Operational Amplifiers

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Overview

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2. Well-known Op-Amp Circuits
3. Op-Amp Circuits
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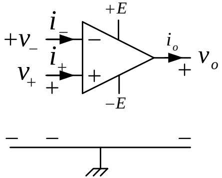
Operational Amplifiers

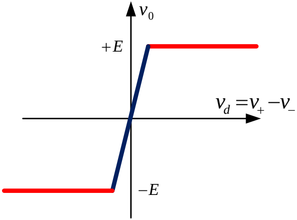
Op-Amp

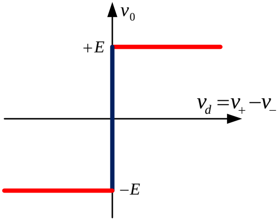
Definition (Op-Amp)

An operational amplifier is a multi-terminal element characterized as

$$v_o = \begin{cases} +E & v_d = v_+ - v_- > 0 \\ \in (-E, +E) & v_d = v_+ - v_- = 0 \approx E\text{sgn}(v_d) \\ -E & v_d = v_+ - v_- < 0 \end{cases}$$



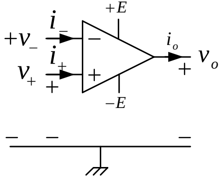


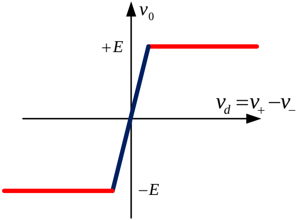


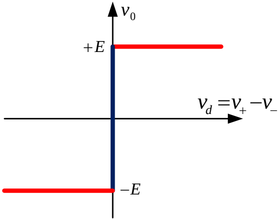
Op-Amp

Statement (Linear and Nonlinear Behaviors of Op-Amp)

An operational amplifier can be considered as a nonlinear analog comparator or can be considered as a linear amplifier with infinite gain $A \approx \infty$ *for* $v_o \in (-E, +E)$.

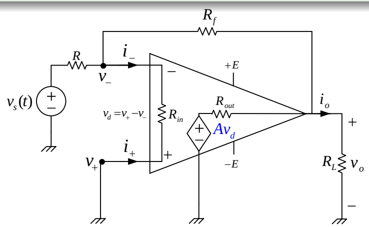






Example (Linear behavior of op-amp)

If an ideal op-amp has negative feedback, it can work as an amplifier provided that $|v_o| < V_{sat} \approx E$ and $|i_o| \lesssim I_{sat}$.



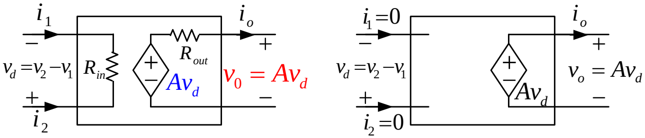
$$\frac{v_- - v_s}{R} + \frac{v_- - v_o}{R_f} + \frac{v_- - 0}{R_{in}} = 0, \quad \frac{v_o - v_-}{R_f} + \frac{v_o - Av_d}{R_{out}} + \frac{v_o - 0}{R_L} = 0$$

$$G = \frac{v_o}{v_s} = \frac{-A + \frac{R_{out}}{R_f}}{\frac{R}{R_f}(1 + A + \frac{R_{out}}{R_{in}} + \frac{R_{out}}{R_L}) + (1 + \frac{R_{out}}{R_L})(1 + \frac{R}{R_{in}}) + \frac{R_{out}}{R_f}} \Rightarrow \lim_{\substack{A\to\infty \\ R_{in}\to\infty \\ R_{out}\to 0}} G = -\frac{R_f}{R}$$

$$v_d = v_+ - v_- = -v_s \frac{RR_{in}G + R_f R_{in}}{R_f R_{in} + RR_{in} + RR_f} \Rightarrow \lim_{\substack{A\to\infty \\ R_{in}\to\infty \\ R_{out}\to 0}} v_d = 0 \Rightarrow \lim_{\substack{A\to\infty \\ R_{in}\to\infty \\ R_{out}\to 0}} i_R = 0$$

Example (Linear model of op-amp)

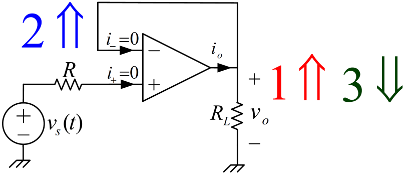
When an ideal op-amp is used as an amplifier, it can be modeled by a dependent voltage source with the implicit assumptions $|v_o| < V_{sat} \approx E$ and $|i_o| \lesssim I_{sat}$.

