# Two-ports

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# Overview

# One-ports

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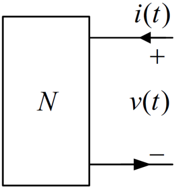



Figure: NTV one-port with the characteristic equation $f(v(t), i(t), t) = 0$.

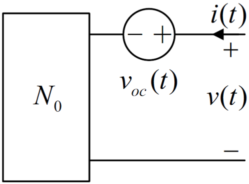



Figure: LTV one-port with the characteristic equation $v(t) = v_{oc}(t) + v_1(t) = v_{oc}(t) + \int_0^t h(t, \tau)i(\tau)d\tau$, where $v_{oc}(t)$ is the open circuit voltage and $v_1(t) = \int_0^t h(t, \tau)i(\tau)d\tau$ describes the in-rest network.

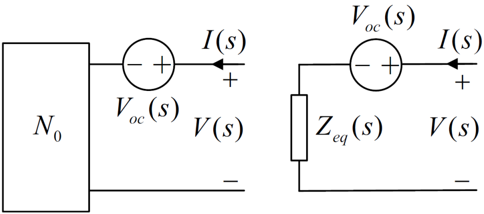



Figure: LTI one-port with the characteristic equation $V(s) = V_{oc}(s) + Z_{eq}(s)I(s)$ or $I(s) = -I_{sc}(s) + Y_{eq}(s)V(s)$, where $V_{oc}(s)$ is the open circuit voltage, $I_{sc}(s)$ is the short circuit current, $Z_{eq}(s)$ is the equivalent impedance, and $Y_{eq}(s)$ is the equivalent admittance. Clearly, $Y_{eq}(s) = Z_{eq}^{-1}(s)$ and $V_{oc}(s) = Z_{eq}(s)I_{sc}(s)$.

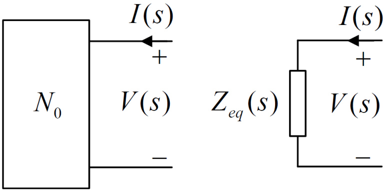



Figure: In-rest LTI one-port with the characteristic equation $V(s) = Z_{eq}(s)I(s)$ or $I(s) = Y_{eq}(s)V(s)$, where $Z_{eq}(s)$ is the equivalent impedance, and $Y_{eq}(s)$ is the equivalent admittance. Clearly, $Y_{eq}(s) = Z_{eq}^{-1}(s)$.

# Two-ports

# Two-ports

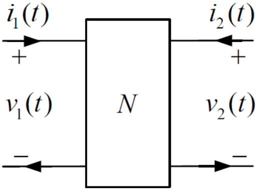



Figure: NTV two-port with the characteristic equation $\begin{cases} f_1(v_1(t), i_1(t), v_2(t), i_2(t), t) = 0 \\ f_2(v_1(t), i_1(t), v_2(t), i_2(t), t) = 0 \end{cases}$.

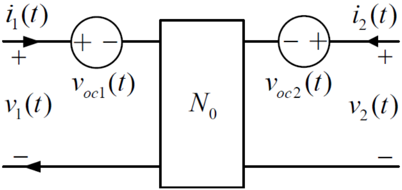



Figure: LTV two-port can be characterized with its open circuit voltages and in-rest network in time domain.

# Two-ports

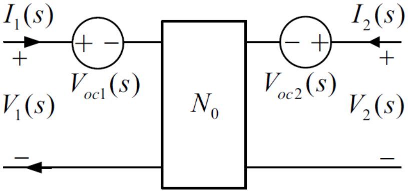



Figure: LTI two-port can be characterized with its open circuit voltages and in-rest network in time or Laplace domain.

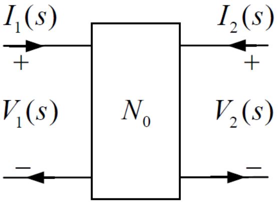



Figure: In-rest LTI two-port can be characterized with its in-rest network in time or Laplace domain.

# Four-terminal

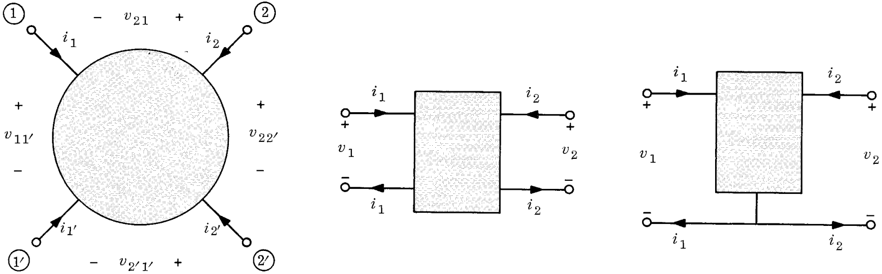



Figure: A Four-terminal element can be characterized by three independent voltages and three independent currents. A two-port is a special four-terminal with extra constraints on its currents. Every three-terminal element can be treated as a two-port.

# Description of Two-ports

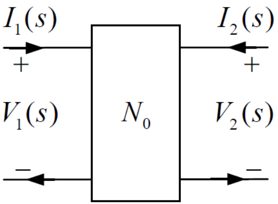



Figure: A Two-port may be described in one of the six common ways.

- Impedance ($Z$-parameters) description: $\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = f_1(\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix})$
- Admittance ($Y$-parameters) description: $\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = f_2(\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix})$
- Hybrid ($H$-parameters) description: $\begin{bmatrix} V_1(s) \\ I_2(s) \end{bmatrix} = f_3(\begin{bmatrix} I_1(s) \\ V_2(s) \end{bmatrix})$
- Hybrid ($G$-parameters) description: $\begin{bmatrix} I_1(s) \\ V_2(s) \end{bmatrix} = f_4(\begin{bmatrix} V_1(s) \\ I_2(s) \end{bmatrix})$
- Transmittance ($ABCD$-parameters) description: $\begin{bmatrix} V_1(s) \\ I_1(s) \end{bmatrix} = f_5(\begin{bmatrix} V_2(s) \\ -I_2(s) \end{bmatrix})$
- Transmittance ($A'B'C'D'$-parameters) description: $\begin{bmatrix} V_2(s) \\ -I_2(s) \end{bmatrix} = f_6(\begin{bmatrix} V_1(s) \\ I_1(s) \end{bmatrix})$

# Impedance Description

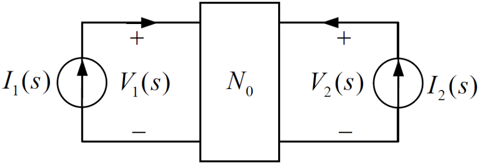


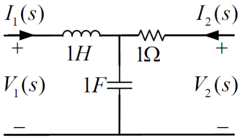

Figure: Impedance description for two-port.

- Impedance description: $\begin{cases} V_1(s) = z_{11}(s)I_1(s) + z_{12}(s)I_2(s) \\ V_2(s) = z_{21}(s)I_1(s) + z_{22}(s)I_2(s) \end{cases}$
- Impedance matrix: $\boldsymbol{Z}(s) = \begin{bmatrix} z_{11}(s) & z_{12}(s) \\ z_{21}(s) & z_{22}(s) \end{bmatrix}$, $\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \boldsymbol{Z}(s) \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$
- First port input impedance: $z_{11}(s) = \frac{V_1(s)}{I_1(s)}\big|_{I_2(s)=0}$
- Second port input impedance: $z_{22}(s) = \frac{V_2(s)}{I_2(s)}\big|_{I_1(s)=0}$
- Transfer impedance from second to first port: $z_{12}(s) = \frac{V_1(s)}{I_2(s)}\big|_{I_1(s)=0}$
- Transfer impedance from first to second port: $z_{21}(s) = \frac{V_2(s)}{I_1(s)}\big|_{I_2(s)=0}$

# Impedance Description

## Example (Impedance description)

The two-port below can be described by its impedance matrix.




$$\left\{\begin{array}{l} z_{11}(s) = \frac{V_1(s)}{I_1(s)}\Big|_{I_2(s)=0} = s + \frac{1}{s} \\ z_{22}(s) = \frac{V_2(s)}{I_2(s)}\Big|_{I_1(s)=0} = 1 + \frac{1}{s} \\ z_{12}(s) = \frac{V_1(s)}{I_2(s)}\Big|_{I_1(s)=0} = \frac{1}{s} \\ z_{21}(s) = \frac{V_2(s)}{I_1(s)}\Big|_{I_2(s)=0} = \frac{1}{s} \end{array}\right. \Rightarrow \boldsymbol{Z} = \begin{bmatrix} \frac{s^2+1}{s} & \frac{1}{s} \\ \frac{1}{s} & \frac{s+1}{s} \end{bmatrix}$$

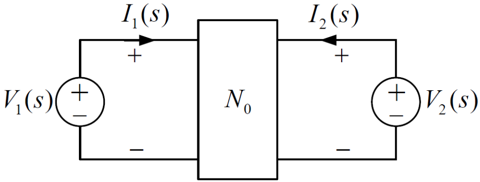


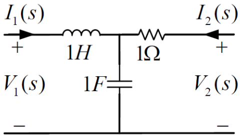

Figure: Admittance description for two-port.

