Simple Circuits

Mohammad Hadi

mohammad.hadi@sharif.edu

@MohammadHadiDastgerdi

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Simple Circuits

Circuit Types

Statement (Linear Circuit)

A linear circuit is a circuit that includes linear elements and/or independent sources.

Statement (LTI Circuit)

An LTI circuit is a circuit that includes LTI elements and/or independent sources.

Statement (Simple Circuit)

A simple circuit is a circuit that includes LTI resistors, linear dependent sources, and independent sources.

- Linear resistors, linear capacitors, linear inductors, linear dependent sources, ... are linear elements.

Definition (Circuit Variables)

Branch currents and branch voltages in a given circuit are called circuit variables.

Definition (Circuit Analysis)

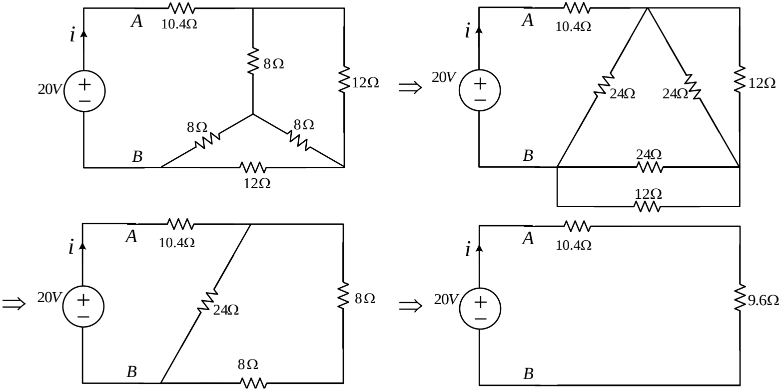
The circuit analysis problem is to determine all or part of the circuit variables for a circuit.

Interconnection Conversion

Interconnection Conversion

Example (Δ/Y/series/parallel conversions)

Interconnection conversions can be used to demonstrate that $i = 1$ A in the circuit below.



Voltage/Current Divider

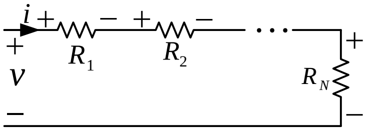


Figure: Resistive voltage divider circuit.

$$v_k = R_k i_k = R_k i = R_k \frac{v}{\sum_{k=1}^{N} R_k} = \frac{R_k}{\sum_{k=1}^{N} R_k} v$$

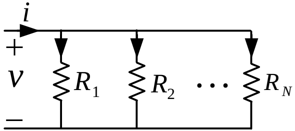


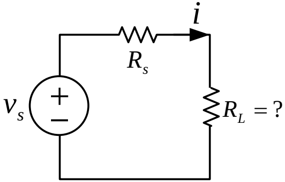
Figure: Resistive current divider circuit.

$$i_k = G_k v_k = G_k v = G_k \frac{i}{\sum_{k=1}^{N} G_k} = \frac{G_k}{\sum_{k=1}^{N} G_k} i$$

Maximum Power Transfer

Example (Maximum power transfer)

In the resistive circuit below, the maximum power is delivered to the load if $R_L = R_s$.



$$v_L = \frac{R_L}{R_s + R_L} v_s \Rightarrow P_L = \frac{v_L^2}{R_L} = \frac{R_L}{(R_s + R_L)^2} v_s^2$$

$$\frac{dP_L}{dR_L} = 0 \Rightarrow R_L = R_s, P_{L_{max}} = \frac{v_s^2}{4R_s}$$

Source Transformation

Source Transformation

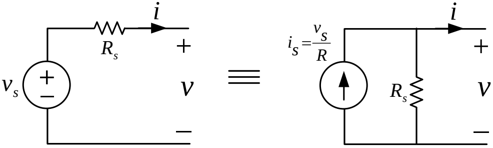


Figure: Independent source transformation.

$$v = v_s - Ri \Rightarrow i = \frac{v_s}{R} - \frac{v}{R}$$

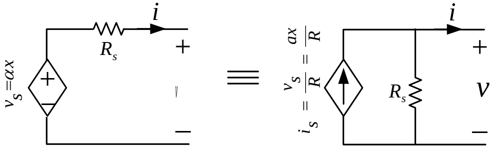
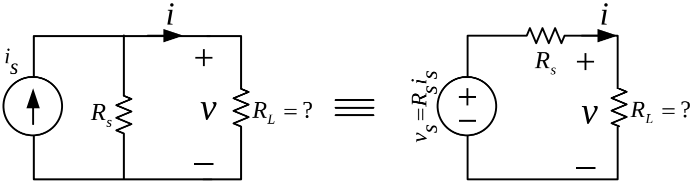


Figure: Dependent source transformation.

$$v = v_s - Ri \Rightarrow i = \alpha x - Ri \Rightarrow i = \frac{v_s}{R} - \frac{\alpha x}{R} = \frac{v_s}{R} - \frac{v_s}{R}$$

Example (Maximum power transfer)

In the resistive circuit below, the maximum power is delivered to the load if $R_L = R_s$.