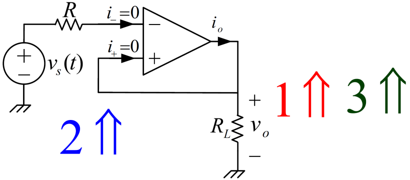


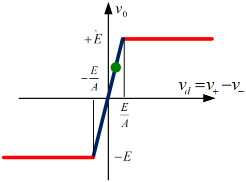
$$A = \infty, R_{in} = \infty, R_{out} = 0 \Rightarrow v_d = 0, \Rightarrow v_- = v_+$$
$$A = \infty, R_{in} = \infty, R_{out} = 0 \Rightarrow i_R = 0 \Rightarrow i_+ = 0, -i_- = 0$$
$$|v_o| < V_{sat} \approx E, \quad |i_o| \lesssim I_{sat}$$

Example (Negative and positive feedback)

Negative feedback makes the circuit stable while the circuit will be unstable when the feedback is positive.





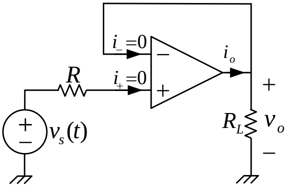


Well-known Op-Amp Circuits

Buffer

Example (Buffer)

A buffer can be implemented using op-amps.



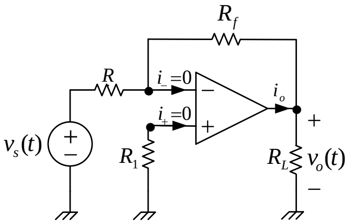
$$v_o(t) = v_-(t) = v_+(t) = v_s(t)$$

$$i_o(t) = \frac{v_o(t)}{R_L}$$

$$|i_o(t)| < I_{sat}, \quad |v_o(t)| < V_{sat} \approx E$$

Example (Inverting amplifier)

An inverting amplifier can be implemented using op-amps.



$$i_+ = 0 \Rightarrow v_+ = 0 \Rightarrow v_- = 0$$

$$\frac{0 - v_s(t)}{R} + \frac{0 - v_o(t)}{R_f} = 0 \Rightarrow v_o(t) = -\frac{R_f}{R} v_s(t)$$

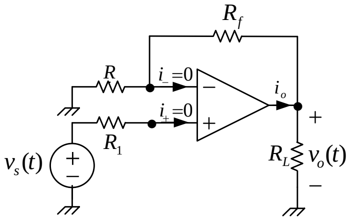
$$i_o(t) = \frac{v_o(t) - 0}{R_f} + \frac{v_o(t)}{R_L}$$

$$|i_o(t)| < I_{sat}, \quad |v_o(t)| < V_{sat} \approx E$$

Non-inverting Amplifier

Example (Non-inverting amplifier)

A non-inverting amplifier can be implemented using op-amps.



$$i_+ = 0 \Rightarrow v_+ = v_s(t) \Rightarrow v_- = v_s(t)$$

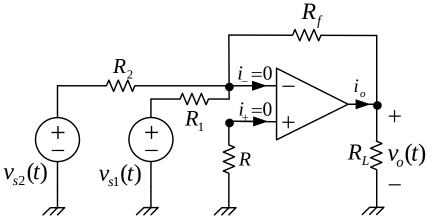
$$\frac{v_s(t) - 0}{R} + \frac{v_s(t) - v_o(t)}{R_f} = 0 \Rightarrow v_o(t) = (1 + \frac{R_f}{R})v_s(t)$$

$$i_o(t) = \frac{v_o(t) - v_s(t)}{R_f} + \frac{v_o(t)}{R_L}$$

$$|i_o(t)| < I_{sat}, \quad |v_o(t)| < V_{sat} \approx E$$

Example (Negative adder)