- Admittance description: $\begin{cases} I_1(s) = y_{11}(s)V_1(s) + y_{12}(s)V_2(s) \\ I_2(s) = y_{21}(s)V_1(s) + y_{22}(s)V_2(s) \end{cases}$
- Admittance matrix: $\boldsymbol{Y}(s) = \begin{bmatrix} y_{11}(s) & y_{12}(s) \\ y_{21}(s) & y_{22}(s) \end{bmatrix} = \boldsymbol{Z}^{-1}(s),\quad \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \boldsymbol{Y}(s) \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix}$
- First port input admittance: $y_{11}(s) = \frac{I_1(s)}{V_1(s)}\Big|_{V_2(s)=0} \neq \frac{1}{z_{11}(s)}$
- Second port input admittance: $y_{22}(s) = \frac{I_2(s)}{V_2(s)}\Big|_{V_1(s)=0} \neq \frac{1}{z_{22}(s)}$
- Transfer admittance from second to first port: $y_{12}(s) = \frac{I_1(s)}{V_2(s)}\Big|_{V_1(s)=0}$
- Transfer admittance from first to second port: $y_{21}(s) = \frac{I_2(s)}{V_1(s)}\Big|_{V_2(s)=0}$

# Admittance Description

## Example (Admittance description)

The two-port below can be described by its admittance matrix.




$$\left\{\begin{array}{l} y_{11}(s) = \frac{I_1(s)}{V_1(s)}\Big|_{V_2(s)=0} = [\frac{1}{s}||1+s]^{-1} = \frac{s+1}{s^2+s+1} \\ y_{22}(s) = \frac{I_2(s)}{V_2(s)}\Big|_{V_1(s)=0} = [\frac{1}{s}||s+1]^{-1} = \frac{s^2+1}{s^2+s+1} \\ y_{12}(s) = \frac{I_1(s)}{V_2(s)}\Big|_{V_1(s)=0} = -\frac{s||\frac{1}{s}}{s||\frac{1}{s}+1}\frac{1}{s} = \frac{-1}{s^2+s+1} \\ y_{21}(s) = \frac{I_2(s)}{V_1(s)}\Big|_{V_2(s)=0} = -\frac{1||\frac{1}{s}}{1||\frac{1}{s}+s}\frac{1}{1} = \frac{-1}{s^2+s+1} \end{array}\right. \Rightarrow \boldsymbol{Y} = \begin{bmatrix} \frac{s+1}{s^2+s+1} & \frac{-1}{s^2+s+1} \\ \frac{-1}{s^2+s+1} & \frac{s^2+1}{s^2+s+1} \end{bmatrix} = \boldsymbol{Z}^{-1}$$

# Hybrid H Description

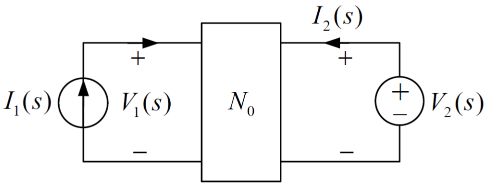


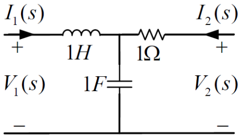

Figure: Hybrid H description for two-port.

- Hybrid H description: $\begin{cases} V_1(s) = h_{11}(s)I_1(s) + h_{12}(s)V_2(s) \\ I_2(s) = h_{21}(s)I_1(s) + h_{22}(s)V_2(s) \end{cases}$
- Hybrid H matrix: $\boldsymbol{H}(s) = \begin{bmatrix} h_{11}(s) & h_{12}(s) \\ h_{21}(s) & h_{22}(s) \end{bmatrix}$, $\begin{bmatrix} V_1(s) \\ I_2(s) \end{bmatrix} = \boldsymbol{H}(s) \begin{bmatrix} I_1(s) \\ V_2(s) \end{bmatrix}$
- First port input impedance: $h_{11}(s) = \frac{V_1(s)}{I_1(s)}\Big|_{V_2(s)=0} = \frac{1}{y_{11}(s)}$
- Second port input admittance: $h_{22}(s) = \frac{I_2(s)}{V_2(s)}\Big|_{I_1(s)=0} = \frac{1}{z_{22}(s)}$
- Voltage gain from second to first port: $h_{12}(s) = \frac{V_1(s)}{V_2(s)}\Big|_{I_1(s)=0}$
- Current gain from first to second port: $h_{21}(s) = \frac{I_2(s)}{I_1(s)}\Big|_{V_2(s)=0}$

# Hybrid H Description

## Example (Hybrid H description)

The two-port below can be described by its hybrid H matrix.




$$\begin{cases} h_{11}(s) = \frac{V_1(s)}{I_1(s)}\Big|_{V_2(s)=0} = s + \frac{1}{s}||1 = \frac{s^2+s+1}{s+1} \\ h_{22}(s) = \frac{I_2(s)}{V_2(s)}\Big|_{I_1(s)=0} = [1+\frac{1}{s}]^{-1} = \frac{s}{s+1} \\ h_{12}(s) = \frac{V_1(s)}{V_2(s)}\Big|_{I_1(s)=0} = \frac{\frac{1}{s}}{\frac{1}{s}+1} = \frac{1}{s+1} \\ h_{21}(s) = \frac{I_2(s)}{I_1(s)}\Big|_{V_2(s)=0} = -\frac{\frac{1}{s}}{\frac{1}{s}+1} = \frac{-1}{s+1} \end{cases} \Rightarrow \boldsymbol{H} = \begin{bmatrix} \frac{s^2+s+1}{s+1} & \frac{1}{s+1} \\ \frac{-1}{s+1} & \frac{s}{s+1} \end{bmatrix}$$

# Hybrid G Description

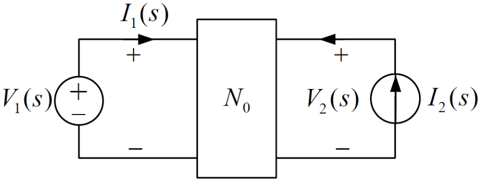


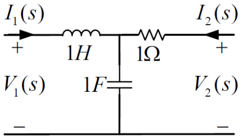

Figure: Hybrid G description for two-port.

- Hybrid G description: $\begin{cases} I_1(s) = g_{11}(s)V_1(s) + g_{12}(s)I_2(s) \\ V_2(s) = g_{21}(s)V_1(s) + g_{22}(s)I_2(s) \end{cases}$
- Hybrid G matrix: $\boldsymbol{G}(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} = \boldsymbol{H}^{-1}(s), \quad \begin{bmatrix} I_1(s) \\ V_2(s) \end{bmatrix} = \boldsymbol{G}(s) \begin{bmatrix} V_1(s) \\ I_2(s) \end{bmatrix}$
- First port input admittance: $g_{11}(s) = \frac{I_1(s)}{V_1(s)}\Big|_{I_2(s)=0} = \frac{1}{z_{11}(s)}$
- Second port input impedance: $g_{22}(s) = \frac{V_2(s)}{I_2(s)}\Big|_{V_1(s)=0} = \frac{1}{y_{22}(s)}$
- Current gain from second to first port: $g_{12}(s) = \frac{I_1(s)}{I_2(s)}\Big|_{V_1(s)=0}$
- Voltage gain from first to second port: $g_{21}(s) = \frac{V_2(s)}{V_1(s)}\Big|_{I_2(s)=0}$

# Hybrid G Description

## Example (Hybrid G description)

The two-port below can be described by its hybrid G matrix.




$$\begin{cases} g_{11}(s) = \frac{I_1(s)}{V_1(s)}\Big|_{I_2(s)=0} = [s + \frac{1}{s}]^{-1} = \frac{s}{s^2+1} \\ g_{22}(s) = \frac{V_2(s)}{I_2(s)}\Big|_{V_1(s)=0} = s||\frac{1}{s} + 1 = \frac{s^2+s+1}{s^2+1} \\ g_{12}(s) = \frac{I_1(s)}{I_2(s)}\Big|_{V_1(s)=0} = -\frac{\frac{1}{s}}{\frac{1}{s}+s} = -\frac{1}{s^2+1} \\ g_{21}(s) = \frac{V_2(s)}{V_1(s)}\Big|_{I_2(s)=0} = \frac{\frac{1}{s}}{\frac{1}{s}+s} = \frac{1}{s^2+1} \end{cases} \Rightarrow \boldsymbol{G} = \begin{bmatrix} \frac{s}{s^2+1} & -\frac{1}{s^2+1} \\ \frac{1}{s^2+1} & \frac{s^2+s+1}{s^2+1} \end{bmatrix} = \boldsymbol{H}^{-1}$$

# Transmittance ABCD Description

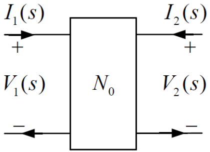


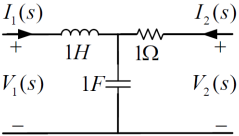

Figure: Transmittance ABCD description for two-port.

