Question 1

Calculate the steady state average power consumed by the resistor in the series RC circuit shown in Fig. 1.

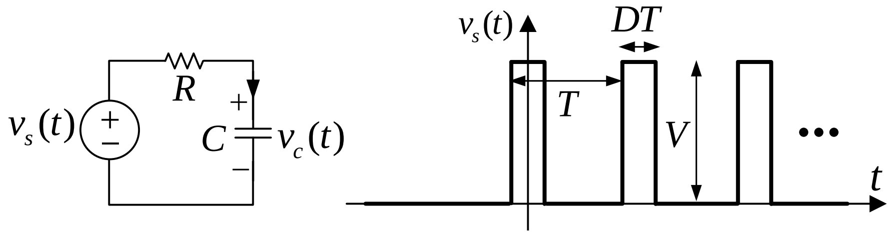

Figure 1: A series RC circuit.

The circuit is derived with $v_s(t) = v_p(t)u(t)$, where $v_p(t)$ is an even periodic pulse having the Fourier series

$$v_p(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\frac{2\pi}{T}t) = AD + \sum_{n=1}^{\infty} \frac{2A}{n\pi}\sin(n\pi D)\cos(n\frac{2\pi}{T}t)$$

So, the circuit is derived with a linear combination of the sinusoidal signals with different frequencies. For a sinusoidal signal $v_{sin}(t)$ with the frequency ω,

$$H(j\omega) = \frac{V_R}{V_{sin}} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{j\omega RC + 1}$$

Because the circuit is LTI, the resistor voltage equals

$$v_R(t) = a_0|H(j0)|\cos(0t + \angle H(j0)) + \sum_{n=1}^{\infty} a_n |H(jn\frac{2\pi}{T})|\cos(n\frac{2\pi}{T}t + \angle H(jn\tfrac{2\pi}{T}))$$

Since the sinusoidal components in the resistor voltage have different frequencies, the average power can be found using superposition as

$$P_{avR} = \frac{a_0^2|H(j0)|^2\cos^2(\angle H(j0))}{R} + \sum_{n=1}^{\infty}\frac{a_n^2|H(jn\frac{2\pi}{T})|^2}{2R}$$

$$P_{avR} = 0 + \sum_{n=1}^{\infty}\frac{a_n^2}{2R}\frac{R^2C^2(n\frac{2\pi}{T})^2}{1 + R^2C^2(n\frac{2\pi}{T})^2} = \sum_{n=1}^{\infty}\frac{8A^2RC^2\sin^2(n\pi D)}{T^2 + 4R^2C^2n^2\pi^2}$$