A negative adder can be implemented using op-amps.



$$i_+ = 0 \Rightarrow v_+ = 0 \Rightarrow v_- = 0$$

$$\frac{0 - v_{s1}(t)}{R_1} + \frac{0 - v_{s2}(t)}{R_2} + \frac{0 - v_o(t)}{R_f} = 0 \Rightarrow v_o(t) = -\frac{R_f}{R_1} v_{s1}(t) - \frac{R_f}{R_2} v_{s2}(t)$$

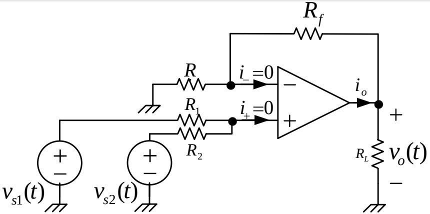
$$i_o(t) = \frac{v_o(t)}{R_f} + \frac{v_o(t)}{R_L}$$

$$|i_o(t)| < I_{sat}, \quad |v_o(t)| < V_{sat} \approx E$$

Positive Adder

Example (Positive adder)

A positive adder can be implemented using op-amps.



$$i_+ = 0 \Rightarrow v_+ = \frac{R_2}{R_1 + R_2} v_{s1}(t) + \frac{R_1}{R_1 + R_2} v_{s2}(t) \Rightarrow v_- = v_+$$

$$\frac{v_-(t) - 0}{R} + \frac{v_-(t) - v_o(t)}{R_f} = 0 \Rightarrow v_o(t) = (1 + \frac{R_f}{R}) v_-(t)$$

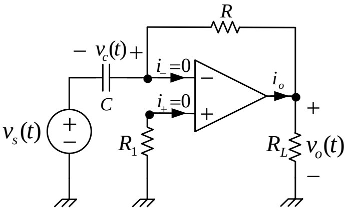
$$v_o(t) = (1 + \frac{R_f}{R})\left[\frac{R_2}{R_1 + R_2} v_{s1}(t) + \frac{R_1}{R_1 + R_2} v_{s2}(t)\right]$$

$$i_o(t) = \frac{v_o(t) - v_-(t)}{R_f} + \frac{v_o(t)}{R_L}$$

$$|i_o(t)| < I_{sat}, \quad |v_o(t)| < V_{sat} \approx E$$

Example (Differentiator)

A differentiator can be implemented using op-amps.



$$i_+ = 0 \Rightarrow v_+ = 0 \Rightarrow v_- = 0 \Rightarrow v_c(t) + v_s(t) = 0$$

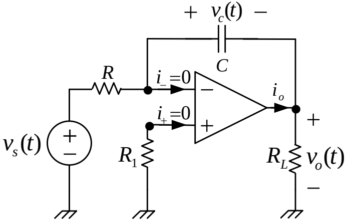
$$C\frac{dv_c}{dt} + \frac{0 - v_o(t)}{R} = 0 \Rightarrow v_o(t) = -RC\frac{dv_s(t)}{dt}$$

$$i_o(t) = \frac{v_o(t)}{R} + \frac{v_o(t)}{R_L}$$

$$|i_o(t)| < I_{sat}, \quad |v_o(t)| < V_{sat} \approx E$$

Example (Integrator)

An integrator can be implemented using op-amps.



$$i_+ = 0 \Rightarrow v_+ = 0 \Rightarrow v_- = 0 \Rightarrow v_c(t) + v_o(t) = 0$$

$$C\frac{dv_c}{dt} + \frac{0 - v_s(t)}{R} = 0 \Rightarrow C\frac{dv_o}{dt} = -\frac{v_s(t)}{R} \Rightarrow v_o(t) = v_o(0) - \frac{1}{RC}\int_0^t v_s(\lambda)d\lambda$$