- Transmittance ABCD description: $\begin{cases} V_1(s) = A(s)V_2(s) + B(s)(-I_2(s)) \\ I_1(s) = C(s)V_2(s) + D(s)(-I_2(s)) \end{cases}$
- Transmittance ABCD matrix: $\boldsymbol{T}(s) = \begin{bmatrix} A(s) & B(s) \\ C(s) & D(s) \end{bmatrix}, \quad \begin{bmatrix} V_1(s) \\ I_1(s) \end{bmatrix} = \boldsymbol{T}(s) \begin{bmatrix} V_2(s) \\ -I_2(s) \end{bmatrix}$
- $A$ parameter: $A(s) = \frac{V_1(s)}{V_2(s)}\Big|_{I_2(s)=0}$
- $B$ parameter: $B(s) = \frac{V_1(s)}{-I_2(s)}\Big|_{V_2(s)=0}$
- $C$ parameter: $C(s) = \frac{I_1(s)}{V_2(s)}\Big|_{I_2(s)=0}$
- $D$ parameter: $D(s) = \frac{I_1(s)}{-I_2(s)}\Big|_{V_2(s)=0}$

# Transmittance ABCD Description

## Example (Transmittance ABCD description)

The two-port below can be described by its transmittance ABCD matrix.




$$\begin{cases} A(s) = \frac{V_1(s)}{V_2(s)}\Big|_{I_2(s)=0} = [\frac{\frac{1}{s}}{\frac{1}{s}+s}]^{-1} = s^2+1 \\ B(s) = \frac{V_1(s)}{-I_2(s)}\Big|_{V_2(s)=0} = [\frac{\frac{1}{s}||1}{\frac{1}{s}||1+s}]^{-1} = s^2+s+1 \\ C(s) = \frac{I_1(s)}{V_2(s)}\Big|_{I_2(s)=0} = [\frac{1}{s}]^{-1} = s \\ D(s) = \frac{I_1(s)}{-I_2(s)}\Big|_{V_2(s)=0} = [\frac{\frac{1}{s}}{\frac{1}{s}+1}]^{-1} = s+1 \end{cases} \Rightarrow \boldsymbol{T} = \begin{bmatrix} s^2+1 & s^2+s+1 \\ s & s+1 \end{bmatrix}$$

# Transmittance A′B′C′D′ Description

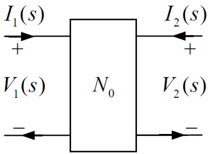


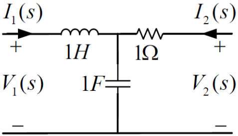

Figure: Transmittance A′B′C′D′ description for two-port.

- Transmittance A′B′C′D′ description: $\begin{cases} V_2(s) = A'(s)V_1(s) + B'(s)I_1(s) \\ -I_2(s) = C'(s)V_1(s) + D'(s)I_1(s) \end{cases}$
- Transmittance A′B′C′D′ matrix:
  $\boldsymbol{T'}(s) = \begin{bmatrix} A'(s) & B'(s) \\ C'(s) & D'(s) \end{bmatrix} = \boldsymbol{T}^{-1}(s), \quad \begin{bmatrix} V_2(s) \\ -I_2(s) \end{bmatrix} = \boldsymbol{T'}(s) \begin{bmatrix} V_1(s) \\ I_1(s) \end{bmatrix}$
- $A'$ parameter: $A'(s) = \frac{V_2(s)}{V_1(s)}\Big|_{I_1(s)=0}$
- $B'$ parameter: $B'(s) = \frac{V_2(s)}{I_1(s)}\Big|_{V_1(s)=0}$
- $C'$ parameter: $C'(s) = \frac{-I_2(s)}{V_1(s)}\Big|_{I_1(s)=0}$
- $D'$ parameter: $D'(s) = \frac{-I_2(s)}{I_1(s)}\Big|_{V_1(s)=0}$

# Transmittance A′B′C′D′ Description

## Example (Transmittance A′B′C′D′ description)

The two-port below can be described by its transmittance A′B′C′D′ matrix.




$$\begin{cases} A'(s) = \frac{V_2(s)}{V_1(s)}\Big|_{I_1(s)=0} = [\frac{\frac{1}{s}}{\frac{1}{s}+1}]^{-1} = s+1 \\ B'(s) = \frac{V_2(s)}{I_1(s)}\Big|_{V_1(s)=0} = -[\frac{\frac{1}{s}||s}{\frac{1}{s}||s+1}\,\frac{1}{s}]^{-1} = -s^2 - s - 1 \\ C'(s) = \frac{-I_2(s)}{V_1(s)}\Big|_{I_1(s)=0} = -[\frac{1}{s}]^{-1} = s \\ D'(s) = \frac{-I_2(s)}{I_1(s)}\Big|_{V_1(s)=0} = [\frac{\frac{1}{s}}{\frac{1}{s}+s}]^{-1} = s+1 \end{cases} \Rightarrow \boldsymbol{T'} = \begin{bmatrix} s+1 & -s^2-s-1 \\ -s & s^2+1 \end{bmatrix} = \boldsymbol{T}^{-1}$$

# Reciprocal Two-ports

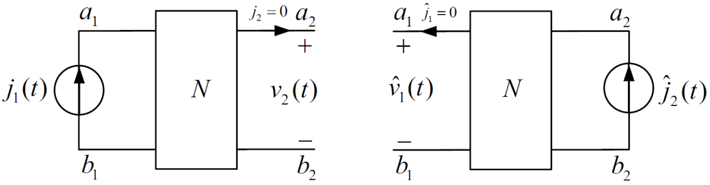



Figure: Reciprocal two-port.

- Impedance ($Z$-parameters) description: $z_{12}(s) = z_{21}(s)$
- Admittance ($Y$-parameters) description: $y_{12}(s) = y_{21}(s)$
- Hybrid ($H$-parameters) description: $h_{12}(s) = -h_{21}(s)$
- Hybrid ($G$-parameters) description: $g_{12}(s) = -g_{21}(s)$
- Transmittance ($ABCD$-parameters) description: $\det[\boldsymbol{T}] = 1$
- Transmittance ($A'B'C'D'$-parameters) description: $\det[\boldsymbol{T'}] = 1$

# Symmetric Two-ports

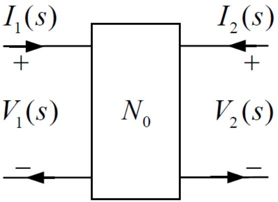



Figure: Symmetric two-port.