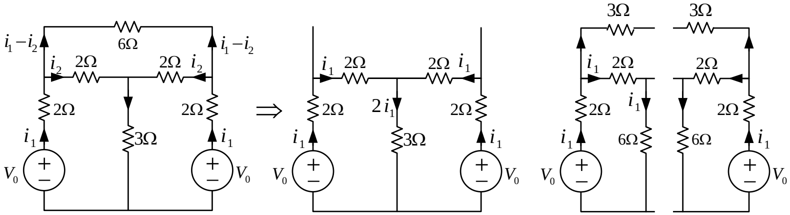


Circuit Symmetry

Circuit Symmetry

Example (Positive Circuit Symmetry)

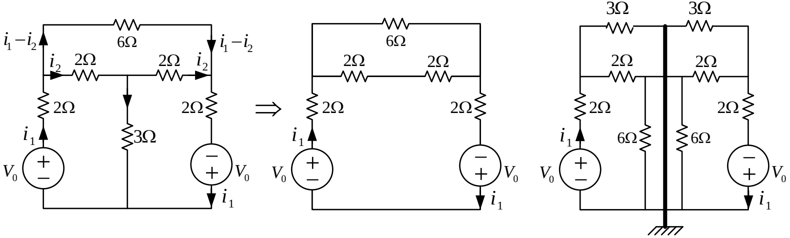
Circuit symmetry can facilitate circuit analysis.



$$i_1 = \frac{V_0}{2+2+6} = \frac{V_0}{10}$$

Example (Negative Circuit Symmetry)

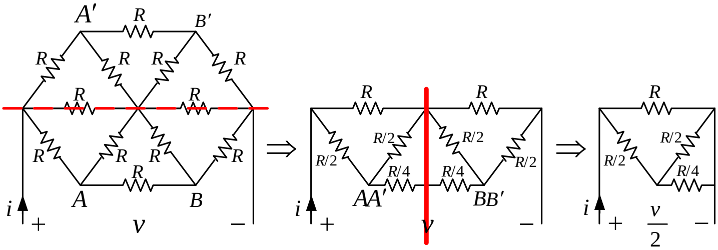
Circuit symmetry can facilitate circuit analysis.



$$i_1 = \frac{V_0}{2 + 2||3} = \frac{V_0}{3.2}$$

Example (Circuit Symmetry)

Circuit symmetry can facilitate circuit analysis.



$$\frac{v/2}{i} = (\frac{R}{4}||\frac{R}{2} + \frac{R}{2})||R = \frac{2}{5}R \Rightarrow R_{in} = \frac{v}{i} = \frac{4}{5}R$$

Node/Mesh Analysis

Example (Simple node analysis)

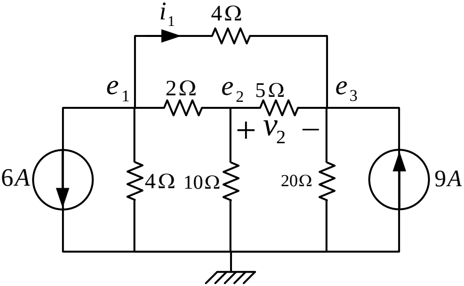
In the circuit below, $v_2 = -15$ V and $i_1 = -4.825$ A.

$$\begin{cases} 6 + \frac{e_1}{4} + \frac{e_1 - e_2}{2} + \frac{e_1 - e_3}{4} = 0 \\ \frac{e_2 - e_1}{2} + \frac{e_2}{10} + \frac{e_2 - e_3}{5} = 0 \\ \frac{e_3 - e_1}{4} + \frac{e_3 - e_2}{5} + \frac{e_3}{20} - 9 = 0 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1 & -0.5 & -0.25 \\ -0.5 & 0.8 & -0.2 \\ -0.25 & -0.2 & 0.5 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 9 \end{bmatrix}$$

$$\Rightarrow \begin{cases} e_1 = 4 \\ e_2 = 8.3 \\ e_3 = 23.3 \end{cases}$$

$$\Rightarrow \begin{cases} i_1 = \frac{e_1 - e_3}{4} = -4.825 \\ v_2 = e_2 - e_3 = -15 \end{cases}$$