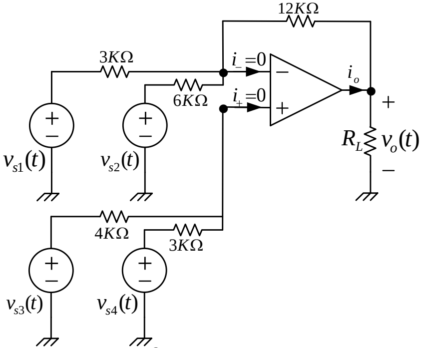
$$i_o(t) = \frac{-v_s(t)}{R} + \frac{v_o(t)}{R_L}$$

$$|i_o(t)| < I_{sat}, \quad |v_o(t)| < V_{sat} \approx E$$

Op-Amp Circuits

Example (Linear voltage combiner)

A linear voltage combiner can be implemented using op-amps.



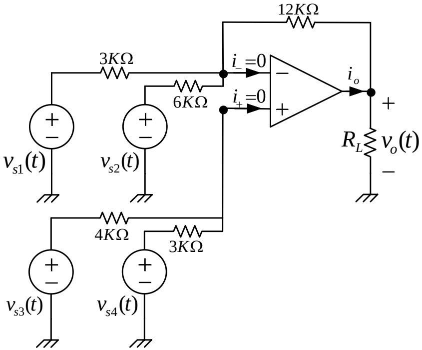
$$v_-(t) = v_+(t) = \frac{3}{3+4} v_{s3}(t) + \frac{4}{3+4} v_{s4}(t)$$

$$\frac{v_-(t) - v_{s1}(t)}{3} + \frac{v_-(t) - v_{s2}(t)}{6} + \frac{v_-(t) - v_o(t)}{12} = 0$$

$$v_o(t) = -4v_{s1}(t) - 2v_{s2}(t) + 3v_{s3}(t) + 4v_{s4}(t)$$

Example (Linear voltage combiner (cont.))

A linear voltage combiner can be implemented using op-amps.



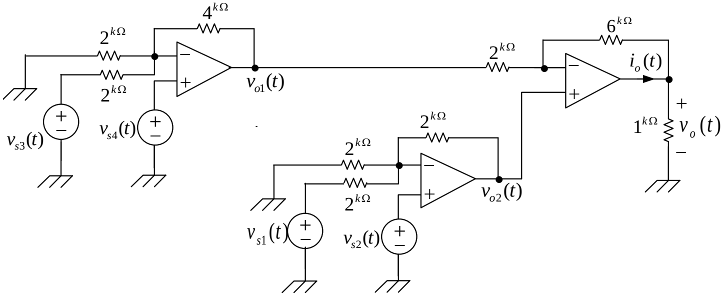
$v_{s1} = 1, v_{s2} = 2, v_{s3} = 3, V_{sat} = 15, I_{sat} = 0.002$

$\Rightarrow |v_o| = | -4 \times 1 - 2 \times 2 + 3 \times 3 + 4v_{s4}| < 15 \Rightarrow -4 < v_{s4}(t) < 3.5$

$v_{s4} = 3 \in [-4, 3.5] \Rightarrow i_o(t) = \dfrac{v_o(t) - v_-(t)}{12} + \dfrac{v_o(t)}{R_L} = \dfrac{10}{12} + \dfrac{13}{R_L} < 2 \Rightarrow R_L > 11.143 \text{ k}\Omega$

Example (Circuit with multiple op-amps)

Ap-amp circuits can be interconnected to create complex circuits.



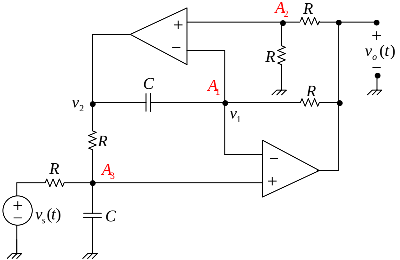
$$\frac{v_{s4}}{2} + \frac{v_{s4} - v_{s3}}{2} + \frac{v_{s4} - v_{o1}}{4} = 0, \quad \frac{v_{s2}}{2} + \frac{v_{s2} - v_{s1}}{2} + \frac{v_{s2} - v_{o2}}{2} = 0$$

$$\frac{v_{o2} - v_{o1}}{2} + \frac{v_{o2} - v_o}{6} = 0, \quad i_o = \frac{v_o - v_{o2}}{6} + \frac{v_o}{1}$$

$$v_o = -4v_{s1} + 12v_{s2} + 6v_{s3} - 15v_{s4}, \quad i_o = -4.5v_{s1} + 13.5v_{s2} + 7v_{s3} - 17.5v_{s4}$$

Example (Circuit with multiple op-amps)

The step response of the ap-amp circuit below has sinusoidal form for $RC = 0.5$.