- Impedance ($Z$-parameters) description: $z_{12}(s) = z_{21}(s), \quad z_{11}(s) = z_{22}(s)$
- Admittance ($Y$-parameters) description: $y_{12}(s) = y_{21}(s), \quad y_{11}(s) = y_{22}(s)$
- Hybrid ($H$-parameters) description: $h_{12}(s) = -h_{21}(s), \quad \det[\boldsymbol{H}] = 1$
- Hybrid ($G$-parameters) description: $g_{12}(s) = -g_{21}(s), \quad \det[\boldsymbol{G}] = 1$
- Transmittance ($ABCD$-parameters) description: $\det[\boldsymbol{T}] = 1, \quad A = D$
- Transmittance ($A'B'C'D'$-parameters) description: $\det[\boldsymbol{T'}] = 1, \quad A' = D'$

# Interrelation of Descriptions

| | $\boldsymbol{Z}$ | $\boldsymbol{Y}$ | $\boldsymbol{H}$ | $\boldsymbol{G}$ | $\boldsymbol{T}$ | $\boldsymbol{T'}$ |
|---|---|---|---|---|---|---|
| $\boldsymbol{Z}$ | $\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$ | $\begin{bmatrix} \frac{y_{22}}{\Delta_Y} & \frac{-y_{12}}{\Delta_Y} \\ \frac{-y_{21}}{\Delta_Y} & \frac{y_{11}}{\Delta_Y} \end{bmatrix}$ | $\begin{bmatrix} \frac{\Delta_H}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$ | $\begin{bmatrix} \frac{1}{g_{11}} & \frac{-g_{12}}{g_{11}} \\ \frac{g_{21}}{g_{11}} & \frac{\Delta_G}{g_{11}} \end{bmatrix}$ | $\begin{bmatrix} \frac{A}{C} & \frac{\Delta_T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$ | $\begin{bmatrix} \frac{D'}{C'} & \frac{1}{C'} \\ \frac{\Delta_{T'}}{C'} & \frac{A'}{C'} \end{bmatrix}$ |
| $\boldsymbol{Y}$ | $\begin{bmatrix} \frac{z_{22}}{\Delta_Z} & \frac{-z_{12}}{\Delta_Z} \\ \frac{-z_{21}}{\Delta_Z} & \frac{z_{11}}{\Delta_Z} \end{bmatrix}$ | $\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$ | $\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_H}{h_{11}} \end{bmatrix}$ | $\begin{bmatrix} \frac{\Delta_G}{g_{22}} & \frac{g_{12}}{g_{22}} \\ \frac{-g_{21}}{g_{22}} & \frac{1}{g_{22}} \end{bmatrix}$ | $\begin{bmatrix} \frac{D}{B} & \frac{-\Delta_T}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix}$ | $\begin{bmatrix} \frac{A'}{B'} & \frac{-1}{B'} \\ \frac{-\Delta_{T'}}{B'} & \frac{D'}{B'} \end{bmatrix}$ |
| $\boldsymbol{H}$ | $\begin{bmatrix} \frac{\Delta_Z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$ | $\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_Y}{y_{11}} \end{bmatrix}$ | $\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$ | $\begin{bmatrix} \frac{g_{22}}{\Delta_G} & \frac{-g_{12}}{\Delta_G} \\ \frac{-g_{21}}{\Delta_G} & \frac{g_{11}}{\Delta_G} \end{bmatrix}$ | $\begin{bmatrix} \frac{B}{D} & \frac{\Delta_T}{D} \\ \frac{-1}{D} & \frac{C}{D} \end{bmatrix}$ | $\begin{bmatrix} \frac{B'}{A'} & \frac{1}{A'} \\ \frac{-\Delta_{T'}}{A'} & \frac{C'}{A'} \end{bmatrix}$ |
| $\boldsymbol{G}$ | $\begin{bmatrix} \frac{1}{z_{11}} & \frac{-z_{12}}{z_{11}} \\ \frac{z_{21}}{z_{11}} & \frac{\Delta_Z}{z_{11}} \end{bmatrix}$ | $\begin{bmatrix} \frac{\Delta_Y}{y_{22}} & \frac{y_{12}}{y_{22}} \\ \frac{-y_{21}}{y_{22}} & \frac{1}{y_{22}} \end{bmatrix}$ | $\begin{bmatrix} \frac{h_{22}}{\Delta_H} & \frac{-h_{12}}{\Delta_H} \\ \frac{-h_{21}}{\Delta_H} & \frac{h_{11}}{\Delta_H} \end{bmatrix}$ | $\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$ | $\begin{bmatrix} \frac{C}{A} & \frac{-\Delta_T}{A} \\ \frac{1}{A} & \frac{B}{A} \end{bmatrix}$ | $\begin{bmatrix} \frac{C'}{D'} & \frac{-1}{D'} \\ \frac{\Delta_{T'}}{D'} & \frac{B'}{D'} \end{bmatrix}$ |
| $\boldsymbol{T}$ | $\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_Z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$ | $\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta_Y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$ | $\begin{bmatrix} \frac{-\Delta_H}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$ | $\begin{bmatrix} \frac{1}{g_{21}} & \frac{g_{22}}{g_{21}} \\ \frac{g_{11}}{g_{21}} & \frac{\Delta_G}{g_{21}} \end{bmatrix}$ | $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ | $\begin{bmatrix} \frac{D'}{\Delta_{T'}} & \frac{B'}{\Delta_{T'}} \\ \frac{C'}{\Delta_{T'}} & \frac{A'}{\Delta_{T'}} \end{bmatrix}$ |
| $\boldsymbol{T'}$ | $\begin{bmatrix} \frac{z_{22}}{z_{12}} & \frac{\Delta_Z}{z_{12}} \\ \frac{1}{z_{12}} & \frac{z_{11}}{z_{12}} \end{bmatrix}$ | $\begin{bmatrix} \frac{-y_{11}}{y_{12}} & \frac{-1}{y_{12}} \\ \frac{-\Delta_Y}{y_{12}} & \frac{-y_{22}}{y_{12}} \end{bmatrix}$ | $\begin{bmatrix} \frac{1}{h_{12}} & \frac{h_{11}}{h_{12}} \\ \frac{h_{22}}{h_{12}} & \frac{\Delta_H}{h_{12}} \end{bmatrix}$ | $\begin{bmatrix} \frac{-\Delta_G}{g_{12}} & \frac{-g_{22}}{g_{12}} \\ \frac{-g_{11}}{g_{12}} & \frac{-1}{g_{12}} \end{bmatrix}$ | $\begin{bmatrix} \frac{D}{\Delta_T} & \frac{B}{\Delta_T} \\ \frac{C}{\Delta_T} & \frac{A}{\Delta_T} \end{bmatrix}$ | $\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}$ |

Table: Interrelation of different descriptions of two-ports. Four elements and one determinant of each description are used in interrelations. If an element or determinant of a description is zero, a corresponding description does not exist.

# Description of Two-ports

## Example (Description of a two-port)

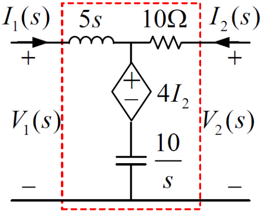
The two-port below can be described using different methods.

$$\begin{cases} V_1 = 5sI_1 + 4I_2 + \frac{10}{s}(I_1 + I_2) \\ V_2 = 10I_2 + 4I_2 + \frac{10}{s}(I_1 + I_2) \end{cases}$$

$$\begin{cases} V_1 = \frac{5s^2+10}{s} I_1 + \frac{4s+10}{s} I_2 \\ V_2 = \frac{10}{s} I_1 + \frac{14s+10}{s} I_2 \end{cases} \Rightarrow \boldsymbol{Z} = \boldsymbol{Y}^{-1} = \begin{bmatrix} \frac{5s^2+10}{s} & \frac{4s+10}{s} \\ \frac{10}{s} & \frac{14s+10}{s} \end{bmatrix}$$

$$\begin{cases} V_1 = \frac{5s^2+10}{s} I_1 + \frac{4s+10}{s} I_2 \\ I_2 = \frac{-10}{14s+10} I_1 + \frac{s}{14s+10} V_2 \end{cases} \Rightarrow \boldsymbol{H} = \boldsymbol{G}^{-1} = \begin{bmatrix} \frac{70s^2+50s+100}{14s+10} & \frac{4s+10}{14s+10} \\ \frac{-10}{14s+10} & \frac{s}{14s+10} \end{bmatrix}$$

$$\begin{cases} V_1 = \frac{5s^2+10}{s} I_1 + \frac{4s+10}{s} I_2 \\ I_1 = \frac{s}{10} V_2 - \frac{14s+10}{10} I_2 \end{cases} \Rightarrow \boldsymbol{T} = \boldsymbol{T'}^{-1} = \begin{bmatrix} \frac{5s^2+10}{10} & \frac{70s^2+50s+100}{10} \\ \frac{s}{10} & \frac{14s+10}{10} \end{bmatrix}$$



# Description of Two-ports

## Example (Description of a two-port)

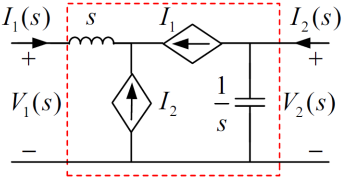
The two-port below has three different descriptions.

$$\begin{cases} I_1 + I_2 + I_1 = 0 \Rightarrow I_2 = -2I_1 \\ V_2 = \frac{1}{s}(I_2 - I_1) = \frac{-3}{s} I_1 \end{cases}$$

$$\boldsymbol{T'} = \begin{bmatrix} 0 & \frac{-3}{s} \\ 0 & 2 \end{bmatrix}$$

$$\boldsymbol{Y} = \begin{bmatrix} 0 & \frac{-s}{3} \\ 0 & \frac{2s}{3} \end{bmatrix}$$

$$\boldsymbol{G} = \begin{bmatrix} 0 & \frac{-1}{2} \\ 0 & \frac{3}{2s} \end{bmatrix}$$



