MATHEMATICAL QUESTIONS

Question 1

Consider an optical transmission system with intensity modulation at a data rate of 5 Gb/s. The average optical signal optical power emitted from the transmitter is $P_{tx} = 6$ dBm, and the loss coefficient of the optical fiber is $\alpha = 0.25$ dB/km. Chromatic dispersion introduces waveform distortion which can be approximated by $\nu_0 = 2 \times 10^{-6}L$ and $\nu_1 = 1 - 2 \times 10^{-6}L$, where v_0 and v_1 are the upper and lower levels of the normalized eye diagram, and L is the fiber length in meters. In the direct detection receiver, a PIN photodiode is used which has a responsivity $\mathcal{R} = 1$ A/W, a load resistance $R_L = 50\,\Omega$, and an electrical bandwidth $B_e = 5$ GHz. Thermal noise is considered as the major noise in the receiver.

(a) Find the maximum fiber length that is allowed for $BER \leq 10^{-12}$ in this system. Numerical solution may be used to solve the final equation.

(b) If chromatic dispersion can be completely compensated so the $\nu_1 = 1$ and $\nu_0 = 0$, what will be the maximum transmission fiber length?

(c) Now, assume that an optical preamplifier is added in front of the photodiode with 6 dB noise figure and 30 dB optical gain. Signal-ASE beat noise is considered as the major noise in the receiver. Operating wavelength is 1550 nm. Find the maximum fiber length that is allowed for $BER \leq 10^{-12}$. If chromatic dispersion can be completely compensated, what will be the maximum transmission fiber length?

Question 2

An optical system has N in-line optical amplifiers each with 6 dB noise figure, and the gain of each optical amplifier exactly compensates the loss of the optical fiber span immediately before it. Assume all the fiber spans have the same length L and the fiber loss coefficient is $\alpha = 0.25$ dB/km. The signal optical power that enters the first fiber span is $P_t = 1$ mW.

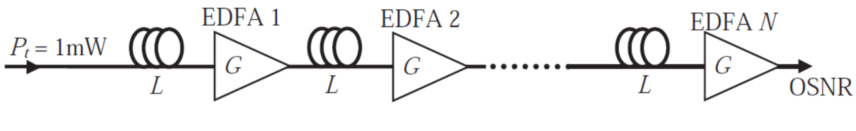

Figure 1: Inline amplification in an optical transmission line.

Question 2 continued on next page...

(a) Derive an expression for optical signal-to-noise ratio (OSNR) at the system output as the function of fiber span length L.

(b) If the total fiber length of the system is $L_T = 1500$ km and $L = L_T/N$, plot OSNR (in logarithm scale) as the function of the number of fiber spans N for $3 < N < 50$.

(c) For an engineering design point of view, discuss how to select the fiber span length between adjacent in-line amplifiers.

Question 3

Consider light reflected at an angle θ from M parallel reflecting planes separated by a distance Λ, as shown in Fig. 2. Assume that only a small fraction of the light is reflected from each plane, so that the amplitudes of the M reflected waves are approximately equal. Show that the reflected waves have a phase difference $\phi = k(2\Lambda \sin(\theta))$ and that the angle θ at which the intensity of the total reflected light is maximum satisfies $2\Lambda \sin(\theta) = \lambda$.

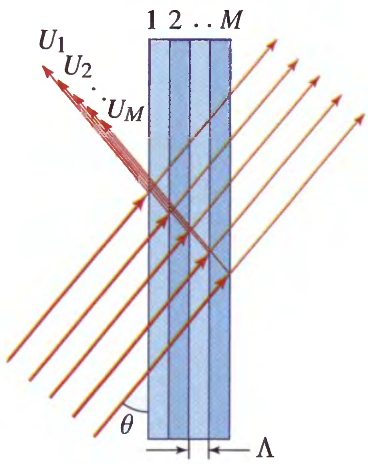

Figure 2: Reflection of a plane wave from M parallel planes separated from each other by a distance Λ.

Question 4 continued on next page...

Question 4

Derive an expression for the Fraunhofer diffraction pattern for an aperture $p(x, y)$ made of $M = 2L + 1$ parallel slits of infinitesimal widths separated by equal distances a as

$$p(x, y) = \sum_{m=-L}^{L} \delta(x - ma)$$

Sketch the pattern as a function of the observation angle $\theta = x/d$, where d is the observation distance. How this aperture can be used as a diffraction grating separating wavelength components of an incident white light?

SOFTWARE QUESTIONS

Question 5

Consider the sample optical network of Fig. 3 and assume that the its topology is described by directional graph $G(N, L)$, where each link $l = (b, e) \in L$ begins at node $b \in N$, ends at node $e \in N$, and has length W_l km. There are $|R|$ traffic requests, where request $r = (s, d) \in R$ originates from source node $S(r) = s \in N$, terminates at destination node $D(r) = d \in N$, and requires fixed transmission rate $B_r = 10$ Gb/s. Switch, transmitter, and receiver connectors have losses α_s, α_t, and α_r, respectively, while fiber loss equals α dB/km. The 10-Gb/s intensity-modulating transmitters provide injected power P_t for each request. The 10-Gb/s direct detection receivers use simple binary modulation without pre-amplification and have electrical bandwidth $B_e = 7.5$ GHz, photodiode responsivity $\mathcal{R} = 0.85$ mA/mW, load resistance $R_L = 50\Omega$, dark current $I_d = 5$nA, working temperature $T = 300$ K, and target BER 10^{-12}. The distortion in the received signal is negligible and the thermal is the dominant source of noise. At most C requests can simultaneously use a fiber link using wavelength-division multiplexing. Let $x_{l,r} = 1$ be a binary decision variable that equals 1 if the request r passes through link l, and 0 otherwise.

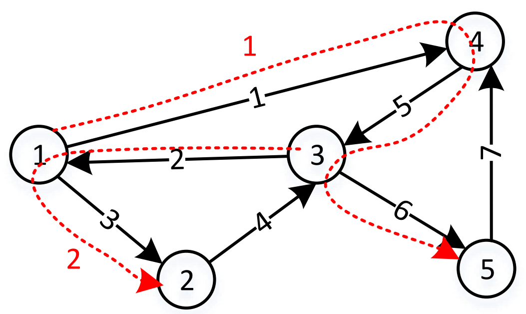

Figure 3: A sample optical network.

Question 5 continued on next page...

(a) Write a resource allocation optimization problem to route traffic requests subject to the satisfaction of BER and capacity constraints.

(b) Write a MATLAB/Python code to solve the formulated optimization problem for the describe system model. You might use YALMIP in MATLAB or Pyomo in Python.

(c) Use the developed code to solve the resource allocation optimization problem for several sample network topologies and traffic requests.

(d) What would happen if the optimization problem is infeasible? How do you handle such a situation?

BONUS QUESTIONS

Question 6

Improve the resource allocation formulation in Question 5 such that the lowest number of requests are blocked to guarantee the feasibility of the resource allocation.

Question 7

Return your answers by filling the LaTeXtemplate of the assignment.