end{cases} \Rightarrow V_{Nn} = 0 \Rightarrow I_{Nn} = 0$

Three-phase Balanced Circuits

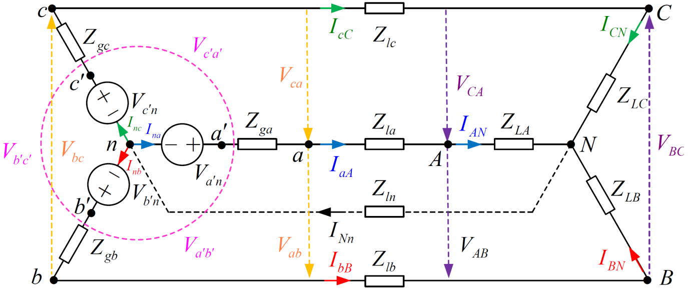


Figure: A three-phase system, connected Y-Y and including a neutral.

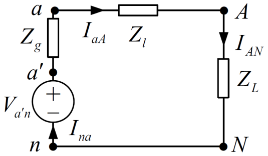


Figure: Single-phase equivalent circuit for a balanced positive-sequence Y-Y three-phase system.

Three-phase Inter-connection

Three-phase Inter-connection

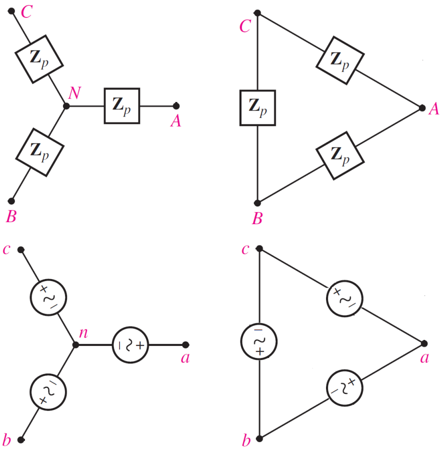


Figure: Typical three-phase connections for source and load.

Phase/Line Quantities

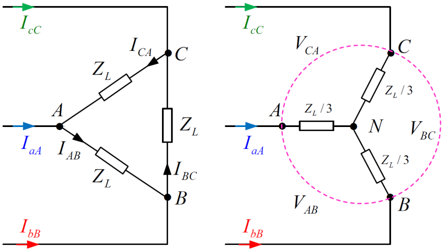


Figure: Phase and line quantities for load in Δ and Y connections.

- Δ load:
  - Line voltages: (V_{AB}, V_{BC}, V_{CA})
  - Line currents: (I_{aA}, I_{bB}, I_{cC})
  - Phase voltages: (V_{AB}, V_{BC}, V_{CA})
  - Phase currents: (I_{AB}, I_{BC}, I_{CA})
- Y load:
  - Line voltages: (V_{AB}, V_{BC}, V_{CA})
  - Line currents: (I_{aA}, I_{bB}, I_{cC})
  - Phase voltages: (V_{AN}, V_{BN}, V_{CN})
  - Phase currents: (I_{aA}, I_{bB}, I_{cC})

Phase/Line Quantities

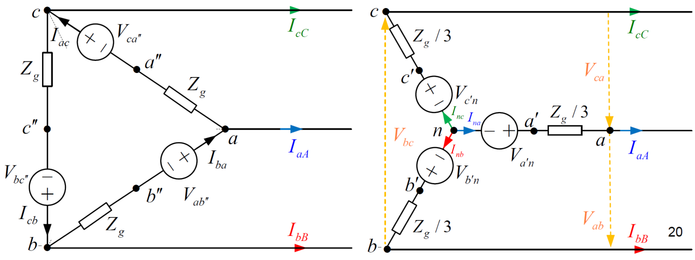


Figure: Phase and line quantities for source in Δ and Y connections.

- Δ source:
  - Line voltages: (V_{ab}, V_{bc}, V_{ca})
  - Line currents: (I_{aA}, I_{bB}, I_{cC})
  - Phase voltages: (V_{ab}, V_{bc}, V_{ca})
  - Phase currents: (I_{ba}, I_{cb}, I_{ac})
- Y source:
  - Line voltages: (V_{ab}, V_{bc}, V_{ca})
  - Line currents: (I_{aA}, I_{bB}, I_{cC})
  - Phase voltages: (V_{an}, V_{bn}, V_{cn})
  - Phase currents: (I_{aA}, I_{bB}, I_{cC})

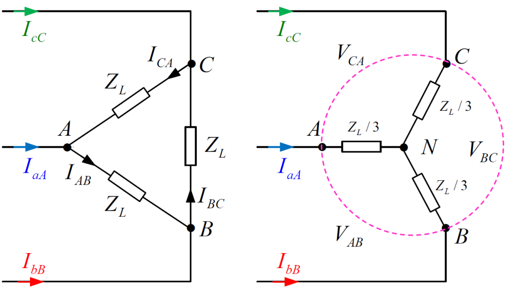


Figure: Δ-Y conversion for balanced load.