## Example (Dependent sources as two-ports)

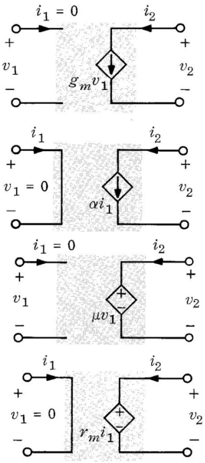
Dependent sources can be modeled by two-ports.

$$\begin{cases} I_1 = 0 \\ I_2 = g_m V_1 \end{cases} \Rightarrow \begin{cases} \boldsymbol{Y} = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix} \\ \boldsymbol{T} = \begin{bmatrix} 0 & \frac{-1}{g_m} \\ 0 & 0 \end{bmatrix} \end{cases}$$

$$\begin{cases} V_1 = 0 \\ I_2 = \alpha I_1 \end{cases} \Rightarrow \begin{cases} \boldsymbol{H} = \begin{bmatrix} 0 & 0 \\ \alpha & 0 \end{bmatrix} \\ \boldsymbol{T} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-1}{\alpha} \end{bmatrix} \end{cases}$$

$$\begin{cases} I_1 = 0 \\ V_2 = \mu V_1 \end{cases} \Rightarrow \begin{cases} \boldsymbol{G} = \begin{bmatrix} 0 & 0 \\ \mu & 0 \end{bmatrix} \\ \boldsymbol{T} = \begin{bmatrix} \frac{1}{\mu} & 0 \\ 0 & 0 \end{bmatrix} \end{cases}$$

$$\begin{cases} V_1 = 0 \\ V_2 = r_m I_1 \end{cases} \Rightarrow \begin{cases} \boldsymbol{Z} = \begin{bmatrix} 0 & 0 \\ r_m & 0 \end{bmatrix} \\ \boldsymbol{T} = \begin{bmatrix} 0 & 0 \\ \frac{-1}{r_m} & 0 \end{bmatrix} \end{cases}$$

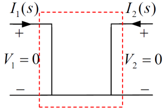


# Description of Two-ports

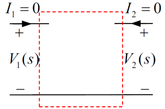
## Example (Two-ports with single description)

A two-port may only have one description.

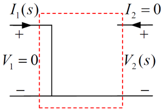
$$\boldsymbol{Z} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$



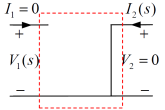

$$\boldsymbol{Y} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$




$$\boldsymbol{H} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$




$$\boldsymbol{G} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

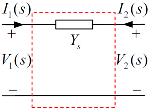


# Description of Two-ports

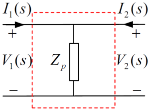
## Example (Some simple two-ports)

Some simple two-ports are shown below.

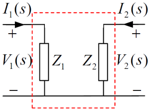
$$\boldsymbol{Y} = \begin{bmatrix} Y_s & -Y_s \\ -Y_s & Y_s \end{bmatrix}$$



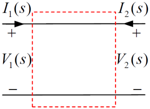

$$\boldsymbol{Z} = \begin{bmatrix} Z_p & Z_p \\ Z_p & Z_p \end{bmatrix}$$




$$\boldsymbol{Z} = \begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix}$$



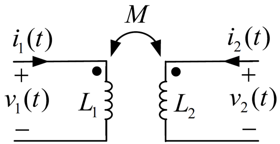

$$\boldsymbol{T} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



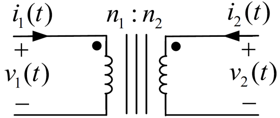
## Example (Well-known two-ports)

Some well-known two-ports are shown below.

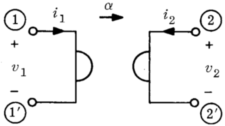
$$\boldsymbol{Z} = \begin{bmatrix} L_1 s & Ms \\ Ms & L_2 s \end{bmatrix}$$




$$\boldsymbol{T} = \begin{bmatrix} \frac{n_1}{n_2} & 0 \\ 0 & \frac{n_2}{n_1} \end{bmatrix}$$




$$\boldsymbol{Z} = \begin{bmatrix} 0 & \alpha \\ -\alpha & 0 \end{bmatrix}$$



# Extension of Two-ports

# Extension of Two-ports

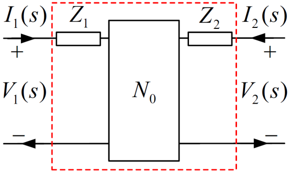



Figure: Adding series impedances in the first and second ports of a two-port results in an extended two-port with the impedance matrix $\boldsymbol{Z} = \begin{bmatrix} z_{11}(s) + Z_1(s) & z_{12}(s) \\ z_{21}(s) & z_{22}(s) + Z_2(s) \end{bmatrix}$.

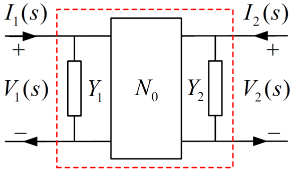



Figure: Adding parallel admittances in the first and second ports of a two-port results in an extended two-port with the admittance matrix $\boldsymbol{Y} = \begin{bmatrix} y_{11}(s) + Y_1(s) & y_{12}(s) \\ y_{21}(s) & y_{22}(s) + Y_2(s) \end{bmatrix}$.

# Extension of Two-ports

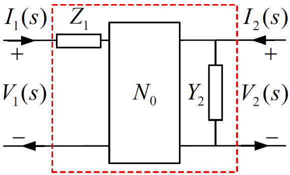



Figure: Adding series impedance in the first port and parallel admittance in the second port of a two-port results in an extended two-port with the hybrid H matrix $\boldsymbol{H} = \begin{bmatrix} h_{11}(s) + Z_1(s) & h_{12}(s) \\ h_{21}(s) & h_{22}(s) + Y_2(s) \end{bmatrix}$.

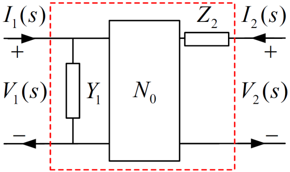



Figure: Adding parallel admittance in the first port and series impedance in the second port of a two-port results in an extended two-port with the hybrid G matrix $\boldsymbol{G} = \begin{bmatrix} g_{11}(s) + Y_1(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) + Z_2(s) \end{bmatrix}$.

# Extension of Two-ports

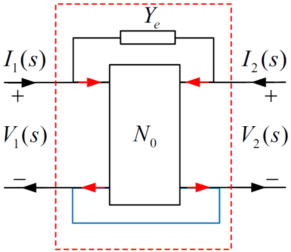



Figure: Connecting the first and second ports of a two-port using an added admittance and a short circuit leads to an extended two-port with the admittance matrix $\boldsymbol{Y} = \begin{bmatrix} y_{11}(s) + Y_e(s) & y_{12}(s) - Y_e(s) \\ y_{21}(s) - Y_e(s) & y_{22}(s) + Y_e(s) \end{bmatrix}$.

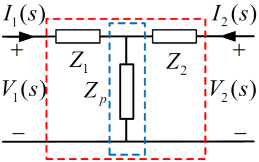
$$I_1 = Y_e(V_1 - V_2) + y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_e(V_2 - V_1) + y_{21}V_1 + Y_{22}V_2$$

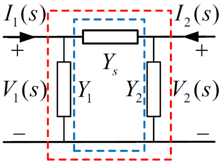
## Example (Π and T two-ports)

Π and T networks can be considered as two-ports.

$$\boldsymbol{Z}_0 = \begin{bmatrix} Z_p & Z_p \\ Z_p & Z_p \end{bmatrix} \Rightarrow \boldsymbol{Z} = \begin{bmatrix} Z_p + Z_1 & Z_p \\ Z_p & Z_p + Z_2 \end{bmatrix}$$



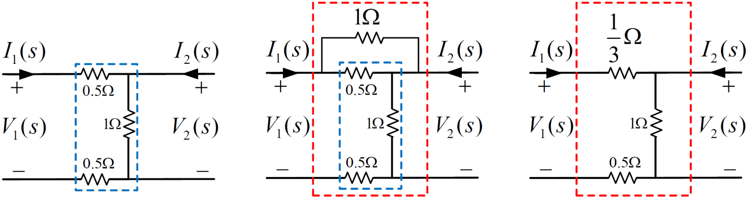

$$\boldsymbol{Y}_0 = \begin{bmatrix} Y_s & -Y_s \\ -Y_s & Y_s \end{bmatrix} \Rightarrow \boldsymbol{Y} = \begin{bmatrix} Y_s + Y_1 & -Y_s \\ -Y_s & Y_s + Y_2 \end{bmatrix}$$



# Extension of Two-ports

## Example (Two-port current condition)

Two-port current condition should be held while extending the two-port.