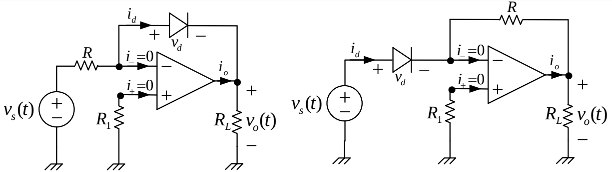


$$\begin{cases} \frac{v_1(t)-v_o(t)}{R} + C\frac{d(v_1(t)-v_2(t))}{dt} = 0 \\ \frac{v_1(t)}{R} + \frac{v_1(t)-v_o(t)}{R} = 0 \\ \frac{v_1(t)-v_2(t)}{R} + \frac{v_1(t)-v_s(t)}{R} + C\frac{dv_1(t)}{dt} = 0 \end{cases}$$

$$\frac{d^2v_o(t)}{dt^2} + 2\frac{dv_o(t)}{dt} + 4v_o(t) = 4\frac{dv_s}{dt} \Rightarrow s(t) = v_o(t) = \frac{4\sqrt{3}}{3}e^{-t}\sin(\sqrt{3}t)u(t)$$

Example (Exponential and logarithmic amplifiers)

Exponential and logarithmic amplifiers can be implemented using op-amps.



$$\frac{0-v_s(t)}{R}+i_d(t)=-\frac{v_s(t)}{R}+I_s(e^{\frac{v_d(t)}{V_T}}-1)\approx-\frac{v_s(t)}{R}+I_se^{\frac{v_d(t)}{V_T}}=-\frac{v_s(t)}{R}+I_se^{-\frac{v_o(t)}{V_T}}=0$$

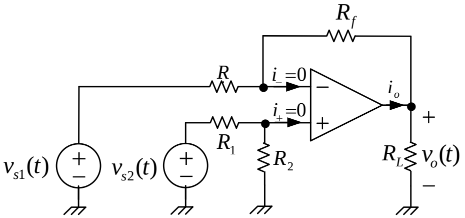
$$\Rightarrow v_o(t)=-V_T\ln(\frac{v_s(t)}{RI_s})$$

$$\frac{0-v_o(t)}{R}-i_d(t)=-\frac{v_o(t)}{R}-I_s(e^{\frac{v_d(t)}{V_T}}-1)\approx-\frac{v_o(t)}{R}-I_se^{\frac{v_d(t)}{V_T}}=-\frac{v_o(t)}{R}-I_se^{\frac{v_s(t)}{V_T}}=0$$

$$\Rightarrow v_o(t)=-RI_se^{\frac{v_s(t)}{V_T}}$$

Example (Op-Amp Circuit Synthesis)

The op-amp circuit below can implement the equation $v_o = -4v_{s1} + 3v_{s2}$.



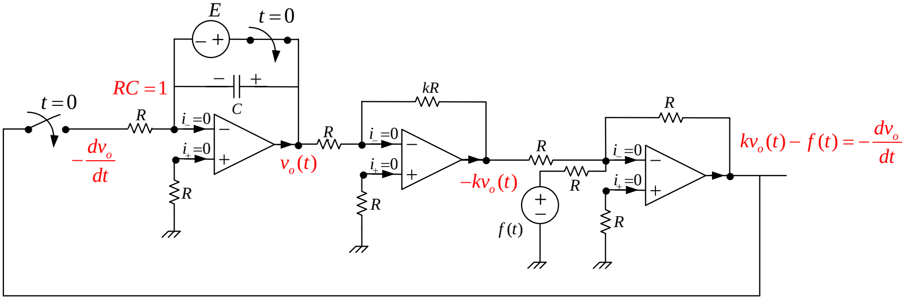
$$v_o(t) = -\frac{R_f}{R} v_{s1}(t) + (1 + \frac{R_f}{R}) \frac{R_2}{R_1 + R_2} v_{s2}(t)$$