Δ-Y Conversion

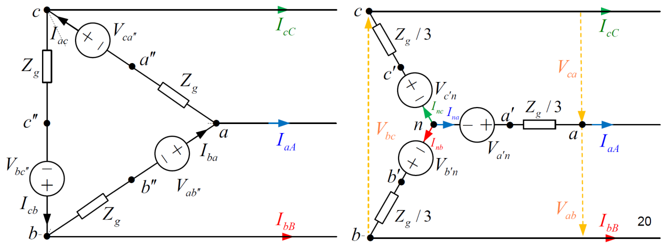


Figure: Δ-Y conversion for balanced source, where $(V_{a'n}, V_{b'n}, V_{c'n}) = (\frac{1}{\sqrt{3}}\angle -30^\circ)(V_{ab''}, V_{bc''}, V_{ca''})$ for positive sequence and $(V_{a'n}, V_{b'n}, V_{c'n}) = (\frac{1}{\sqrt{3}}\angle 30^\circ)(V_{ab''}, V_{bc''}, V_{ca''})$ for negative sequence.

$$\begin{cases} V_{c'n} - V_{a'n} = V_{ca''} \\ V_{a'n} - V_{b'n} = V_{ab''} \\ V_{b'n} - V_{c'n} = V_{bc''} \\ V_{a'n} + V_{b'n} + V_{c'n} = 0 \end{cases} \Rightarrow \begin{cases} V_{a'n} = \frac{1}{3}V_{ab''} - \frac{1}{3}V_{ca''} \\ V_{b'n} = \frac{1}{3}V_{bc''} - \frac{1}{3}V_{ab''} \\ V_{c'n} = \frac{1}{3}V_{ca''} - \frac{1}{3}V_{bc''} \\ V_{ab''} + V_{bc''} + V_{ca''} = 0 \end{cases}$$

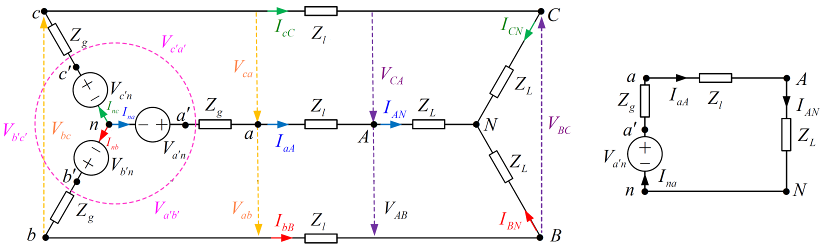


Figure: Single-phase equivalent circuit for a balanced positive-sequence Y-Y three-phase system.

- At source:
  - $|V_{ab}| = \sqrt{3}|V_{an}|$
  - $|I_{aA}|$
  - $|S_S| = \sqrt{3}|V_{ab}||I_{aA}|$
- At load:
  - $|V_{AB}| = \sqrt{3}|V_{AN}|$
  - $|I_{aA}|$
  - $|S_L| = \sqrt{3}|V_{AB}||I_{aA}|$

Y-Y Balanced Connection

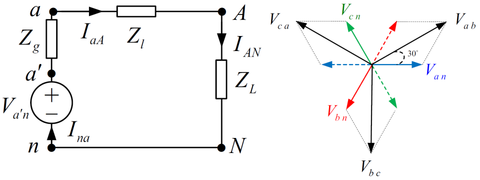


Figure: Single-phase equivalent circuit for a balanced positive-sequence Y-Y three-phase system.

- Source:
  - KVL: $I_{aA} = \frac{V_{a'n}}{Z_g + Z_l + Z_L}$
  - KVL: $V_{an} = V_{a'n} - I_{na} Z_g$
  - KVL: $V_{ab} = V_{an} - V_{bn} = V_{an} - V_{an}\angle{-120^\circ} = V_{an}\sqrt{3}\angle{30^\circ}$
  - Line currents: $(I_{aA}, I_{bB}, I_{cC}) = I_{aA}(1, 1\angle{-120^\circ}, 1\angle{+120^\circ})$
  - Line voltage: $(V_{ab}, V_{bc}, V_{ca}) = V_{ab}(1, 1\angle{-120^\circ}, 1\angle{+120^\circ})$
  - Phase currents: (I_{aA}, I_{bB}, I_{cC})
  - Phase voltage: $(V_{an}, V_{bn}, V_{cn}) = \frac{1}{\sqrt{3}}\angle{-30^\circ}(V_{ab}, V_{bc}, V_{ca})$

Y-Y Balanced Connection

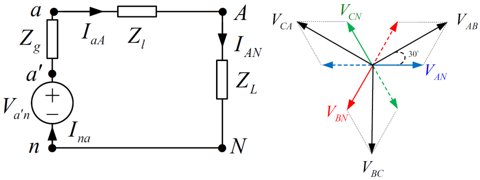


Figure: Single-phase equivalent circuit for a balanced positive-sequence Y-Y three-phase system.

