MATHEMATICAL QUESTIONS

Question 1

Consider the forward Kolmogrov equation

$$\frac{dp(n,t)}{dt} = [(n-1)a + c]p(n-1,t) + [(n+1)b]p(n+1,t) - [n(a+b) + c]p(n,t)$$

and show that it leads to the equivalent equation

$$\frac{\partial \Phi(z,t)}{\partial t} = (z-1)\big[(az-b)\big]\frac{\partial \Phi(z,t)}{\partial z} + c\Phi(z,t)\big]$$

in terms of the probability generating function $\Phi(z,t) = \sum_{n=-\infty}^{\infty} p(n,t)z^n$.

Question 2

Consider the three-level pumping system shown in Fig. 1. Write the rate equations and find the population difference $N = N_2 - N_1$. Assume that $\tau_{32} \ll \tau_{31}$ so that the pumping serves to populate level 2. Further, the thermally excited population of level 2 is assumed to be negligible. Moreover, since τ_{32} is very short, level 3 retains a negligible steady-state population and therefore, all of the atoms that are raised to it immediately decay to level 2. Thus, the totoal atomic density is $N_a \approx N_1 + N_2$. In addition, nonradiative decay from level 2 to level 1 is assumed negligible, i.e., $t_{sp} \ll \tau_{nr}$. Finally, let the pumping involve a transition between the ground state and level 3 with transition probability W.

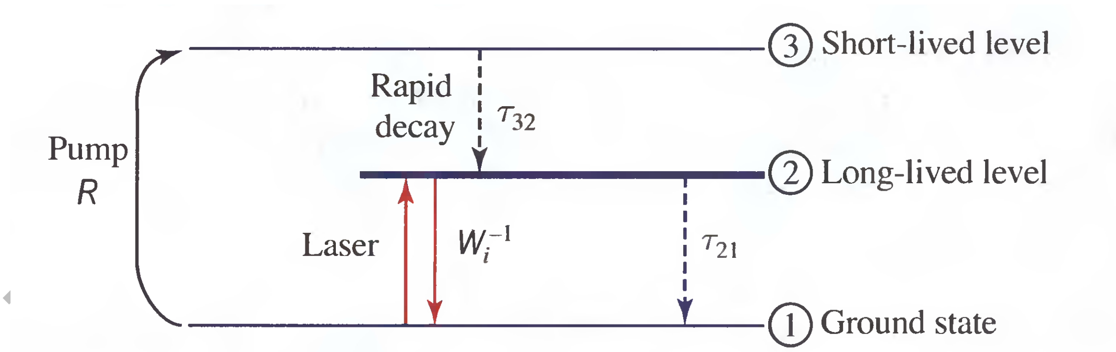

Figure 1: 3-level pumping schemes.

Question 3 continued on next page. . .

Question 3

Consider the Kolmogrov equation

$$\frac{\partial \Phi(z,t)}{\partial t} = (z-1)\big[(az-b)\big]\frac{\partial \Phi(z,t)}{\partial z} + c\Phi(z,t)\big]$$

with the birth, death, and immigration rates a, b, and c, respectively.

(a) Find $\Phi(z,t)$ for the zero immigration rate and initial condition $\Phi(z,0) = z$.

(b) Find $\Phi(z,t)$ for the initial condition $\Phi(z,0) = z$.

(c) Find $\Phi(z,t)$ for the initial condition $\Phi(z,0) = z^l$.

SOFTWARE QUESTIONS

Question 4

Consider the sample optical network of Fig. 2 and assume that the its topology is described by directional graph $G(N,L)$, where each link $l = (b,e) \in L$ begins at node $b \in N$, ends at node $e \in N$, and has length W_l km. Link l has $\lceil \frac{W_l}{K_s} \rceil + 1$ optical amplifiers to compensate for fiber and switching attenuation, where L_s is the span length. There are $|R|$ traffic requests, where request $r = (s,d) \in R$ originates from source node $S(r) = s \in N$, terminates at destination node $D(r) = d \in N$, and requires transmission rate B_r. The transmitters have linewidth σ_λ. The dominant dispersion in the fibers is material dispersion with the coefficient D_λ. At most C requests can simultaneously use a fiber link using wavelength-division multiplexing. Let $x_{l,r} = 1$ be a binary decision variable that equals 1 if the request r passes through link l, and 0 otherwise. Further, assume that the binary variable $a_l = 1$ if at least a request is routed over link l.

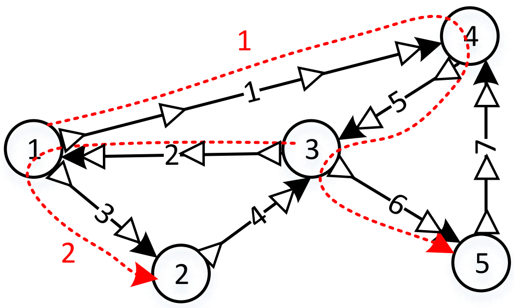

Figure 2: A sample optical network.

(a) Write a binary linear resource allocation optimization problem to route traffic requests while using the lowest possible number of optical amplifiers subject to the satisfaction of dispersion constraint.

(b) Write a MATLAB/Python code to solve the formulated optimization problem for the describe system model.

(c) Use the developed code to solve the resource allocation optimization problem for several sample network topologies and traffic requests.

(d) What does happen if the optimization problem is infeasible? How do you handle such a situation?

BONUS QUESTIONS

Question 5

Improve the resource allocation formulation in Question 4 such that the lowest number of requests are blocked to guarantee the feasibility of the resource allocation.

Question 6

Return your answers by filling the LaTeXtemplate of the assignment.