Question 1

Verify Tellegen's theorem for the circuit of Fig. 1. Provide full description for each step of calculations.

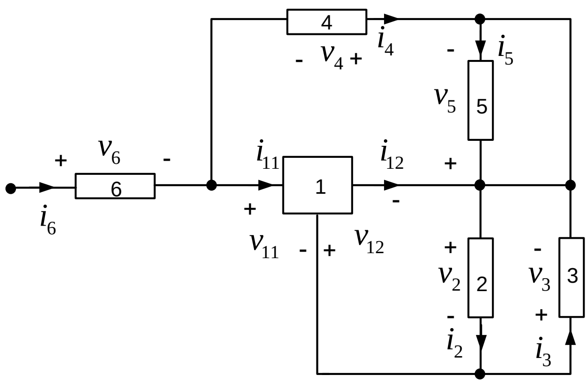

Figure 1: A lumped circuit.

The sum of absorbed powers should be zero by Tellegen's theorem. So,

$$\sum_{i=1}^{6} p_i = p_1+p_2+p_3+p_4+p_5+p_6 = v_{11}i_{11}+(-v_{12})(-i_{12})+v_2i_2+v_3i_3+v_4(-i_4)+v_5(-i_5)+v_6i_6$$

Since $i_6 = 0$ and $v_5 = 0$ according to KCL and KVL, respectively,

$$\sum_{i=1}^{6} p_i = v_{11}i_{11} + v_{12}i_{12} + v_2i_2 + v_3i_3 - v_4i_4$$

According to KVL $v_{12} = -v_2 = v_3$ and $v_4 = -v_5 - v_{12} - v_{11} = -v_{12} - v_{11}$. So,

$$\sum_{i=1}^{6} p_i = v_{11}i_{11} + v_{12}i_{12} - v_{12}i_2 + v_{12}i_3 + (v_{11} + v_{12})i_4$$

According to KCL, $i_4 = i_6 - i_{11} = -i_{11}$. Thus,

$$\sum_{i=1}^{6} p_i = v_{11}i_{11} + v_{12}i_{12} - v_{12}i_2 + v_{12}i_3 - (v_{11} + v_{12})i_{11}$$

Which gives

$$\sum_{i=1}^{6} p_i = v_{11}(i_{11} - i_{11}) + v_{12}(i_{12} - i_2 + i_3 - i_{11}) = v_{12}(i_{12} - i_2 + i_3 - i_{11})$$

The expression in the parenthesis is zero. In fact, a KCL for the current i_{13} of the down leg of the three-terminal element yields

$$i_{13} = i_{11} - i_{12} = -i_2 + i_3 \Rightarrow i_{12} - i_{11} - i_2 + i_3 = 0$$

Finally,

$$\sum_{i=1}^{6} p_i = 0$$