- Load:
  - KVL: $I_{aA} = \frac{V_{a'n}}{Z_g + Z_l + Z_L}$
  - KVL: $V_{AN} = I_{AN} Z_L$
  - KVL: $V_{AB} = V_{AN} - V_{BN} = V_{AN} - V_{AN}\underline{/-120^\circ} = V_{AN}\sqrt{3}\underline{/30^\circ}$
  - Line currents: $(I_{aA}, I_{bB}, I_{cC}) = I_{aA}(1, 1\underline{/-120^\circ}, 1\underline{/+120^\circ})$
  - Line voltage: $(V_{AB}, V_{BC}, V_{CA}) = V_{AB}(1, 1\underline{/-120^\circ}, 1\underline{/+120^\circ})$
  - Phase currents: (I_{aA}, I_{bB}, I_{cC})
  - Phase voltage: $(V_{AN}, V_{BN}, V_{CN}) = \frac{1}{\sqrt{3}}\underline{/-30^\circ}(V_{AB}, V_{BC}, V_{CA})$

Y-Y Connection

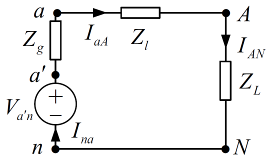


Figure: Single-phase equivalent circuit for a balanced positive-sequence Y-Y three-phase system.

- Delivered instantaneous power:

$$\begin{aligned}
p_S(t) &= v_{an}(t)i_{na}(t) + v_{bn}(t)i_{nb}(t) + v_{cn}(t)i_{nc}(t) \\
&= |V_{an}||I_{aA}|[\cos(\angle V_{an} - \angle I_{aA}) + \cos(2\omega t + \angle V_{an} + \angle I_{aA})] \\
&\quad + |V_{bn}||I_{bB}|[\cos(\angle V_{bn} - \angle I_{bB}) + \cos(2\omega t + \angle V_{bn} + \angle I_{bB})] \\
&\quad + |V_{cn}||I_{cC}|[\cos(\angle V_{cn} - \angle I_{cC}) + \cos(2\omega t + \angle V_{cn} + \angle I_{cC})] \\
&= |V_{an}||I_{aA}|[\cos(2\omega t + \angle V_{an} + \angle I_{aA}) + \cos(2\omega t + \angle V_{an} + \angle I_{aA} - 240^\circ) \\
&\quad + \cos(2\omega t + \angle V_{an} + \angle I_{aA} + 240^\circ)] + 3|V_{an}||I_{aA}|\cos(\angle V_{an} - \angle I_{aA}) \\
&= 3|V_{an}||I_{aA}|\cos(\angle V_{an} - \angle I_{aA}) \\
&= \sqrt{3}|V_{ab}||I_{aA}|\cos(\angle V_{an} - \angle I_{aA})
\end{aligned}$$

Y-Y Connection

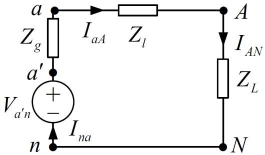


Figure: Single-phase equivalent circuit for a balanced positive-sequence Y-Y three-phase system.

- Delivered apparent power:

$$\begin{aligned} S_S &= V_{an} I_{na}^* + V_{bn} I_{nb}^* + V_{cn} I_{nc}^* \\ &= |V_{an}||I_{aA}|(\angle V_{an} - \angle I_{aA}) + |V_{bn}||I_{bB}|(\angle V_{bn} - \angle I_{bB}) + |V_{cn}||I_{cC}|(\angle V_{cn} - \angle I_{cC}) \\ &= 3|V_{an}||I_{aA}|(\angle V_{an} - \angle I_{aA}) = \sqrt{3}|V_{ab}||I_{aA}|(\angle V_{an} - \angle I_{aA}) \end{aligned}$$

- Delivered real power:

$$P_S = 3|V_{an}||I_{aA}|\cos(\angle V_{an} - \angle I_{aA}) = \sqrt{3}|V_{ab}||I_{aA}|\cos(\angle V_{an} - \angle I_{aA})$$

- Delivered reactive power:

$$Q_S = 3|V_{an}||I_{aA}|\sin(\angle V_{an} - \angle I_{aA}) = \sqrt{3}|V_{ab}||I_{aA}|\sin(\angle V_{an} - \angle I_{aA})$$

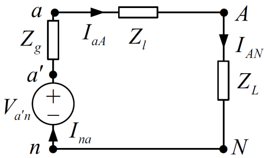


Figure: Single-phase equivalent circuit for a balanced positive-sequence Y-Y three-phase system.