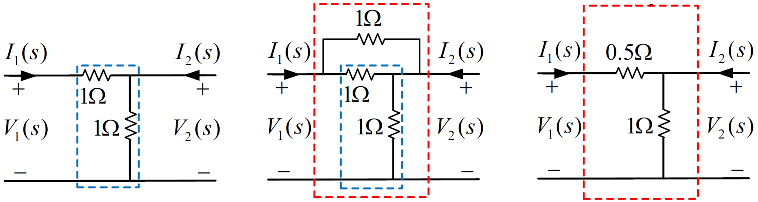

$$\boldsymbol{Z}_0 = \begin{bmatrix} 1+0.5+0.5 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow \boldsymbol{Y}_0 = \boldsymbol{Z}_0^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\boldsymbol{Z} = \begin{bmatrix} 1+0.5+\frac{1}{3} & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow \boldsymbol{Y} = \boldsymbol{Z}^{-1} \begin{bmatrix} \frac{6}{5} & -\frac{6}{5} \\ -\frac{6}{5} & \frac{11}{5} \end{bmatrix} \neq \begin{bmatrix} 1+1 & -1-1 \\ -1-1 & 2+1 \end{bmatrix}$$

# Extension of Two-ports

## Example (Two-port current condition)

Two-port current condition should be held while extending the two-port.




$$\boldsymbol{Z}_0 = \begin{bmatrix} 1+1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow \boldsymbol{Y}_0 = \boldsymbol{Z}_0^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\boldsymbol{Z} = \begin{bmatrix} 1+0.5 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow \boldsymbol{Y} = \boldsymbol{Z}^{-1} \begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 1+1 & -1-1 \\ -1-1 & 2+1 \end{bmatrix}$$

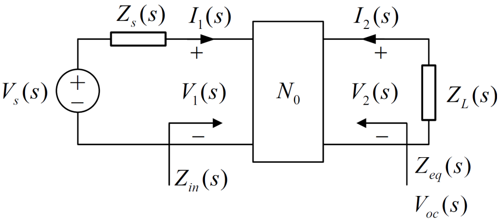


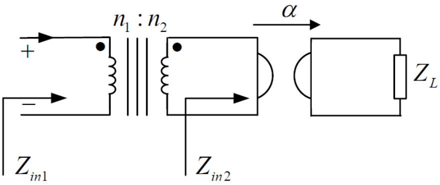

Figure: A terminated two-port with the open circuit voltage $V_{oc}(s) = V_2(s)|_{I_2=0} = \frac{z_{21}(s)}{z_{11}(s)+Z_s(s)} V_s(s)$.

| | **Z** | **Y** | **H** | **G** | **T** | **T′** |
|---|---|---|---|---|---|---|
| $Z_{in}$ | $z_{11} - \frac{z_{12}z_{21}}{z_{22}+Z_L}$ | $\left[y_{11} - \frac{y_{12}y_{21}}{y_{22}+y_L}\right]^{-1}$ | $h_{11} - \frac{h_{12}h_{21}}{h_{22}+Y_L}$ | $\left[g_{11} - \frac{g_{12}g_{21}}{g_{22}+Z_L}\right]^{-1}$ | $\frac{AZ_L+B}{CZ_L+D}$ | $\frac{B'+D'Z_L}{A'+C'Z_L}$ |
| $Z_{eq}$ | $z_{22} - \frac{z_{12}z_{21}}{z_{11}+Z_s}$ | $\left[y_{22} - \frac{y_{12}y_{21}}{y_{11}+y_s}\right]^{-1}$ | $\left[h_{22} - \frac{h_{12}h_{21}}{h_{11}+Z_s}\right]^{-1}$ | $g_{22} - \frac{g_{12}g_{21}}{g_{11}+Y_s}$ | $\frac{B+DZ_s}{A+CZ_s}$ | $\frac{A'Z_s+B'}{C'Z_s+D'}$ |

Table: Input impedanceand equivalent impedance for a terminated two-port.

## Example (Terminated two-port)

The input impedance for a terminated two-port can be found using two-port descriptions.




$$\boldsymbol{Z}_2 = \begin{bmatrix} 0 & \alpha \\ -\alpha & 0 \end{bmatrix} \Rightarrow \boldsymbol{T}_2 = \begin{bmatrix} 0 & -\alpha \\ -\frac{1}{\alpha} & 0 \end{bmatrix} \Rightarrow Z_{in2}(s) = \frac{AZ_L + B}{CZ_L + D} = \frac{-\alpha}{-\frac{1}{\alpha} Z_L} = \frac{\alpha^2}{Z_L(s)} \Rightarrow Z_{in1}(s) = (\frac{n_1}{n_2})^2 Z_{in2}(s)$$

$$\boldsymbol{T}_1 = \begin{bmatrix} \frac{n_1}{n_2} & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix} \Rightarrow \boldsymbol{T} = \boldsymbol{T}_1 \boldsymbol{T}_2 = \begin{bmatrix} 0 & -\alpha \frac{n_1}{n_2} \\ -\frac{1}{\alpha} \frac{n_1}{n_2} & 0 \end{bmatrix} \Rightarrow Z_{in1}(s) = \frac{AZ_L + B}{CZ_L + D} = \frac{-\alpha \frac{n_1}{n_2}}{-\frac{1}{\alpha} \frac{n_1}{n_2} Z_L} = (\frac{n_1}{n_2})^2 \frac{\alpha^2}{Z_L(s)}$$

# Interconnection of Two-ports

# Interconnection of Two-ports

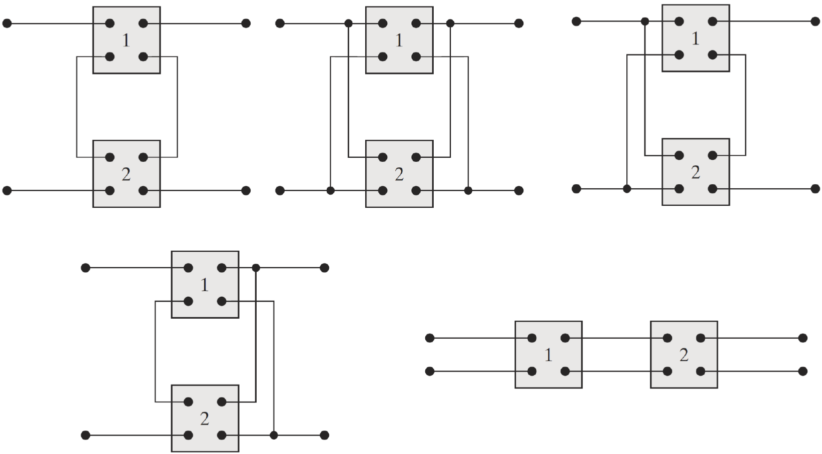



Figure: Various interconnections of two-ports including series-series, parallel-parallel, series-parallel, parallel-series, and cascade connections. Brune test provides a sufficient condition for possibility of each connection. For reciprocal two-ports, the Brune test specifies a sufficient and necessary condition.

# Series-Series Connection

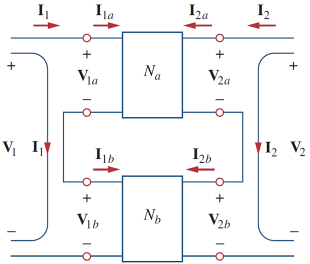



Figure: The overall impedance matrix in series-series connection is $\boldsymbol{Z} = \boldsymbol{Z}_a + \boldsymbol{Z}_b$.

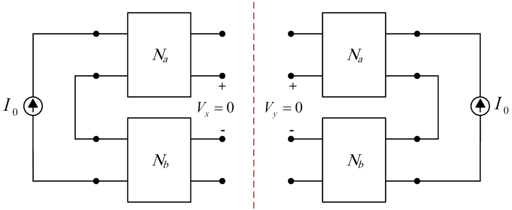



Figure: The Brune test configuration for checking the validity of the current condition for the overall series-series connected two-port.

# Parallel-Parallel Connection

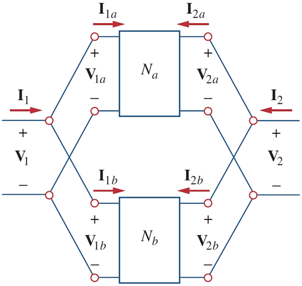



Figure: The overall admittance matrix in parallel-parallel connection is $\boldsymbol{Y} = \boldsymbol{Y}_a + \boldsymbol{Y}_b$.