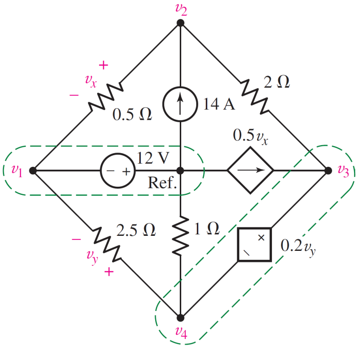


Example (Node analysis with supernode)

In the circuit below, $v_1 = -12$ V, $v_2 = -4$ V, $v_3 = 0$ V, and $v_4 = -2$ V.

$$\begin{cases} v_1 = -12 \\ v_3 - v_4 = 0.2v_y \\ \dfrac{v_1 - v_2}{0.5} + \dfrac{v_3 - v_2}{2} + 14 = 0 \\ \dfrac{v_1 - v_4}{2.5} + \dfrac{-v_4}{1} + \dfrac{v_2 - v_3}{2} + 0.5v_x = 0 \end{cases}$$

$$\Rightarrow \begin{cases} v_1 = -12 \\ v_3 - v_4 = 0.2v_4 - 0.2v_1 \\ \dfrac{v_1 - v_2}{0.5} + \dfrac{v_3 - v_2}{2} + 14 = 0 \\ \dfrac{v_1 - v_4}{2.5} + \dfrac{-v_4}{1} + \dfrac{v_2 - v_3}{2} + 0.5(v_2 - v_1) = 0 \end{cases}$$



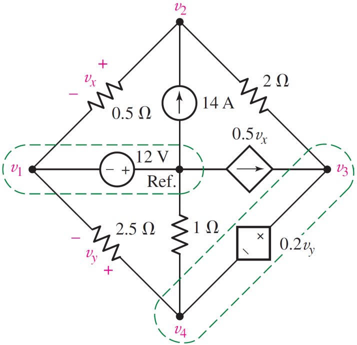
Example (Node analysis with supernode (cont.))

In the circuit below, $v_1 = -12$ V, $v_2 = -4$ V, $v_3 = 0$ V, and $v_4 = -2$ V.

$$\Rightarrow \begin{cases} -2v_1 + 2.5v_2 - 0.5v_3 = 14 \\ 0.1v_1 - v_2 + 0.5v_3 + 1.4v_4 = 0 \\ v_1 = -12 \\ 0.2v_1 + v_3 - 1.2v_4 = 0 \end{cases}$$

$$\Rightarrow \begin{bmatrix} -2 & 2.5 & -0.5 & 0 \\ 0.1 & -1 & 0.5 & 1.4 \\ 1 & 0 & 0 & 0 \\ 0.2 & 0 & 1 & -1.2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 14 \\ 0 \\ -12 \\ 0 \end{bmatrix}$$

$$\Rightarrow v_2 = \frac{\begin{vmatrix} -2 & 14 & -0.5 & 0 \\ 0.1 & 0 & 0.5 & 1.4 \\ 1 & -12 & 0 & 0 \\ 0 & 0 & 1 & -1.2 \end{vmatrix}}{\begin{vmatrix} -2 & 2.5 & -0.5 & 0 \\ 0.1 & -1 & 0.5 & 1.4 \\ 1 & 0 & 0 & 0 \\ 0.2 & 0 & 1 & -1.2 \end{vmatrix}} = -4$$



Node Analysis

Node analysis procedures:

1. Count the number of nodes (N nodes).
2. Designate a reference node (usually, a high-degree node).
3. Label the nodal voltages ($N-1$ labels).
4. Form a supernode about each voltage source and relate its voltage to nodal voltages.
5. Write a KCL equation for each nonreference node and for each supernode that does not contain the reference node. Use element equations to express the currents in terms of nodal voltages.
6. Express any additional unknowns in terms of appropriate nodal voltages (occurs for dependent sources).
7. Organize the equations.
8. Solve the system of equations for the nodal voltages ($N-1$ equations).

✓ Handy node analysis: appropriate the circuits with a low number of nodes.

