MATHEMATICAL QUESTIONS

Question 1

For the lumped circuit in Fig. 1.

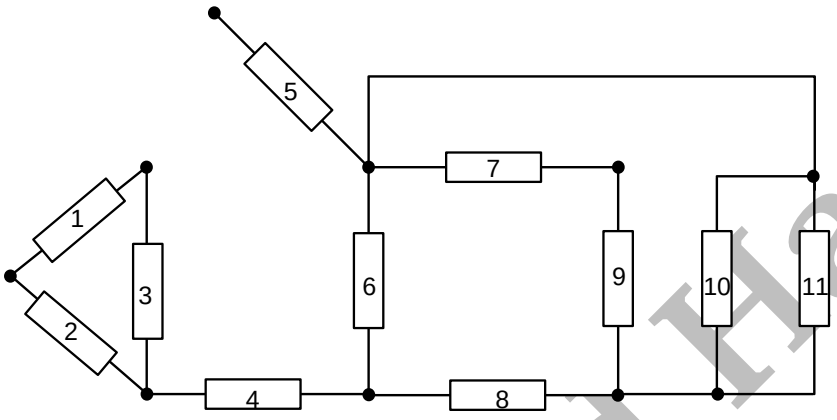

Figure 1: A sample lumped circuit.

(a) Choose a suitable voltage polarity and current direction for each element such that the passive sign convention is held.

(b) Draw the equivalent circuit graph.

(c) Determine the number of nodes, branches, and meshes.

(d) Write node KCL equations.

(e) Write mesh KVL equations.

Question 1 continued on next page...

(f) Introduce a set of linearly independent KCLs that has the largest possible cardinality.

(g) Introduce a set of linearly independent KVLs that has the largest possible cardinality.

(h) Introduce a set of linearly independent currents that has the largest possible cardinality.

(i) Introduce a set of linearly independent voltages that has the largest possible cardinality.

(j) Write a KCL for the left node of branch 5*.*

(k) Write a KCL for a Gaussian surface crossing branch 4*.*

(l) Write a KVL for a closed chain passing over branches 9 *and* 10*.*

(m) Write a KVL for a closed chain passing over branch 5*.*

(n) Verify that Tellegen's theorem is held for this circuit.

Question 2

An FM receiver is connected to its antenna by a piece of cable 2 **m long. Considering that the receiver is tuned to** 100 **MHz, can you say that the instantaneous currents at the input of the receiver and at the antenna terminals are equal? If not, for what approximate cable lengths would they be equal?**

Question 3 continued on next page...

Question 3

Consider the lumped circuit of Fig. 2.

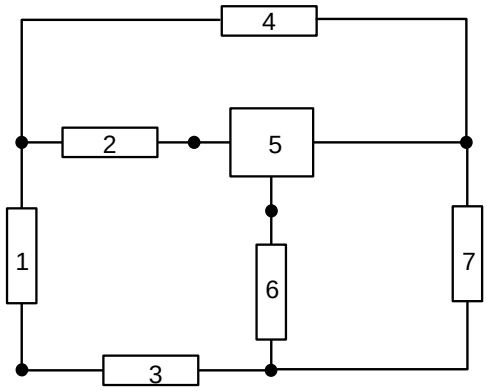

Figure 2: A lumped circuit with a three-terminal element.

(a) Label voltages and currents of different elements such that at least for two elements the passive sign convention is violated.

(b) Write node KCL equations.

(c) Write mesh KVL equations.

(d) Verify the integrity of Tellegen's theorem for the circuit.

Question 4

Consider the lumped circuit of Fig. 3.

Question 4 continued on next page...

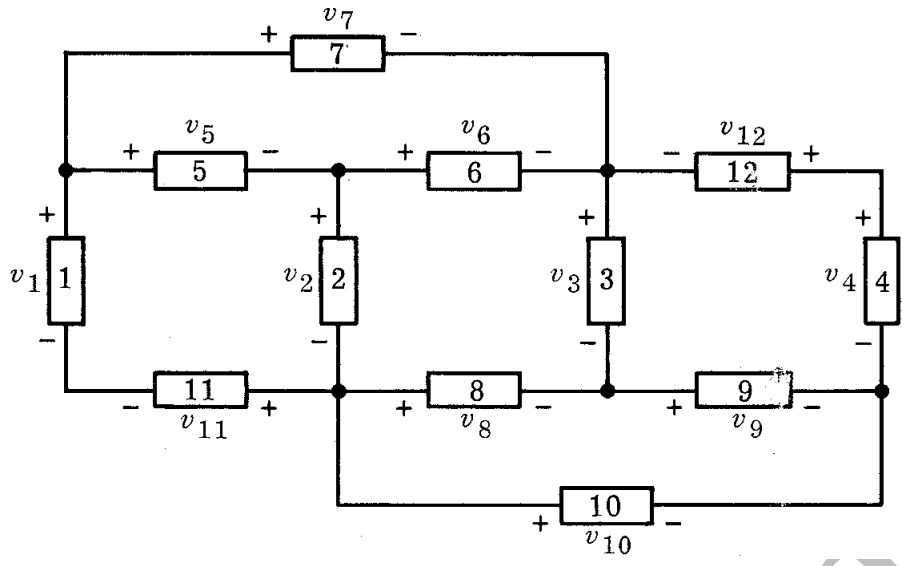

Figure 3: A planar lumped circuit with 12 elements.

(a) Assume that the voltages $v_1 = 10$, $v_2 = 5$, $v_4 = -3$, $v_6 = 2$, $v_7 = -3$, and $v_{12} = 8$ V are given. Determine as many branch voltages as possible.

(b) Assume that the passive sign convention is held and let $i_1 = 2$, $i_7 = -5$, $i_4 = 5$, $i_{10} = -3$, and $i_3 = 1$ A. Is it possible to determine the remaining branch currents? Determine as many as you can.

(c) Prove that $i_1 + i_2 + i_3 + i_4 = 0$ and $i_7 + i_6 + i_8 + i_{10} = 0$.

(d) Assuming the given voltages and currents in the previous parts, determine as many absorbed powers as possible.

SOFTWARE QUESTIONS

Question 5 continued on next page...

Question 5

A directed graph can be represented by its adjacency matrix. In fact, for the graph $G(\mathbf{N} = \{1, 2, \cdots, n\}, \mathbf{E})$ with n nodes, the adjacency matrix is $A_{n\times n} = [a_{ij}]$, where a_{ij} is 1 if $(i, j) \in \mathbf{E}$, and 0 otherwise. Write a MATLAB/Python function that takes the adjacency matrix of a connected planar directed graph, draws the corresponding graph, and determines the number of nodes, branches, and meshes of the graph. For simplicity, assume that there is no parallel edges with the same source and destination nodes.

BONUS QUESTIONS

Question 6

Prove that in a connected planar graph, the number of meshes is given by $l = b - n + 1$, where b and n are the number of branches and nodes of the graph, respectively.

Question 7

Return your answers by filling the LaTeXtemplate of the assignment.