- Absorbed instantaneous power:

$$
\begin{aligned}
p_L(t) &= v_{AN}(t)i_{AN}(t) + v_{BN}(t)i_{BN}(t) + v_{CN}(t)i_{CN}(t) \\
&= |V_{AN}||I_{aA}|[\cos(\angle V_{AN} - \angle I_{aA}) + \cos(2\omega t + \angle V_{AN} + \angle I_{aA})] \\
&\quad + |V_{BN}||I_{bB}|[\cos(\angle V_{BN} - \angle I_{bB}) + \cos(2\omega t + \angle V_{BN} + \angle I_{bB})] \\
&\quad + |V_{CN}||I_{cC}|[\cos(\angle V_{CN} - \angle I_{cC}) + \cos(2\omega t + \angle V_{CN} + \angle I_{cC})] \\
&= |V_{AN}||I_{aA}|[\cos(2\omega t + \angle V_{AN} + \angle I_{aA}) + \cos(2\omega t + \angle V_{AN} + \angle I_{aA} - 240^\circ) \\
&\quad + \cos(2\omega t + \angle V_{AN} + \angle I_{aA} + 240^\circ)] + 3|V_{AN}||I_{aA}|\cos(\angle V_{AN} - \angle I_{aA}) \\
&= 3|V_{AN}||I_{aA}|\cos(\angle V_{AN} - \angle I_{aA}) \\
&= \sqrt{3}|V_{AB}||I_{aA}|\cos(\angle V_{AN} - \angle I_{aA}) \\
&= \sqrt{3}|V_{AB}||I_{aA}|\cos(\angle Z_L)
\end{aligned}
$$

Y-Y Connection

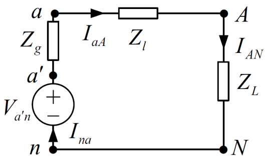


Figure: Single-phase equivalent circuit for a balanced positive-sequence Y-Y three-phase system.

- Absorbed apparent power:

$$\begin{aligned} S_L &= V_{AN} I_{AN}^* + V_{BN} I_{BN}^* + V_{CN} I_{CN}^* \\ &= |V_{AN}||I_{aA}|(\angle V_{AN} - \angle I_{aA}) + |V_{BN}||I_{bB}|(\angle V_{BN} - \angle I_{bB}) + |V_{CN}||I_{cC}|(\angle V_{CN} - \angle I_{cC}) \\ &= 3|V_{AN}||I_{aA}|(\angle V_{AN} - \angle I_{aA}) = \sqrt{3}|V_{AB}||I_{aA}|(\angle V_{AN} - \angle I_{AN}) = \sqrt{3}|V_{AB}||I_{aA}|(\angle Z_L) \end{aligned}$$

- Absorbed real power:

$$P_L = 3|V_{AN}||I_{aA}|\cos(\angle V_{AN} - \angle I_{aA}) = \sqrt{3}|V_{AB}||I_{aA}|\cos(\angle V_{AN} - \angle I_{aA}) = \sqrt{3}|V_{AB}||I_{aA}|\cos(\angle Z_L)$$

- Absorbed reactive power:

$$Q_L = 3|V_{AN}||I_{aA}|\sin(\angle V_{AN} - \angle I_{aA}) = \sqrt{3}|V_{AB}||I_{aA}|\sin(\angle V_{AN} - \angle I_{aA}) = \sqrt{3}|V_{AB}||I_{aA}|\sin(\angle Z_L)$$

Y-Δ Balanced Connection

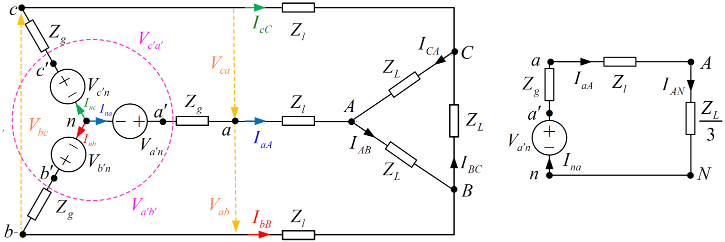


Figure: Single-phase equivalent circuit for a balanced positive-sequence Y-Δ three-phase system.

- At source:
  - $|V_{ab}| = \sqrt{3}|V_{an}|$
  - $|I_{aA}|$
  - $|S_S| = \sqrt{3}|V_{ab}||I_{aA}|$
- At load:
  - $|V_{AB}|$
  - $|I_{aA}| = \sqrt{3}|I_{AB}|$
  - $|S_L| = \sqrt{3}|V_{AB}||I_{aA}|$

Y-Δ Balanced Connection

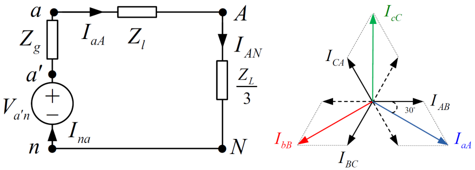


Figure: Single-phase equivalent circuit for a balanced positive-sequence Y-Δ three-phase system.