Example (Simple mesh analysis)

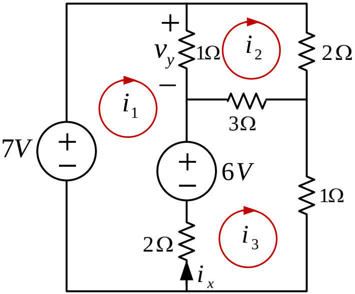
In the circuit below, $i_x = 0$ A and $v_y = 1$ V.

$$\begin{cases} -7 + 1(i_1 - i_2) + 6 + 2(i_1 - i_3) = 0 \\ 1(i_2 - i_1) + 2(i_2) + 3(i_2 - i_3) = 0 \\ 2(i_3 - i_1) - 6 + 3(i_3 - i_2) + 1(i_3) = 0 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

$$\Rightarrow \begin{cases} i_1 = 3 \\ i_2 = 2 \\ i_3 = 3 \end{cases}$$

$$\Rightarrow \begin{cases} i_x = i_3 - i_1 = 0 \\ v_y = 1(i_1 - i_2) = 1 \end{cases}$$



Example (Mesh analysis with supermesh)

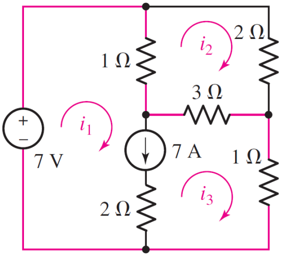
In the circuit below, $i_1 = 9$ A, $i_2 = 2.5$ A, and $i_3 = 2$ A.

$$\begin{cases} i_1 - i_3 = 7 \\ (i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0 \\ (i_1 - i_2) + 3(i_3 - i_2) + (i_3) - 7 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} i_1 \qquad - \; i_3 = 7 \\ -i_1 + 6i_2 - 3i_3 = 0 \\ i_1 - 4i_2 + 4i_3 = 7 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ -1 & 6 & -3 \\ 1 & -4 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \\ 7 \end{bmatrix}$$

$$\Rightarrow i_2 = \frac{\begin{vmatrix} 1 & 7 & -1 \\ -1 & 0 & -3 \\ 1 & 7 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & -1 \\ -1 & 6 & -3 \\ 1 & -4 & 4 \end{vmatrix}} = 2.5$$



Mesh analysis procedures:

1. Make sure that the circuit is planar.
2. Count the number of meshes (M meshes).
3. Label the mesh currents (M labels).
4. Form a supermesh to enclose the meshes shares a current source and relate its current to mesh currents.
5. Write a KVL equation around each mesh and supermesh. Use element equations to express the voltages in terms of mesh currents.
6. Express any additional unknowns in terms of appropriate mesh currents (occurs for dependent sources).
7. Organize the equations.
8. Solve the system of equations for the mesh currents (M equations).

✓ Handy mesh analysis: appropriate the for the planar circuits with a low number of meshes.

Thevenin/Norton Equivalent Circuits

Thevenin/Norton Equivalent Circuits

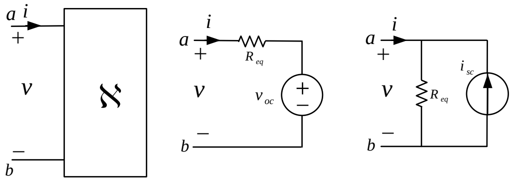


Figure: Any linear one-port can be replaced with its equivalent Thevenin or Norton one-port.

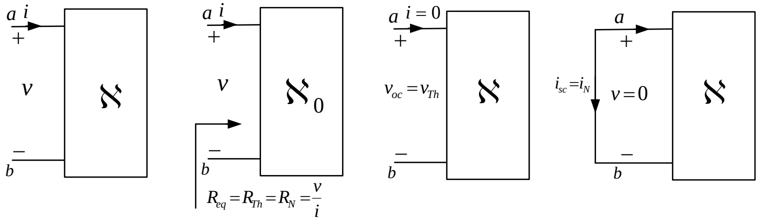


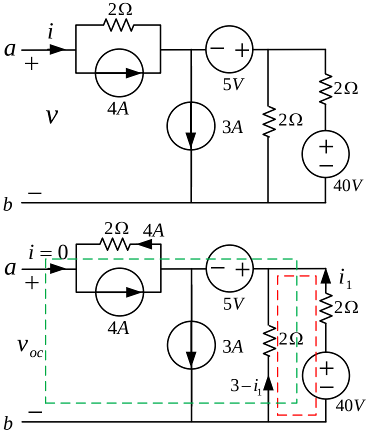
Figure: Open circuit voltage, short circuit current, and equivalent resistor are related by $v_{oc} = R_{eq} i_{sc}$.