$$\frac{R_f}{R} = 4, \quad (1 + \frac{R_f}{R}) \frac{R_2}{R_1 + R_2} = 3$$

$$R_f = 4R \Rightarrow \frac{R_2}{R_1 + R_2} = \frac{3}{5} \Rightarrow R_2 = 3R, R_1 = 2R$$

Example (Op-Amp Circuit Synthesis)

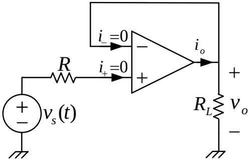
The op-amp circuit below can solve the differential equation $\frac{dv_o}{dt} + kv_o = f(t), t > 0; v_o(0^+) = E$.

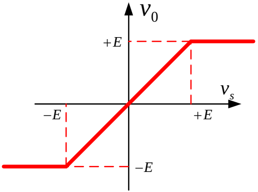


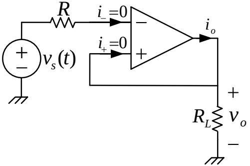
$$\frac{dv_o}{dt} + kv_o = f(t) \Rightarrow -\frac{dv_o}{dt} = kv_o - f(t)$$

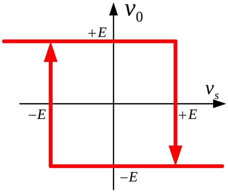
Example (Buffer with negative and positive feedback)

Negative and positive feedback lead to completely different behavior in a buffer circuit.



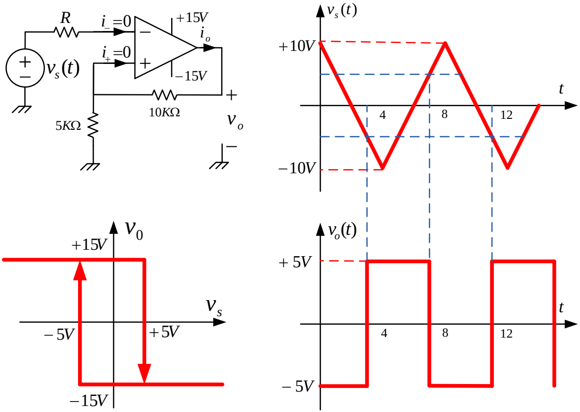






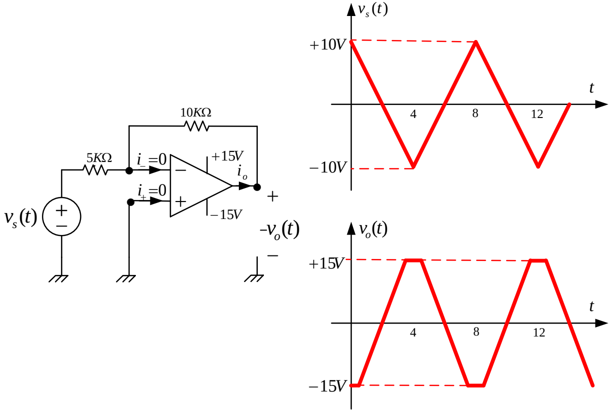
Example (Schmitt Trigger)

A Schmitt trigger circuit can be implemented using an op-amp with positive feedback.



Example (Saturated inverting amplifier)

An inverting op-amp amplifier is saturated if the input voltage amplitude is unacceptably high.

