Question 1

Calculate the response of $v_o(t)$ in Fig. 1 when $i_s(t) = \delta(t)$, $v_c(0^-) = 0$, and $i_L(0^-) = 0$.

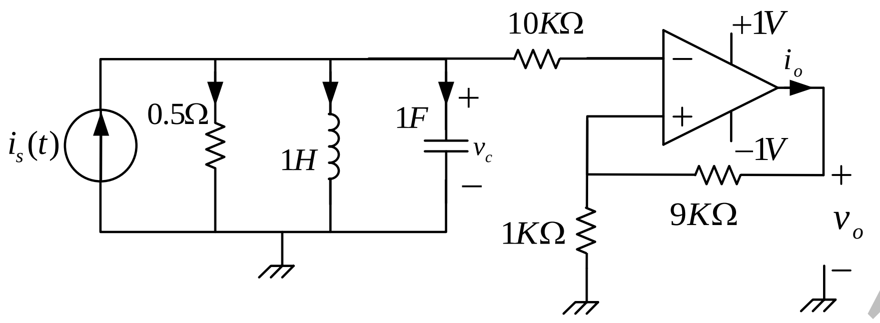

Figure 1: An op-amp circuit.

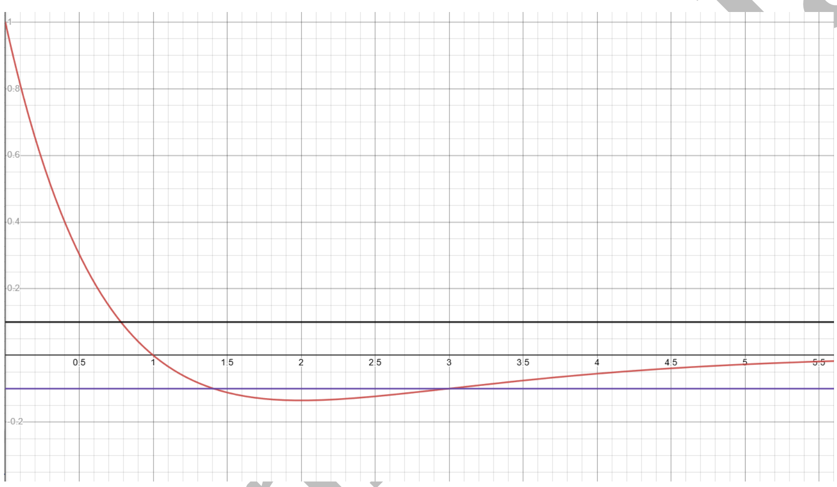

Figure 2: Capacitor voltage curve along with the horizontal lines ± 0.1.

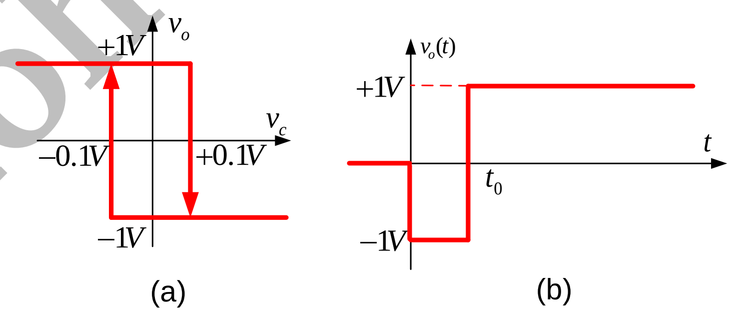

Figure 3: (a) Schmitt trigger characteristic curve. (b) Response of $v_o(t)$ versus time.

Question 1 continued on next page...

The differential equation of the parallel RLC circuit is obtained as

$$\frac{dv_c(t)}{dt} + \frac{v_c(t)}{0.5} + i_L(0^-) + \int_{0^-}^{t} v_c(\lambda)d\lambda = i_s(t)$$

$$\frac{d^2v_c(t)}{dt^2} + 2\frac{dv_c(t)}{dt} + v_c(t) = \frac{di_s(t)}{dt}$$

Considering $i_s(t) = \delta(t)$, $v_c(0^-) = 0$, and $i_L(0^-) = 0$, we have

$$\frac{d^2v_c(t)}{dt^2} + 2\frac{dv_c(t)}{dt} + v_c(t) = \delta'(t), \quad i_L(0^-) = 0, v_c(0^-) = 0$$

Since the differentiation order in the left hand side of the differential equation is higher than the right side,

$$s^2 + 2s + 1 = 0 \Rightarrow s_{1,2} = -1 \Rightarrow v_c(t) = (K_1e^{-t} + K_2te^{-t})u(t)$$

Substituting the response into the differential equation,

$$\big(-(K_2 - K_1)e^{-t} - K_2e^{-t} + K_2te^{-t}\big)u(t) + (K_2 - K_1)\delta(t) + K_1\delta'(t)$$

$$+2\big(-K_1e^{-t} + K_2e^{-t} - K_2te^{-t}\big)u(t) + 2K_1\delta(t) + \big(K_1e^{-t} + K_2te^{-t}\big)u(t) = \delta'(t)$$

$$\begin{cases} K_1 = 1 \\ K_2 + K_1 = 0 \Rightarrow K_2 = -1 \end{cases} \Rightarrow v_c(t) = (e^{-t} - te^{-t})u(t)$$

The capacitor voltage is plotted in Fig. 2. The op-amp part of the circuit is a Schmitt trigger with the characteristic curve shown in Fig. 3(a). The Schmitt trigger changes its output when the capacitor voltage is ascending and equal to $\frac{1}{1+9} \times 1 = 0.1$ V or when it is descending and equal to $-\frac{1}{1+9} \times 1 = -0.1$ V. According to the capacitor voltage curve in Fig. 2, the only possible time for output change is

$$h_{v_c}(t_0) = -0.1 \Rightarrow t_0 \approx 1.41$$

The Schmitt trigger output jumps to -1 at $t = 0^+$ since $v_c(0^+) = 1$. The output holds its -1 voltage until the time t_0, where a jump to $+1$ occurs. After that, no jump condition happens and the output voltage remains $+1$. The output response $v_o(t)$ is plotted in Fig. 3(b) and is described by

$$h_o(t) = -[u(t) - u(t - t_0)] + u(t - t_0) = 2u(t - t_0) - u(t) \approx 2u(t - 1.41) - u(t)$$