Example (Thevenin/Norton equivalent circuits)

In the circuit below, $v_{oc} = 4$ V, $i_{sc} = \frac{4}{3}$ A, and $R_{eq} = 3\ \Omega$.

$$2(3 - i_1) = -40 + 2i_1 \Rightarrow i_1 = \frac{23}{2}$$

$$v_{oc} = -2(4) - 5 - 2i_1 + 40 = 4$$

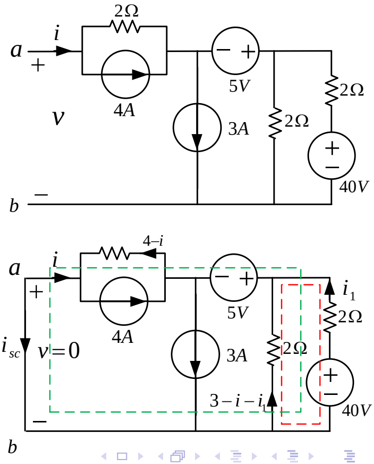


Example (Thevenin/Norton equivalent circuits (cont.))

In the circuit below, $v_{oc} = 4$ V, $i_{sc} = \frac{4}{3}$ A, and $R_{eq} = 3\ \Omega$.

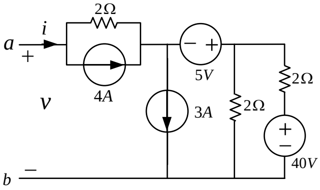
$$\begin{cases} 2(3 - i_1 - i) = -40 + 2i_1 \Rightarrow i_1 = \frac{23}{2} \\ -2(4 - i) - 5 - 2i_1 + 40 = 0 \end{cases}$$

$$\Rightarrow i = -\frac{4}{3} \Rightarrow i_{sc} = \frac{4}{3}$$

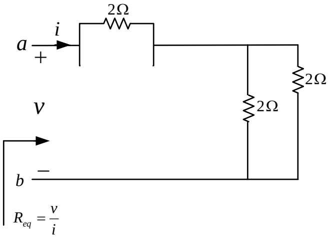


Example (Thevenin/Norton equivalent circuits (cont.))

In the circuit below, $v_{oc} = 4$ V, $i_{sc} = \frac{4}{3}$ A, and $R_{eq} = 3\ \Omega$.

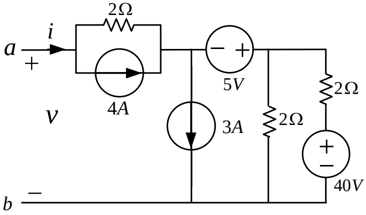


$$R_{eq} = 2 + (2||2) = 3$$

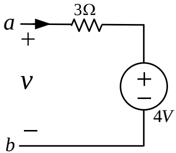


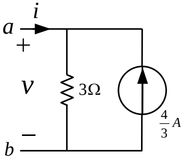
Example (Thevenin/Norton equivalent circuits (cont.))

In the circuit below, $v_{oc} = 4$ V, $i_{sc} = \frac{4}{3}$ A, and $R_{eq} = 3\ \Omega$.



$$v_{oc} = 4$$
$$i_{sc} = \frac{4}{3}$$
$$R_{eq} = 3$$





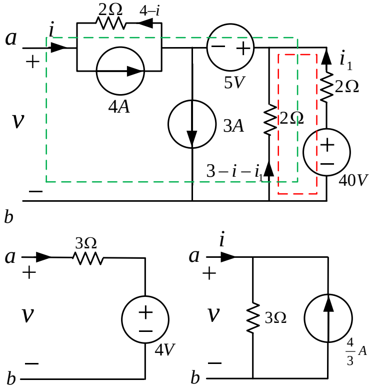
Example (Thevenin/Norton equivalent circuits)

In the circuit below, $v_{oc} = 4$ V, $i_{sc} = \frac{4}{3}$ A, and $R_{eq} = 3\ \Omega$.

$$2(3 - i_1 - i) = -40 + 2i_1 \Rightarrow i_1 = \frac{23 - i}{2}$$

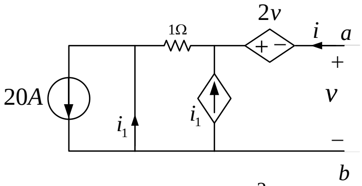
$$v = -2(4 - i) - 5 - 2i_1 + 40 = 3i + 4$$

$$i = \frac{1}{3}v - \frac{4}{3}$$



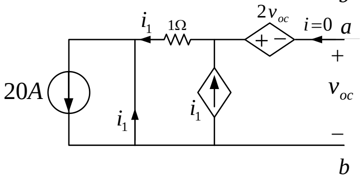
Example (Thevenin/Norton equivalent circuits)

In the circuit below, $v_{oc} = \frac{10}{3}$ V, $i_{sc} = 20$ A, and $R_{eq} = \frac{1}{6}$ Ω.



