Question 1

A capacitor is described by the characteristic curve $\ln(v^2) + \sinh^{-1}(\frac{q}{v^2}) = 0$.

(a) Is the capacitor linear?

We have

$$
\begin{aligned}
\sinh^{-1}(\frac{q}{v^2}) &= -\ln(v^2) \\
\frac{q}{v^2} &= \sinh(-\ln(v^2)) \\
\frac{q}{v^2} &= \frac{e^{-\ln(v^2)} - e^{\ln(v^2)}}{2} \\
\frac{q}{v^2} &= \frac{e^{\ln(v^{-2})} - e^{\ln(v^2)}}{2} \\
\frac{q}{v^2} &= \frac{v^{-2} - v^2}{2} \\
q &= \frac{1 - v^4}{2}
\end{aligned}
$$

Clearly, q is not a linear function of v and the capacitor is not linear.

(b) Is the capacitor time-invariant?

The characteristic curve does not change with time and therefore, the capacitor is time-invariant.

(c) Is the capacitor voltage-controlled?

$$q = \frac{1 - v^4}{2}$$

Clearly, q is a function of v and the capacitor is voltage-controlled.

(d) Is the capacitor charge-controlled?

$$
\begin{aligned}
q &= \frac{1 - v^4}{2} \\
v &= \pm\sqrt[4]{1 - 2q}, \quad q \leq 0.5
\end{aligned}
$$

Clearly, v is not a function of q and the capacitor is not charge-controlled.

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(e) Is the capacitor passive?

Since the characteristic curve is not confined within the first and third quarters of the qv plane, the capacitor is not passive. In fact,

$$q = \frac{1 - v^4}{2}$$
$$qv = v\frac{1 - v^4}{2}$$

Since the product of qv is not always non-negative, the capacitor is not passive. For example, for $v = 2$, $qv = -15 < 0$.

(f) Find the voltage-current equation of the capacitor.

$$q = \frac{1 - v^4}{2}$$
$$\frac{dq}{dt} = -\frac{1}{2}4v^3\frac{dv}{dt}$$
$$i = -2v^3v'$$

(g) Calculate the absorbed power of the capacitor.

$$p = vi = -2v^4v'$$

(h) Calculate the absorbed energy of the capacitor during the time interval $[0, t]$.

$$w(0, t) = \int_0^t p(\lambda)d\lambda$$
$$w(0, t) = \int_0^t -2v^4(\lambda)v'(\lambda)d\lambda$$
$$w(0, t) = \int_{v(0)}^{v(t)} -2u^4du$$
$$w(0, t) = -\frac{2}{5}u^5\Big|_{v(0)}^{v(t)} = -\frac{2}{5}\left[v^5(t) - v^5(0)\right]$$