- Load:
  - KVL: $I_{aA} = \frac{V_{a'n}}{Z_g + Z_l + Z_L/3}$
  - KVL: $V_{AN} = I_{AN} Z_L / 3$
  - KVL: $I_{aA} = I_{AB} - I_{CA} = (V_{AB} - V_{AB}\angle{+120^\circ})/Z_L = I_{AB}\sqrt{3}\angle{-30^\circ}$
  - Line currents: $(I_{aA}, I_{bB}, I_{cC}) = I_{aA}(1, 1\angle{-120^\circ}, 1\angle{+120^\circ})$
  - Line voltage: $(V_{AB}, V_{BC}, V_{CA}) = V_{AB}(1, 1\angle{-120^\circ}, 1\angle{+120^\circ})$
  - Phase currents: $(I_{AB}, I_{BC}, I_{CA}) = \frac{1}{\sqrt{3}}\angle{30^\circ}(I_{aA}, I_{bB}, I_{cC})$
  - Phase voltage: (V_{AB}, V_{BC}, V_{CA})

Y-Δ Connection

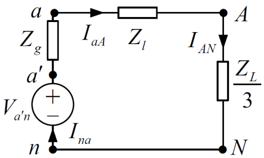


Figure: Single-phase equivalent circuit for a balanced positive-sequence Y-Δ three-phase system.

- Absorbed instantaneous power:

$$p_L(t) = 3|V_{AB}||I_{AB}|\cos(\angle V_{AB} - \angle I_{AB}) = \sqrt{3}|V_{AB}||I_{aA}|\cos(\angle V_{AB} - \angle I_{AB}) = \sqrt{3}|V_{AB}||I_{aA}|\cos(\angle Z_L)$$

- Absorbed apparent power:

$$S_L = 3|V_{AB}||I_{AB}|(\angle V_{AB} - \angle I_{AB}) = \sqrt{3}|V_{AB}||I_{aA}|(\angle V_{AB} - \angle I_{AB}) = \sqrt{3}|V_{AB}||I_{aA}|(\angle Z_L)$$

- Absorbed real power:

$$P_L = 3|V_{AB}||I_{AB}|\cos(\angle V_{AB} - \angle I_{AB}) = \sqrt{3}|V_{AB}||I_{aA}|\cos(\angle V_{AB} - \angle I_{AB}) = \sqrt{3}|V_{AB}||I_{aA}|\cos(\angle Z_L)$$

- Absorbed reactive power:

$$Q_L = 3|V_{AB}||I_{AB}|\sin(\angle V_{AB} - \angle I_{AB}) = \sqrt{3}|V_{AB}||I_{aA}|\sin(\angle V_{AB} - \angle I_{AB}) = \sqrt{3}|V_{AB}||I_{aA}|\sin(Z_L)$$

Δ-Y Balanced Connection

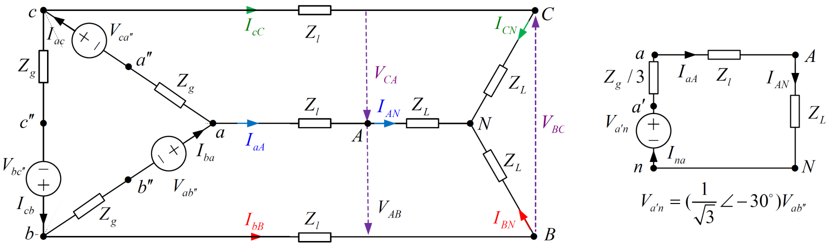


Figure: Single-phase equivalent circuit for a balanced positive-sequence Δ-Y three-phase system.

- At source:
  - $|V_{ab}|$
  - $|I_{aA}| = \sqrt{3}|I_{ba}|$
  - $|S_S| = \sqrt{3}|V_{ab}||I_{aA}|$
- At load:
  - $|V_{AB}| = \sqrt{3}|V_{AN}|$
  - $|I_{aA}|$
  - $|S_L| = \sqrt{3}|V_{AB}||I_{aA}|$

Δ-Δ Balanced Connection

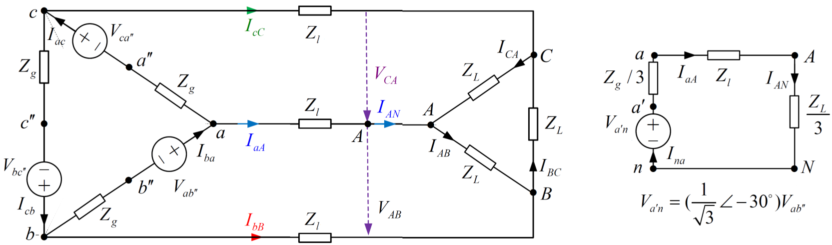


Figure: Single-phase equivalent circuit for a balanced positive-sequence Δ-Δ three-phase system.