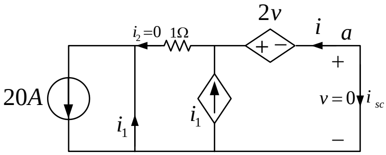
$$i_1 + i_1 = 20 \Rightarrow i_1 = 10$$

$$v_{oc} = -2v_{oc} + i_1 + 0 \Rightarrow v_{oc} = \frac{10}{3}$$



Example (Thevenin/Norton equivalent circuits (cont.))

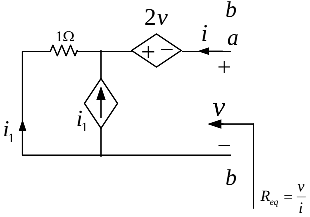
In the circuit below, $v_{oc} = \frac{10}{3}$ V, $i_{sc} = 20$ A, and $R_{eq} = \frac{1}{6}$ Ω.



$$i_1 = 20 \Rightarrow i_{sc} = i_1 = 20$$

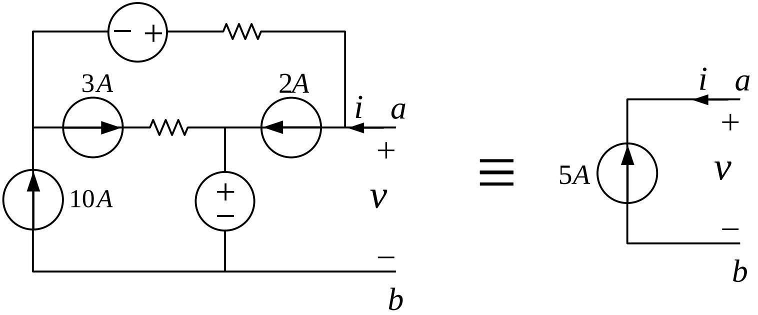
$$i_1 + i_1 + i = 0 \Rightarrow i_1 = \frac{i}{2}$$

$$v = -2v - i_1 = -2v + \frac{i}{2} \Rightarrow R_{eq} = \frac{v}{i} = \frac{1}{6}$$



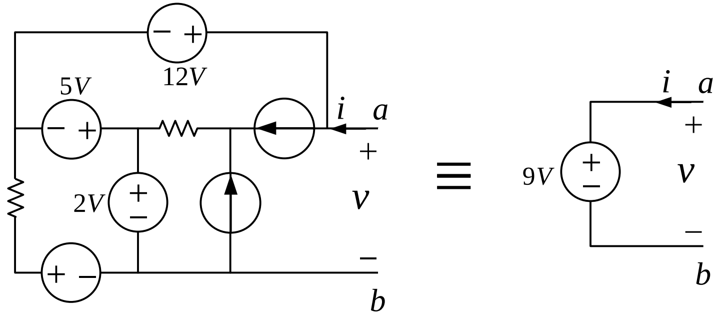
Example (Norton equivalent circuits)

For the circuit below, $i_{sc} = 5$ A and $R_{eq} = \infty$ Ω and therefore, only Norton equivalent circuit exists.



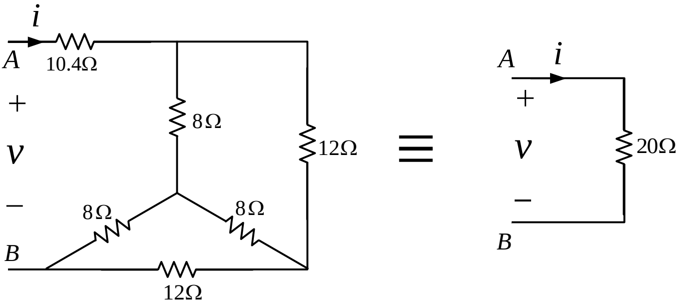
Example (Thevenin equivalent circuits)

For the circuit below, $v_{oc} = 9$ V and $R_{eq} = 0\ \Omega$ and therefore, only Thevenin equivalent circuit exists.



Example (Thevenin equivalent circuits)

For the circuit below, $R_{eq} = 20\ \Omega$ and therefore, the Thevenin/Norton equivalent circuit includes only a resistor.



