Student Name: .....
Student Number: .....

Question 1

The characteritic curves of two nonlinear inductors are shown in Fig. 1, where each inductor and its corresponding characteristic curve have the same color. Let the initial magnetic fluxes be $\phi_1(0) = \Phi_1 = 0$ and $\phi_2(0) = \Phi_2 = -6$.

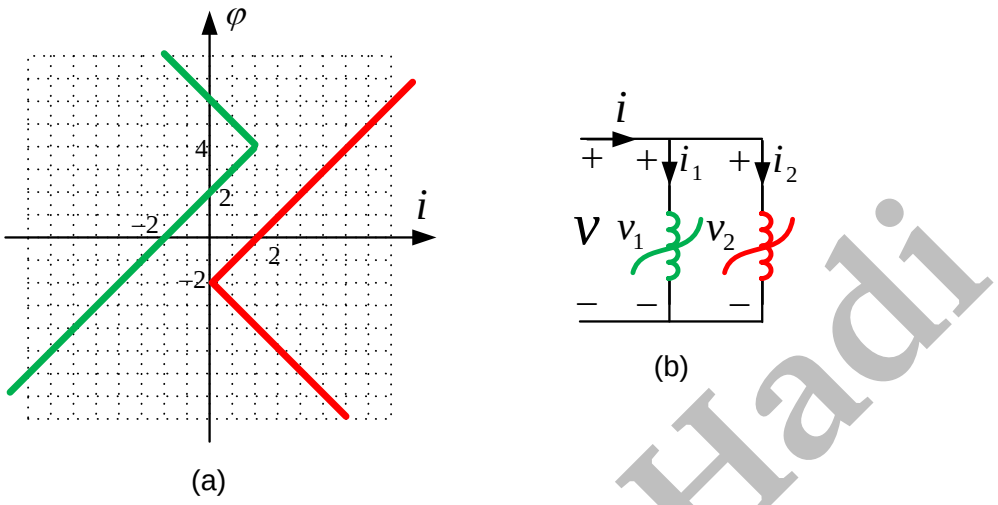

Figure 1: Two NTI inductors (a) their characteristic curves (b) their parallel connection.

(a) Find an expression for the characteristic curve of the parallel connection of the two inductors.

Clearly,

$$i_1(\phi_1) = \begin{cases} -\phi_1 + 6, & \phi_1 \geq 4 \\ \phi_1 - 2, & \phi_1 < 4 \end{cases}, \quad i_2(\phi_2) = \begin{cases} \phi_2 + 2, & \phi_2 \geq -2 \\ -\phi_2 - 2, & \phi_2 < -2 \end{cases}$$

As discussed in the class, for parallel connection,

$$v(t) = v_1(t) = v_2(t) \Rightarrow \frac{d\phi(t)}{dt} = \frac{d\phi_1(t)}{dt} = \frac{d\phi_2(t)}{dt} \Rightarrow \phi(t) - \phi(0) = \phi_1(t) - \phi_1(0) = \phi_2(t) - \phi_2(0)$$

Let the initial flux of the equivalent inductor be $\phi(0) = \Phi = 0$. Now,

$$\phi_1(t) = \phi(t) - \phi(0) + \phi_1(0) = \phi(t) - \Phi + \Phi_1 = \phi(t)$$

$$\phi_2(t) = \phi(t) - \phi(0) + \phi_2(0) = \phi(t) - \Phi + \Phi_2 = \phi(t) - 6$$

According to KCL, $i = i_1 + i_2$, so,

$$i(\phi) = i_1(\phi_1) + i_2(\phi_2) = i_1(\phi) + i_2(\phi - 6)$$

$$i(\phi) = \begin{cases} -\phi + 6, & \phi \geq 4 \\ \phi - 2, & \phi < 4 \end{cases} + \begin{cases} \phi - 6 + 2, & \phi - 6 \geq -2 \\ -(\phi - 6) - 2, & \phi - 6 < -2 \end{cases} = 2$$

Finally, the characteristic curve is the constant function $i(\phi) = 2$.

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(b) Find an expression for the current signal $i(t)$ if $v(t) = \sin(t)$.

Clearly, $i = 2$ regardless of the applied voltage.