MATHEMATICAL QUESTIONS

Question 1

Solve the nonlinear Schrodinger equation

$$\frac{\partial \mathcal{A}}{\partial z} + \frac{j\beta_2}{2}\frac{\partial^2 \mathcal{A}}{\partial t^2} + \frac{\alpha}{2}\mathcal{A} - j\gamma|\mathcal{A}|^2\mathcal{A} = 0$$

for the special case of $\beta_2 = 0$, $\alpha \neq 0$, and $\gamma \neq 0$.

Question 2

Consider the graded-index cylindrical optical fiber shown in Fig. 1 with the refractive index profile

$$n(r) = \begin{cases} n_1\sqrt{1-(\frac{r}{a})^p\Delta}, & r \leq a \\ n_2, & r > a \end{cases}$$

, where $\Delta = \frac{n_1-n_2}{n_1} \approx \frac{n_1^2-n_2^2}{2n_1^2}$ is the relative refractive index, a is the fiber core radius, and p is the grade profile parameter. Such a fiber can be analyzed using the so-called approximate WKB (Wentzel-Kramers-Brillouin) technique.

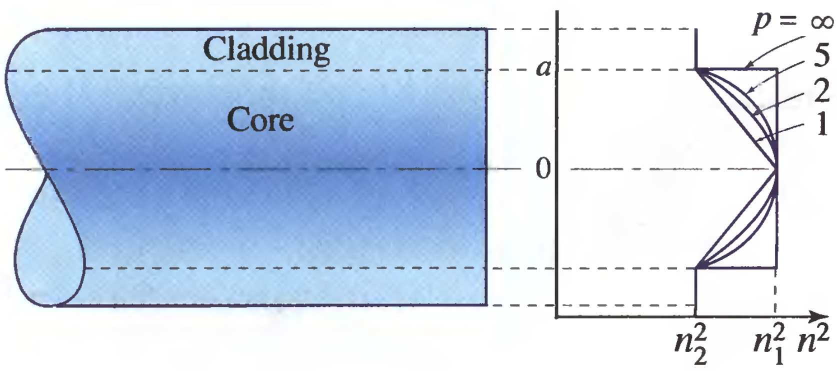

Figure 1: Graded-index cylindrical optical fiber with power-law refractive-index profile.

(a) Determine the approximated number of modes propagated in this fiber.

(b) Determine the group velocity of each propagated mode. Ignore material dispersion.

Question 2 continued on next page...

(c) Considering the modal dispersion, determine the approximated upper-bound of the rate-distance product.

SOFTWARE QUESTIONS

Question 3

Consider the sample optical network of Fig. 2 and assume that the its topology is described by directional graph $G(N, L)$, where each link $l = (b, e) \in L$ begins at node $b \in N$, ends at node $e \in N$, and has length W_l km. There are $|R|$ traffic requests, where request $r = (s, d) \in R$ originates from source node $S(r) = s \in N$, terminates at destination node $D(r) = d \in N$, and requires transmission rate B_r. Switch, transmitter, and receiver connectors have the losses α_s, α_t, and α_r, respectively, while fiber loss equals α dB/km. The transmitters have linewidth σ_λ and provide injected power P_t for each request. The receiver power sensitivity is P_r. The dominant dispersion in the fibers is material dispersion with the coefficient D_λ. At most C requests can simultaneously use a fiber link using wavelength-division multiplexing. Let $x_{l,r} = 1$ be a binary decision variable that equals 1 if the request r passes through link l, and 0 otherwise.

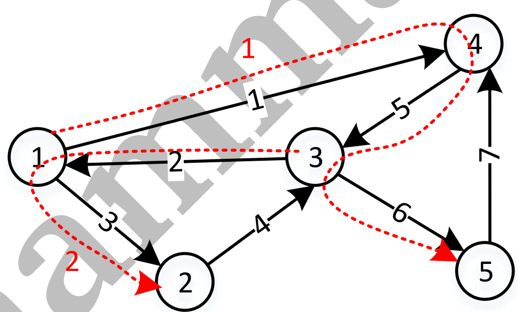

Figure 2: A sample optical network.

(a) Write a binary linear resource allocation optimization problem to route traffic requests subject to the satisfaction of attenuation, dispersion, and capacity constraints.

(b) Write a MATLAB/Python code to solve the formulated optimization problem for the describe system model.

Question 3 continued on next page. . .

(c) Use the developed code to solve the resource allocation optimization problem for several sample network topologies and traffic requests.

(d) What does happen if the optimization problem is infeasible? How do you handle such a situation?

BONUS QUESTIONS

Question 4

Improve the resource allocation formulation in Question 3 such that the lowest number of requests are blocked to guarantee the feasibility of the resource allocation.

Question 5

Return your answers by filling the LaTeXtemplate of the assignment.