Superposition

Superposition

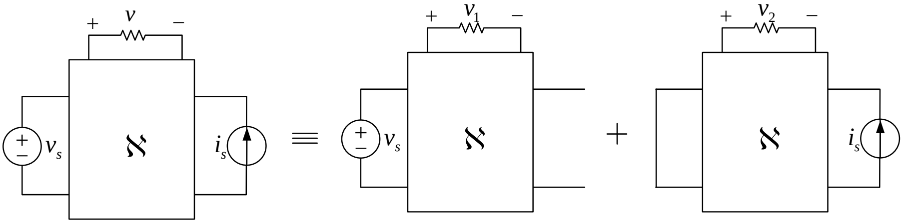


Figure: For any linear circuit, the voltage across or the current through any element may be calculated by adding algebraically all the individual voltages or currents caused by the separate independent sources acting alone, with all other independent voltage sources replaced by short circuits and all other independent current sources replaced by open circuits.

Example (Superposition)

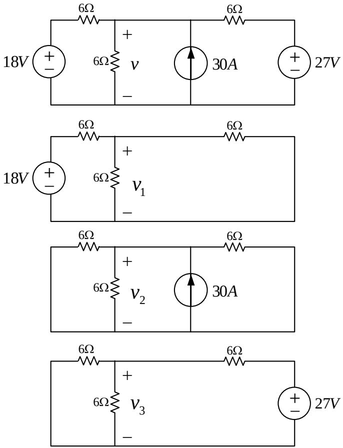
Superposition can facilitate circuit analysis.

$$v_1 = \frac{6||6}{6||6+6}18 = 6$$

$$iv_2 = 6\frac{1/6}{1/6+1/6+1/6}30 = 60$$

$$v_3 = \frac{6||6}{6||6+6}27 = 9$$

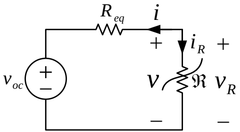
$$v = v_1 + v_2 + v_3 = 75$$

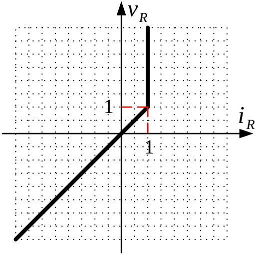


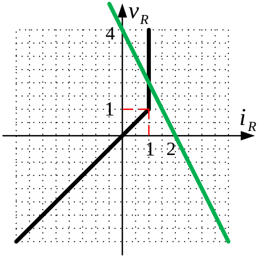
Small Signal Analysis

Example (Operating Point)

Thevenin equivalent circuit can facilitate analysis of a circuit with a single nonlinear element.







$$v_{oc} = 4, R_e q = 2 \Rightarrow v = V_{oc} + R_{eq} i = 4 + 2i$$

$$v = 2i + 4 \Rightarrow v_R = -2i_R + 4 \Rightarrow i_R = 1, v_R = 2$$

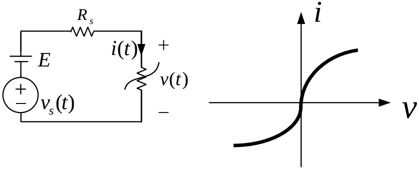
Example (Small-signal analysis)

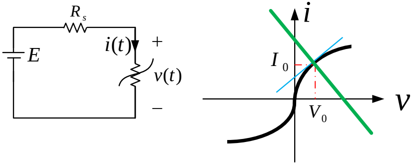
Circuits with nonlinear elements can be investigated using small-signal analysis.

$$|v_s(t)| \ll E$$

$$\begin{cases} v(t) = -R_s i(t) + E \\ i(t) = f(v(t)) \end{cases}$$

$$\Rightarrow \begin{cases} v(t) = V_0 = -R_s I_0 + E \\ i(t) = I_0 = f(V_0) \end{cases}$$





Small-signal Analysis

Example (Small-signal analysis (cont.))

Circuits with nonlinear elements can be investigated using small-signal analysis.

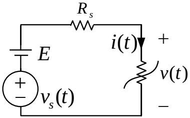
$|v_s(t)| \ll E$

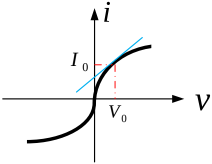
$$\begin{cases} V_0 + v_1(t) = -R_s(I_0 + i_1(t)) + E + v_s(t) \\ I_0 + i_1(t) = f(V_0 + v_1(t)) \end{cases}$$

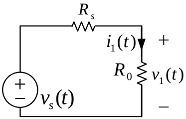
$$\Rightarrow \begin{cases} V_0 + v_1(t) = -R_s(I_0 + i_1(t)) + E + v_s(t) \\ I_0 + i_1(t) \approx f(V_0) + \frac{df}{dv}|_{(V_0,I_0)} v_1(t) \end{cases}$$