- At source:
  - $|V_{ab}|$
  - $|I_{aA}| = \sqrt{3}|I_{ba}|$
  - $|S_S| = \sqrt{3}|V_{ab}||I_{aA}|$
- At load:
  - $|V_{AB}|$
  - $|I_{aA}| = \sqrt{3}|I_{AB}|$
  - $|S_L| = \sqrt{3}|V_{AB}||I_{aA}|$

Example (Delta-Wye connection for induction motors)

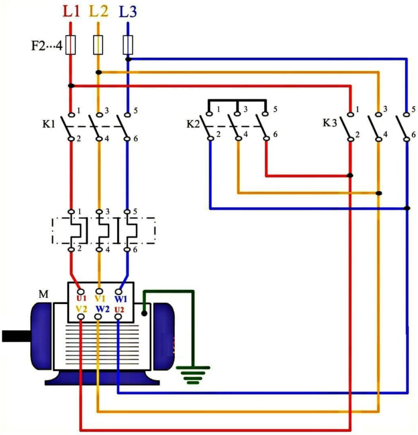
Induction motors provide much torque for Δ connection.

- Y connection:

$$
\begin{aligned}
P_Y &= \sqrt{3}|V_{AB}||I_{aA}|\cos(\angle Z_L) \\
&= \sqrt{3}|V_{AB}|\frac{|V_{AN}|}{|Z_L|}\cos(\angle Z_L) \\
&= \frac{|V_{AB}|^2}{|Z_L|}\cos(\angle Z_L)
\end{aligned}
$$

- Δ connection:

$$
\begin{aligned}
P_\Delta &= \sqrt{3}|V_{AB}||I_{aA}|\cos(\angle Z_L) \\
&= \sqrt{3}|V_{AB}|\sqrt{3}|I_{AB}|\cos(\angle Z_L) \\
&= 3|V_{AB}|\frac{|V_{AB}|}{|Z_L|}\cos(\angle Z_L) = 3P_Y
\end{aligned}
$$



Balanced Three-phase Interconnection

Example (Reactive power compensation)

The pure leading capacitive load Z_1 can compensate for the reactive power absorbed by the lagging inductive load Z_2 with $\text{PF}_2 = 0.75$ and $|S_2| = 30$ kVA, where the line voltage is 381 V_{rms}.

- Inductive load:

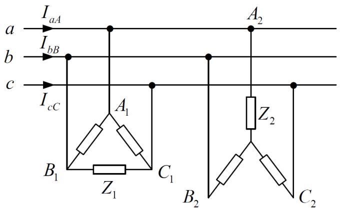
$$S_2 = 30\angle{+\cos^{-1}(0.75)} = 30\angle{41.41^\circ} = 22.5 + j19.84 \text{ kVA}$$

- Capacitive load:

$$S_1 = 3V_{A_1B_1}I^*_{A_1B_1} = 3(jB_c)^*|V_{A_1B_1}|^2 = -j3B_c \times 381^2 \text{ VA}$$

- Overall load:

$$S = S_1 + S_2 = 22.5 + j19.84 - j435.5B_c$$
$$\Im S = 0 \Rightarrow B_c = 0.0456 \ \mho$$
$$f = 50 \text{ Hz} \Rightarrow B_c = \omega C = 2\pi f C = 0.0286 \Rightarrow C = 145 \ \mu\text{F}$$



Example (Reactive power compensation)

The pure leading capacitive load Z_1 can compensate for the reactive power absorbed by the lagging inductive load Z_2 with $\text{PF}_2 = 0.75$ and $|S_2| = 30$ kVA, where the line voltage is 381 V_{rms}.

- Inductive load:

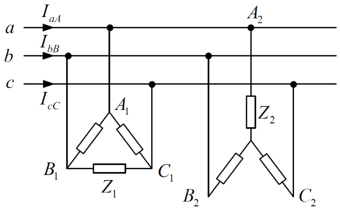
$$S_2 = 3V_{A_2N}I^*_{aA_2},\ V_{A_2N} = \frac{381}{\sqrt{3}}\underline{/0^\circ}$$
$$\Rightarrow I_{aA_2} = 45.5\underline{/-41.41^\circ}$$

- Capacitive load:

$$I_{aA_1} = (jB_c)|V_{A_1B_1}|\underline{/30^\circ}(\sqrt{3}\underline{/-30^\circ}) = 30.1\underline{/90^\circ}$$