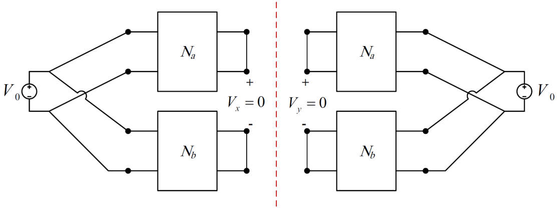



Figure: The Brune test configuration for checking the validity of the current condition for the overall parallel-parallel connected two-port.

# Series-Parallel Connection

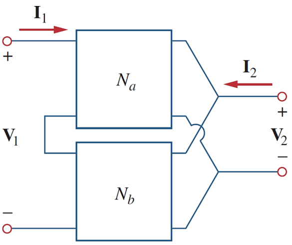



Figure: The overall hybrid H matrix in series-parallel connection is $\boldsymbol{H} = \boldsymbol{H}_a + \boldsymbol{H}_b$.

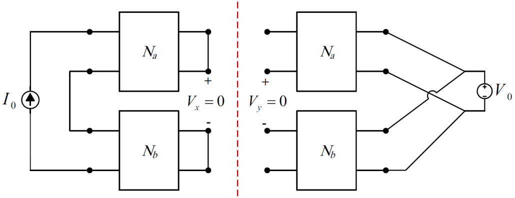



Figure: The Brune test configuration for checking the validity of the current condition for the overall series-parallel connected two-port.

# Parallel-Series Connection

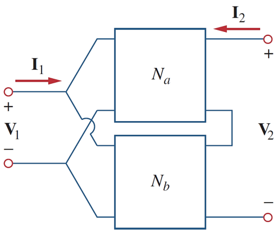



Figure: The overall hybrid G matrix in parallel-series connection is $\boldsymbol{G} = \boldsymbol{G}_a + \boldsymbol{G}_b$.

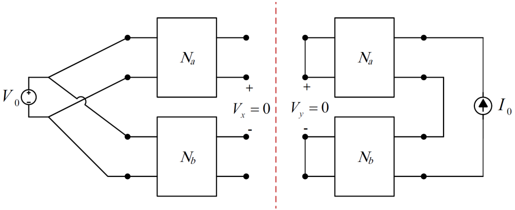



Figure: The Brune test configuration for checking the validity of the current condition for the overall parallel-series connected two-port.

# Cascade Connection

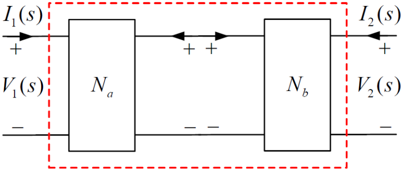



Figure: The overall transmittance ABCD matrix in cascade connection is $\boldsymbol{T} = \boldsymbol{T}_a\boldsymbol{T}_b$.

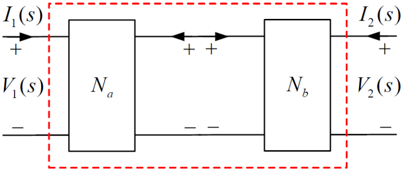
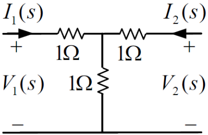
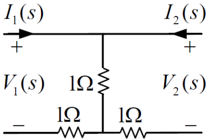


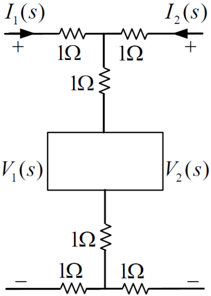

Figure: The overall transmittance A′B′C′D′ matrix in cascade connection is $\boldsymbol{T'} = \boldsymbol{T'}_b\boldsymbol{T'}_a$.

## Example (Series-series connection)

Interconnection rules can be used to find a suitable description for a complex two-port.



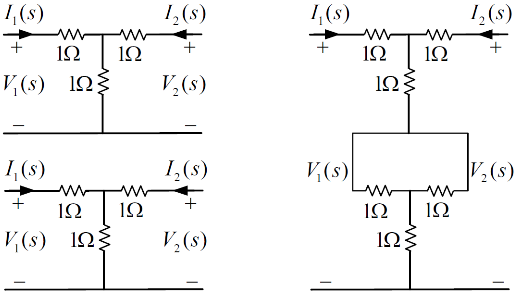




$$\boldsymbol{Z}_a = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad \boldsymbol{Z}_b = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \Rightarrow \boldsymbol{Z} = \boldsymbol{Z}_a + \boldsymbol{Z}_b = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$

# Interconnection of Two-ports

## Example (Current condition violation)

Current condition may be violated while interconnecting two-ports.




$$\boldsymbol{Z}_a = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad \boldsymbol{Z}_b = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad \boldsymbol{Z} = \begin{bmatrix} 3.5 & 2.5 \\ 2.5 & 3.5 \end{bmatrix} \neq \boldsymbol{Z}_a + \boldsymbol{Z}_b = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$

# Two-ports Models

# Two-ports Models

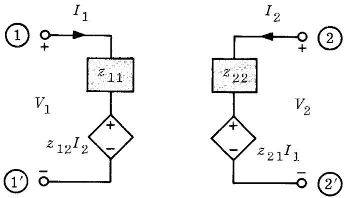



Figure: An equivalent circuit of a two-port in terms of the open circuit impedance parameters.

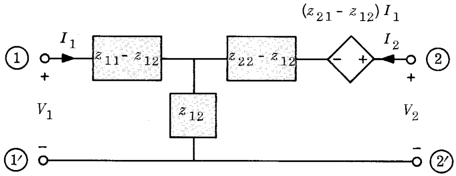



Figure: T equivalent circuit of a two-port in terms of the open circuit impedance parameters. Note that the terminals $1'$ and $2'$ have the same voltage. If the tow-port is reciprocal, the dependent current source vanishes.

# Two-ports Models

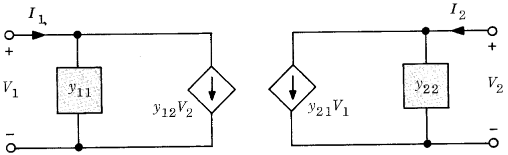



Figure: An equivalent circuit of a two-port in terms of the short circuit admittance parameters.

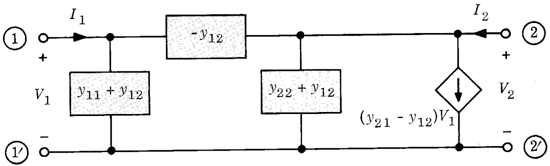



Figure: Π equivalent circuit of a two-port in terms of the short circuit admittance parameters. Note that the terminals 1′ and 2′ have the same voltage. If the tow-port is reciprocal, the dependent current source vanishes.

# Two-ports Models

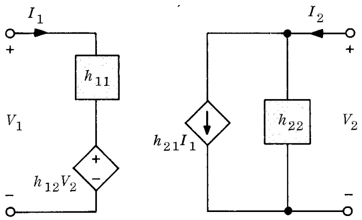
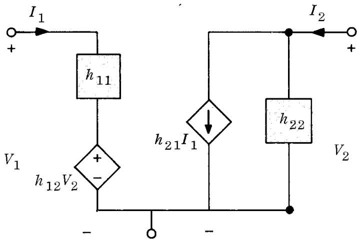



Figure: An equivalent circuit of a two-port in terms of the H parameters.

## Example (BJT transistor)

A BJT transistor can be modeled using its small-signal hybrid H parameters.




$$\begin{cases} h_{ie} = r_\pi = h_{11} \approx 1.0\text{–}10 \text{ k}\Omega \\ h_{re} = h_{12} \approx 0.5\text{–}8.0 \times 10^{-4} \\ h_{fe} = \beta = h_{21} \approx 100\text{–}400 \\ h_{oe} = h_{22} \approx .0\text{–}40\ \mu\mho \end{cases}$$

# Natural Frequencies

# Natural Frequencies

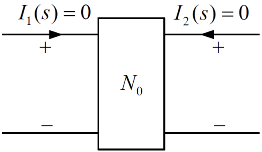



Figure: The poles of the impedance matrix elements are the natural frequencies of the circuit obtained by making the first and second ports open circuit.

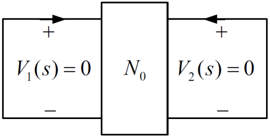



Figure: The poles of the admittance matrix elements are the natural frequencies of the circuit obtained by making the first and second ports short circuit.

# Natural Frequencies

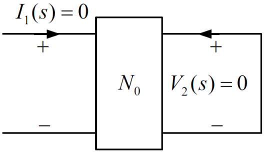



Figure: The poles of the hybrid H matrix elements are the natural frequencies of the circuit obtained by making the first port open circuit and the second port short circuit.