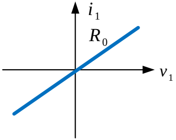
$$\Rightarrow \begin{cases} v_1(t) = -R_s i_1(t) + v_s(t) \\ i_1(t) \approx \frac{1}{R_0} v_1(t) \end{cases}$$

$$\Rightarrow \begin{cases} v_1(t) = -R_s i_1(t) + v_s(t) \\ i_1(t) \approx \frac{1}{R_0} v_1(t),\ G_0 = \frac{1}{R_0} = \frac{df}{dv}|_{(V_0,I_0)} \end{cases}$$









Circuits with Ideal Diodes

Circuits with Ideal Diodes

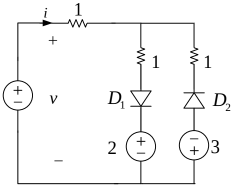
Example (Circuits with ideal diodes)

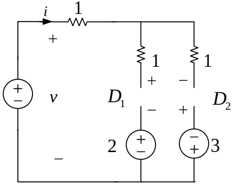
In circuits with ideal diodes, the circuit should be analyzed for various configurations of diodes.

$$i = 0$$

$$v_{D_1} = v - 2 \leq 0 \Rightarrow v \leq 2$$

$$v_{D_2} = -v - 3 \leq 0 \Rightarrow v \geq -3$$

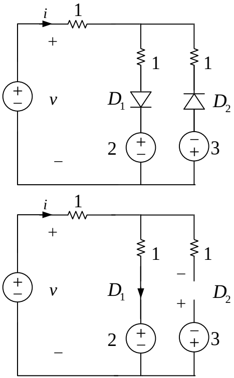




Example (Circuits with ideal diodes (cont.))

In circuits with ideal diodes, the circuit should be analyzed for various configurations of diodes.

$$i = \frac{v-2}{2}$$
$$i_{D_1} = i \geq 0 \Rightarrow v \geq 2$$
$$v_{D_2} = -v - 3 + \frac{v-2}{2} \leq 0 \Rightarrow v \geq -8$$



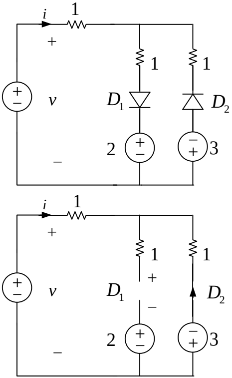
Example (Circuits with ideal diodes (cont.))

In circuits with ideal diodes, the circuit should be analyzed for various configurations of diodes.

$$i = \frac{v+3}{2}$$

$$v_{D_1} = v - 2 - \frac{v+3}{2} \leq 0 \Rightarrow v \leq 7$$

$$i_{D_2} = -i \geq 0 \Rightarrow v \leq -3$$



Circuits with Ideal Diodes

Example (Circuits with ideal diodes (cont.))

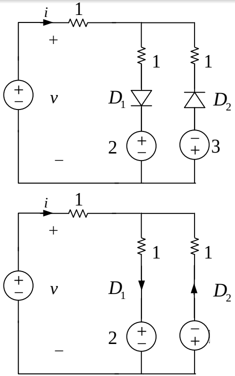
In circuits with ideal diodes, the circuit should be analyzed for various configurations of diodes.

$$\frac{e-v}{1}+\frac{e-2}{1}+\frac{e+3}{1}=0 \Rightarrow e=\frac{v-1}{3}$$

$$i=\frac{v-e}{1}=\frac{2v+1}{3}$$

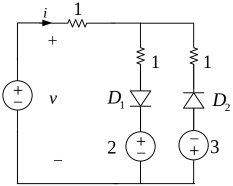
$$i_{D_1}=e-2=\frac{v-7}{3}\geq 0 \Rightarrow v\geq 7$$

$$i_{D_2}=-(e+3)=-\frac{v+8}{3}\geq 0 \Rightarrow v\leq -8$$

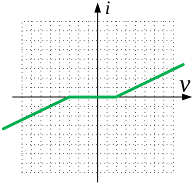


Example (Circuits with ideal diodes (cont.))

In circuits with ideal diodes, the circuit should be analyzed for various configurations of diodes.



$$i = \begin{cases} \frac{v+3}{2}, & v \leq -3 \\ 0, & -3 < v \leq 2 \\ \frac{v-2}{2}, & v > 2 \end{cases}$$



The End