- Overall load:

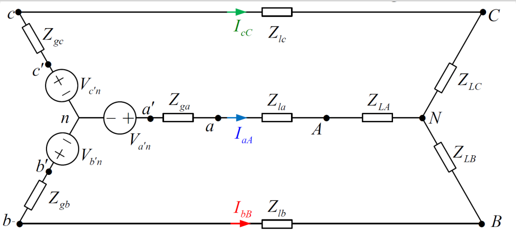
$$I_{aA} = I_{aA_1} + I_{aA_2} = 34.1 - j0.0 = 34.1\underline{/0^\circ}$$



Imbalanced Three-phase Interconnection

Example (Imbalanced three-phase circuit analysis)

Nodal or mesh analysis can be used to analyze the imbalanced three-phase circuit below.



$$(V_{a'n}, V_{b'n}, V_{c'n}) = 1000(1, 1\angle-120^\circ, 1\angle 120^\circ)\ \text{V}_{\text{rms}}$$

$$Z_{ga} = Z_{gb} = Z_{gc} = 2 + j8,\ Z_{la} = Z_{lb} = Z_{lc} = 1 + j2,\ Z_{LA} = 19 + j18,\ Z_{LB} = 49 - j2,\ Z_{LC} = 29 + j50\ \Omega$$

$$\begin{cases} (z_{gc} + Z_{lc} + Z_{LC})I_{cC} + (Z_{LA} + Z_{la} + Z_{ga})(I_{cC} + I_{bB}) = V_{c'n} - V_{a'n} \\ (z_{gb} + Z_{lb} + Z_{LB})I_{bB} + (Z_{LA} + Z_{la} + Z_{ga})(I_{cC} + I_{bB}) = V_{b'n} - V_{a'n} \end{cases}$$

$$\Rightarrow \begin{cases} I_{aA} = 13.15 - j19.15 \\ I_{bB} = -19.95 - j10.38 \\ I_{cC} = 6.8 + j19.53 \end{cases} \Rightarrow S = Z_{LA}|I_{aA}|^2 + Z_{LB}|I_{bB}|^2 + Z_{LC}|I_{cC}|^2 = 42057 + 24994\ \text{VA, Lag}$$

Power Measurement

Ideal Power Meter

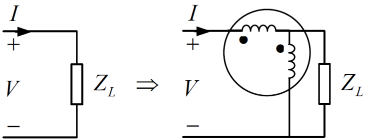


Figure: Ideal single-phase wattmeter with having no voltage drop and no drawn current in series and parallel measuring branch, respectively.

$$W = \Re\{VI^*\} = |V||I|\text{PF} = P_L$$

Three-phase Power Measurement

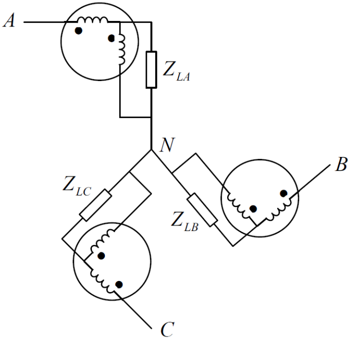


Figure: Three-phase power measurement using three wattmeters.

$$W_1 + W_2 + W_3 = \Re\{V_{AN} I_{AN}^*\} + \Re\{V_{BN} I_{BN}^*\} + \Re\{V_{CN} I_{CN}^*\} = P_L$$

Three-phase Power Measurement

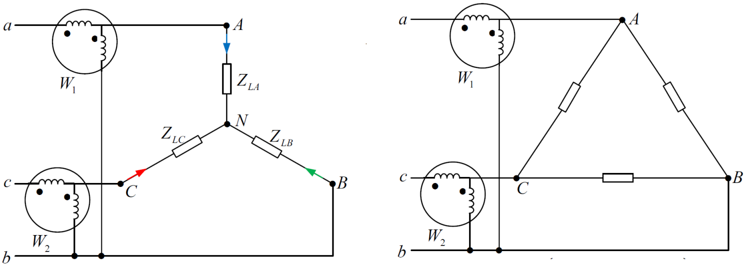


Figure: Three-phase power measurement using two wattmeters.

$$
\begin{aligned}
W_1 + W_2 &= \Re\{V_{AB} I_{aA}^* + V_{CB} I_{cC}^*\} \\
&= \Re\{(V_{AN} + V_{NB}) I_{aA}^* + (V_{CN} + V_{NB}) I_{cC}^*\} \\
&= \Re\{V_{AN} I_{aA}^* + V_{NB}(I_{aA} + I_{cC})^* + V_{CN} I_{cC}^*\} \\
&= \Re\{V_{AN} I_{aA}^* + V_{BN} I_{bB}^* + V_{CN} I_{cC}^*\} = P_L
\end{aligned}
$$

$$
\begin{aligned}
W_1 + W_2 &= \Re\{V_{AB} I_{aA}^* + V_{CB} I_{cC}^*\} \\
&= \Re\{V_{AB}(I_{AB} - I_{CA})^* + V_{CB}(I_{CA} - I_{BC})^*\} \\
&= \Re\{V_{AB} I_{AB}^* + (V_{CB} - V_{AB}) I_{CA}^* + V_{BC} I_{BC}^*\} \\
&= \Re\{V_{AB} I_{AB}^* + V_{CA} I_{CA}^* + V_{BC} I_{BC}^*\} = P_L
\end{aligned}
$$