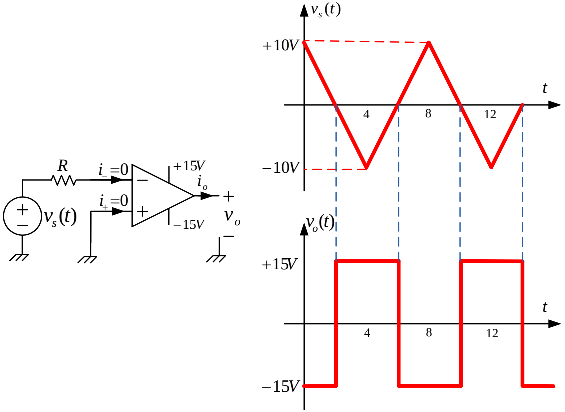


$$v_o = -2v_s \Rightarrow |v_o| = 2|v_s| < 15 \Rightarrow |v_s| < 7.5$$

Negative and Positive Feedback

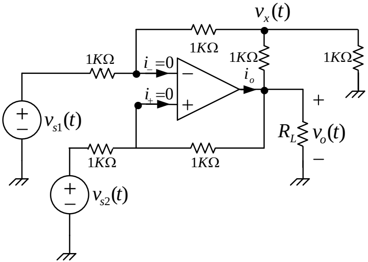
Example (Op-amp without feedback)

An op-amp circuit may have no feedback.



Example (Op-amp with negative and positive feedback)

For an op-amp circuit with both positive and negative feedback, the negative feedback is usually assumed dominant.



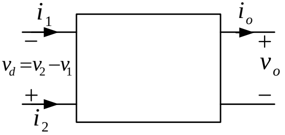
$$\begin{cases} \frac{v_-(t)-v_{s1}(t)}{1} + \frac{v_-(t)-v_x(t)}{1} = 0 \\ \frac{v_+(t)-v_{s2}(t)}{1} + \frac{v_+(t)-v_o(t)}{1} = 0 \\ \frac{v_x(t)-v_-(t)}{1} + \frac{v_x(t)-v_o(t)}{1} + \frac{v_x(t)-0}{1} = 0 \\ v_+(t) = v_-(t) \end{cases} \Rightarrow v_o(t) = 2v_{s1}(t) - \frac{5}{3}v_{s2}(t)$$

Amplifiers

Amplifier Types

Statement (Amplifier Types)

An amplifier can be modeled as a two-port and may have one of the four main types including voltage/voltage, voltage/current, current/voltage, and currrent/current amplifier.

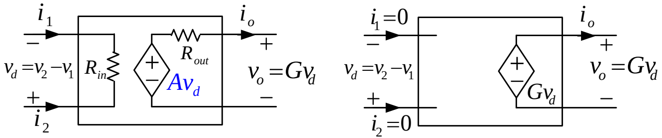


$$G_{vv} = \frac{v_o}{v_d}, \quad G_{vi} = \frac{i_o}{v_d}, \quad G_{iv} = \frac{v_o}{i_1}, \quad G_{ii} = \frac{i_o}{i_1}$$

Amplifier Models

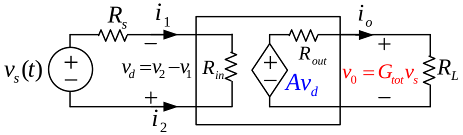
Statement (Amplifier Models)

An amplifier can be described by its total gain, internal gain, input resistance, and output resistance.



Example (Ideal amplifier)

Input and output resistance degrade the amplifier performance.



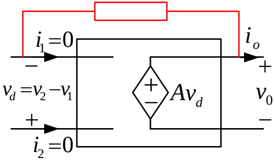
$$v_o = \frac{R_L}{R_L + R_{out}} A v_d(t) = \frac{R_L}{R_L + R_{out}} A \frac{-R_{in}}{R_{in} + R_s} v_s \Rightarrow G_{tot} = \frac{v_o}{v_s} = -\frac{R_L}{R_L + R_{out}} \frac{R_{in}}{R_{in} + R_s} A$$

$R_{in} \to \infty, R_o \to 0 \Rightarrow G_{tot} = -A$

Amplifier Feedback

Statement (Negative Feedback)

Negative feedback can be used to stabilize the total gain for an amplifier with high internal gain, high input resistance, and low output resistance.



$$A \to \infty, R_{in} \to \infty, R_o \to 0$$

The End