BER Calculations

Mohammad Hadi

mohammad.hadi@sharif.edu

@MohammadHadiDastgerdi

Fall 2021

Overview

System Model

Binary IMDD

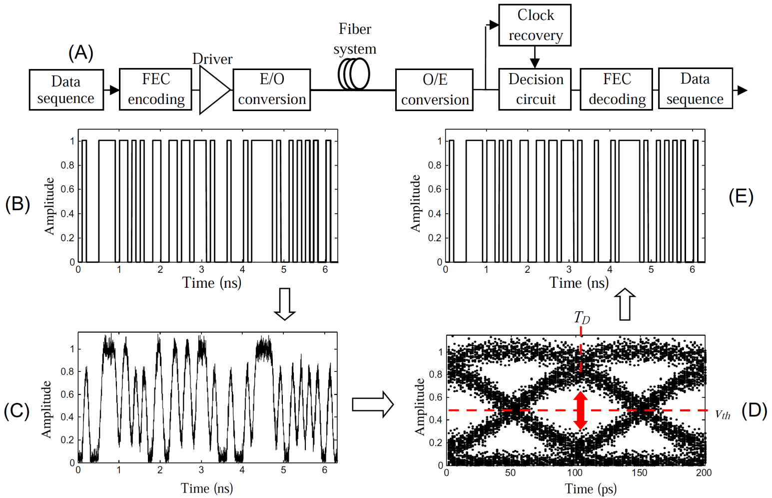

Figure: Schematic diagram of data flow through the components of an IMDD link. (A) Block diagram of an optical transmission system, (B) ideal binary data sequence, (C) distorted signal waveform after transmission, (D) eye-diagram of the received signal, and (E) recovered data sequence after the decision circuit.

Binary IMDD

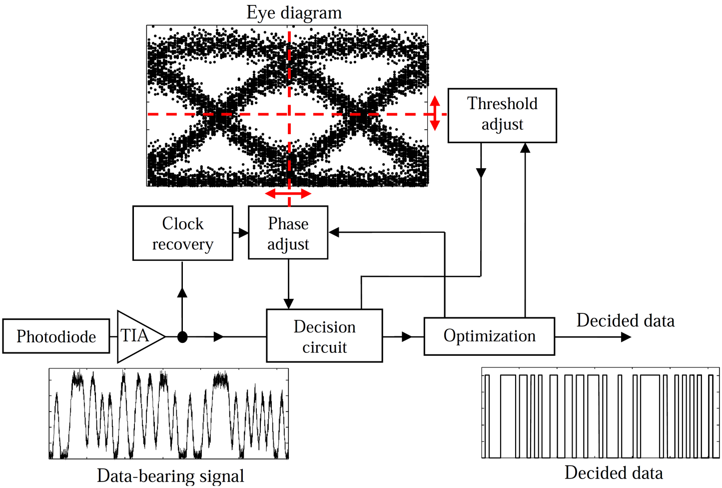

Figure: Illustration of bit decision in a binary receiver.

BER Performance

BER

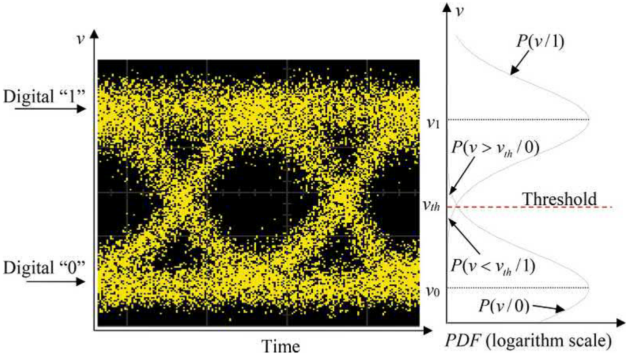

Figure: Probability distribution function (PDF) of the eye diagram.

- Bit error rate: $BER = P(0)P(v > v_{th}|0) + P(1)P(v > v_{th}|1) = 0.5[P(v > v_{th}|0) + P(v < v_{th}|1)]$
- Misdetected 0 bit: $P(v > v_{th}|0) = \frac{1}{\sqrt{2\pi\sigma_0^2}} \int_{v_0}^{\infty} \exp\left(-\frac{(v-v_0)^2}{2\sigma_0^2}\right) dv = \frac{1}{\sqrt{2\pi}} \int_{Q_0}^{\infty} \exp\left(-\frac{\zeta^2}{2}\right) d\zeta$
- Misdetected 1 bit: $P(v < v_{th}|1) = \frac{1}{\sqrt{2\pi\sigma_1^2}} \int_{-\infty}^{v_1} \exp\left(-\frac{(v-v_1)^2}{2\sigma_1^2}\right) dv = \frac{1}{\sqrt{2\pi}} \int_{Q_1}^{\infty} \exp\left(-\frac{\zeta^2}{2}\right) d\zeta$
- Bit error rate: $BER = 0.25[\mathrm{erfc}(\frac{Q_0}{\sqrt{2}}) + \mathrm{erfc}(\frac{Q_1}{\sqrt{2}})]$, $Q_0 = \frac{v_{th} - v_0}{\sigma_0}$, $Q_1 = \frac{v_1 - v_{th}}{\sigma_1}$