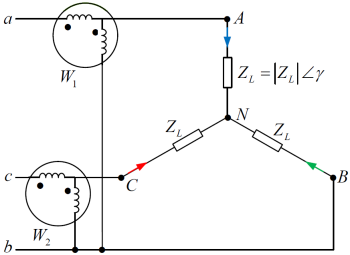


Figure: Three-phase balanced power measurement using two wattmeters for positive phase sequence.

$$W_1 + W_2 = \Re\{V_{AB} I_{aA}^* + V_{CB} I_{cC}^*\}$$
$$= \Re\{(\sqrt{3}\angle 30^\circ) V_{aA} I_{AN}^* + (\sqrt{3}\angle -30^\circ) V_{CN} I_{cC}^*\}$$
$$= \Re\{[(\sqrt{3}\angle 30^\circ) + (\sqrt{3}\angle -30^\circ)] V_{AN} I_{aA}^*\}$$
$$= \Re\{2\sqrt{3}\cos(30^\circ) V_{AN} I_{AN}^*\}$$
$$= 3\Re\{V_{AN} I_{aA}^*\} = P_L$$

$$W_2 - W_1 = \Re\{V_{CB} I_{cC}^* - V_{AB} I_{aA}^*\}$$
$$= \Re\{(\sqrt{3}\angle -30^\circ) V_{CN} I_{cC}^* - (\sqrt{3}\angle 30^\circ) V_{AN} I_{aA}^*\}$$
$$= \Re\{[(\sqrt{3}\angle -30^\circ) - (\sqrt{3}\angle 30^\circ)] V_{AN} I_{aA}^*\}$$
$$= \Re\{-j2\sqrt{3}\sin(30^\circ) V_{AN} I_{aA}^*\}$$
$$= \frac{1}{\sqrt{3}} 3\Im\{V_{AN} I_{aA}^*\} = \frac{Q}{\sqrt{3}}$$

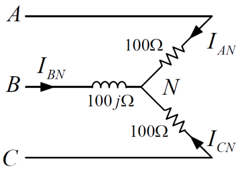
$$PF = \cos\left(\tan^{-1}\left(\sqrt{3}(W_2 - W_1)/(W_1 + W_2)\right)\right)$$

Phase Sequence Determination

Phase Sequence Determination

Example (Phase sequence determination by imbalance load)

An imbalanced load of $100(1, \underline{/90^\circ}, 1)$ can be used to determine the phase sequence in the circuit below, where is absolute phase voltage is 220 V_{rms}.



Positive phase sequence:

$$\begin{cases} 100I_{AN} + j100(I_{AN} + I_{CN}) = V_{AB} = 381\angle 0^\circ \\ -100I_{CN} - j100(I_{AN} + I_{CN}) = V_{BC} = 381\angle -120^\circ \end{cases}$$

$$\Rightarrow \begin{cases} I_{AN} = 1.9\angle -48.45^\circ \\ I_{CN} = 0.51\angle 71.55^\circ \end{cases} \Rightarrow |V_{AN}| > |V_{CN}|$$

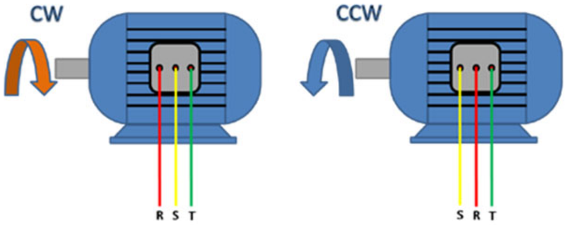
Negative phase sequence:

$$\begin{cases} 100I_{AN} + j100(I_{AN} + I_{CN}) = V_{AB} = 381\angle 0^\circ \\ -100I_{CN} - j100(I_{AN} + I_{CN}) = V_{BC} = 381\angle 120^\circ \end{cases}$$

$$\Rightarrow \begin{cases} I_{AN} = 0.51\angle 11.55^\circ \\ I_{CN} = 1.9\angle -108.45^\circ \end{cases} \Rightarrow |V_{AN}| < |V_{CN}|$$

Example (Phase sequence determination by induction motors)

The direction of rotation in three-phase induction motors is an indicator of the phase sequence.



The End