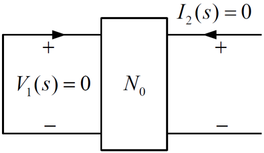
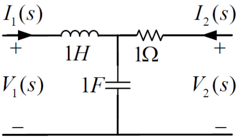



Figure: The poles of the hybrid G matrix elements are the natural frequencies of the circuit obtained by making the first port short circuit and the second port open circuit.

# Natural Frequencies

## Example (Natural frequencies)

Natural frequencies can be found using different two-port descriptions.




$$\mathbf{Z} = \begin{bmatrix} \frac{s^2+1}{s} & \frac{1}{s} \\ \frac{1}{s} & \frac{s+1}{s} \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} \frac{s+1}{s^2+s+1} & \frac{-1}{s^2+s+1} \\ \frac{-1}{s^2+s+1} & \frac{s^2+1}{s^2+s+1} \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} \frac{s^2+s+1}{s+1} & \frac{1}{s+1} \\ \frac{-1}{s+1} & \frac{s}{s+1} \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \frac{s}{s^2+1} & -\frac{1}{s^2+1} \\ \frac{1}{s^2+1} & \frac{s^2+s+1}{s^2+1} \end{bmatrix}$$

$$s_1 = 0, \qquad s_{1,2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}, \qquad s_1 = -1, \qquad s_{1,2} = \pm j$$

# Calculation Techniques

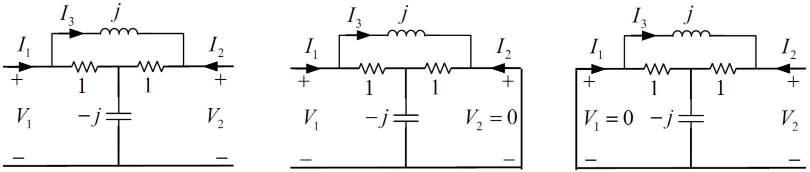
# Calculation Techniques

- Element definitions
- Circuit analysis
- Two-port extension
- Two-port interconnection
- Description interrelation

# Calculation Techniques

## Example (Element definition technique)

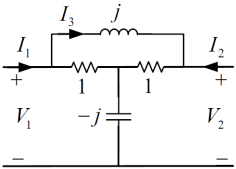
Two-port description may be found using its element definitions.




$$\begin{cases} y_{11}(j\omega) = \frac{I_1(j\omega)}{V_1(j\omega)}\Big|_{V_2(j\omega)=0} = \frac{3+6j}{5} \\ y_{22}(j\omega) = \frac{I_2(j\omega)}{V_2(j\omega)}\Big|_{V_1(j\omega)=0} = \frac{3+6j}{5} \\ y_{12}(j\omega) = \frac{I_1(j\omega)}{V_2(j\omega)}\Big|_{V_1(j\omega)=0} = \frac{-2-4j}{5} \\ y_{21}(j\omega) = \frac{I_2(j\omega)}{V_1(j\omega)}\Big|_{V_2(j\omega)=0} = \frac{-2-4j}{5} \end{cases} \Rightarrow \boldsymbol{Y} = \begin{bmatrix} \frac{3+6j}{5} & \frac{-2-4j}{5} \\ \frac{-2-4j}{5} & \frac{3+6j}{5} \end{bmatrix}$$

## Example (Circuit analysis technique)

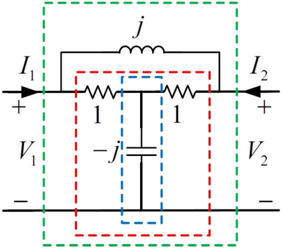
Two-port description may be found using circuit analysis.




$$\begin{bmatrix} 1-j & j & -1 \\ j & 1-j & -1 \\ -1 & -1 & j+2 \end{bmatrix} \begin{bmatrix} I_1 \\ -I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{3+6j}{5} & \frac{-2-4j}{5} \\ \frac{-2-4j}{5} & \frac{3+6j}{5} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \Rightarrow \boldsymbol{Y} = \begin{bmatrix} \frac{3+6j}{5} & \frac{-2-4j}{5} \\ \frac{-2-4j}{5} & \frac{3+6j}{5} \end{bmatrix}$$

## Example (Two-port extension)

Two-port description may be found by extending a simple two-port.



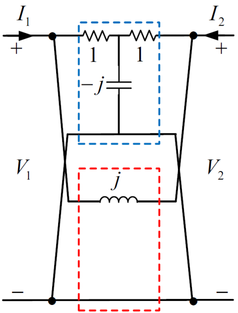

$$\boldsymbol{Z}_1 = \begin{bmatrix} -j & -j \\ -j & -j \end{bmatrix} \Rightarrow \boldsymbol{Z}_2 = \begin{bmatrix} 1-j & -j \\ -j & 1-j \end{bmatrix} \Rightarrow \boldsymbol{Y}_2 = \begin{bmatrix} \frac{3+j}{5} & \frac{-2+j}{5} \\ \frac{-2+j}{5} & \frac{3+j}{5} \end{bmatrix}$$

$$\boldsymbol{Y} = \begin{bmatrix} \frac{3+j}{5}+j & \frac{-2+j}{5}-j \\ \frac{-2+j}{5}-j & \frac{3+j}{5}+j \end{bmatrix} = \begin{bmatrix} \frac{3+6j}{5} & \frac{-2-4j}{5} \\ \frac{-2-4j}{5} & \frac{3+6j}{5} \end{bmatrix}$$

# Calculation Techniques

## Example (Two-port interconnection)

Two-port description may be found by interconnecting several simple two-ports.



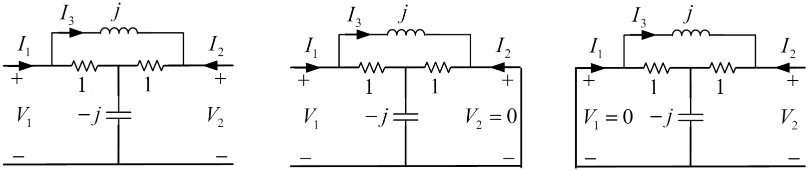

$$\boldsymbol{Z}_1 = \begin{bmatrix} -j & -j \\ -j & -j \end{bmatrix} \Rightarrow \boldsymbol{Z}_a = \begin{bmatrix} 1-j & -j \\ -j & 1-j \end{bmatrix} \Rightarrow \boldsymbol{Y}_a = \begin{bmatrix} \frac{3+j}{5} & \frac{-2+j}{5} \\ \frac{-2+j}{5} & \frac{3+j}{5} \end{bmatrix}, \quad \boldsymbol{Y}_b = \begin{bmatrix} j & -j \\ -j & j \end{bmatrix}$$

$$\boldsymbol{Y} = \boldsymbol{Y}_a + \boldsymbol{Y}_b = \begin{bmatrix} \frac{3+6j}{5} & \frac{-2-4j}{5} \\ \frac{-2-4j}{5} & \frac{3+6j}{5} \end{bmatrix}$$

# Calculation Techniques

## Example (Description interrelation)

Two-port description may be found using description interrelations from another available description.




$$\boldsymbol{Y} = \begin{bmatrix} \frac{3+6j}{5} & \frac{-2-4j}{5} \\ \frac{-2-4j}{5} & \frac{3+6j}{5} \end{bmatrix} \Rightarrow \boldsymbol{Z} = \boldsymbol{Y}^{-1} = \begin{bmatrix} \frac{3-6j}{5} & \frac{-2-4j}{5} \\ \frac{-2-4j}{5} & \frac{3-6j}{5} \end{bmatrix}$$

# Multi-ports

# Multi-ports

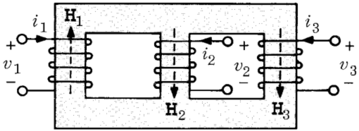



Figure: Three-winding coupled inductors create a three-port with $\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} L_1 s & M_{12} s & M_{13} s \\ M_{21} s & L_2 s & M_{23} s \\ M_{31} s & M_{32} s & L_3 s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$.

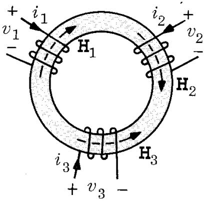



Figure: Three-winding ideal transformers create a three-port with $\begin{bmatrix} V_1 \\ V_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{n_1}{n_3} \\ 0 & 0 & \frac{n_2}{n_3} \\ -\frac{n_1}{n_3} & -\frac{n_2}{n_3} & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ V_3 \end{bmatrix}$.

# The End