Q-value

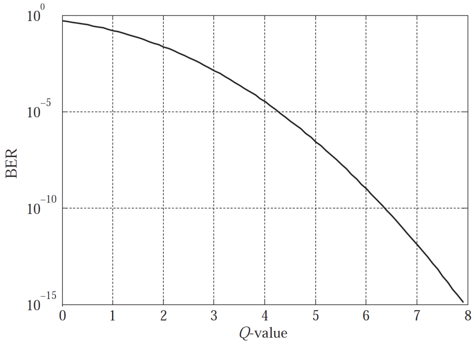

Figure: BER as a function of receiver Q-value. As a rule of thumb, $Q = 6$, 7, and 8 correspond to the BER of approximately 10^{-9}, 10^{-12}, and 10^{-15}, respectively.

- Optimum threshold: $v_{th} = \frac{v_0\sigma_1 + v_1\sigma_0}{\sigma_0 + \sigma_1}$
- Minimum bit error rate: $BER = 0.5\text{erfc}(\frac{Q}{\sqrt{2}})$
- Q-value: $Q = \frac{v_1 - v_0}{\sigma_0 + \sigma_1}$

Q-value

- Q-value: $Q = \frac{v_1 - v_0}{\sigma_0 + \sigma_1} = \frac{2M\mathcal{R}P_{ave}}{\sqrt{(\sigma_{th}^2 + \sigma_{sh}^2 + \sigma_{dk}^2 + \sigma_{S-ASE}^2 + \sigma_{ASE-ASE}^2 + \sigma_{RIN}^2)B_e} + \sqrt{(\sigma_{th}^2 + \sigma_{dk}^2 + \sigma_{ASE-ASE}^2)B_e}}$
- Signal 1 power level: P_1
- Signal 0 power level: $P_0 = 0$
- Average power: $P_{ave} = 0.5[P_1 + P_0]$
- Photodiode gain: $M > 1$ for avalanche PD and $M = 1$ for PID PD
- Photodiode responsivity: $\mathcal{R}$
- Electric bandwidth: B_e

Q-value

- Thermal noise PSD: $\sigma_{th}^2 = \frac{4kT}{R_L}$
  - Load resistance: R_L
  - Boltzmann's constant: k
  - Absolute temperature: T
- Dark current PSD: $\sigma_{dk}^2 = M^2 F_M 2qI_{dk}$
  - Electron charge: q
  - Photodiode dark current: I_{dk}
  - Photodiode noise figure: $F_M > 1$ for avalanche PD and $F_M = 1$ for PID PD
  - Photodiode gain: $M > 1$ for avalanche PD and $M = 1$ for PID PD
- Shot noise PSD: $\sigma_{sh}^2 = M^2 F_M 2q\mathcal{R}P_1$
  - Electron charge: q
  - Photodiode responsivity: $\mathcal{R}$
  - Signal 1 power level: P_1
  - Photodiode noise figure: $F_M > 1$ for avalanche PD and $F_M = 1$ for PID PD
  - Photodiode gain: $M > 1$ for avalanche PD and $M = 1$ for PID PD

Q-value

- Signal-ASE beat noise PSD: $\sigma^2_{S-ASE} = M^2 F_M 2\mathcal{R}^2 P_1 \rho_{ASE}$
    - Accumulated ASE noise at the photodiode input: ρ_{ASE}
    - Photodiode responsivity: $\mathcal{R}$
    - Signal 1 power level: P_1
    - Photodiode noise figure: $F_M > 1$ for avalanche PD and $F_M = 1$ for PID PD
    - Photodiode gain: $M > 1$ for avalanche PD and $M = 1$ for PID PD
- ASE-ASE beat noise PSD: $\sigma^2_{ASE-ASE} \approx M^2 F_M \mathcal{R}^2 \rho^2_{ASE} B_o/2$
    - Accumulated ASE noise at the photodiode input: ρ_{ASE}
    - Photodiode responsivity: $\mathcal{R}$
    - Optical bandwidth: B_o
    - Photodiode noise figure: $F_M > 1$ for avalanche PD and $F_M = 1$ for PID PD
    - Photodiode gain: $M > 1$ for avalanche PD and $M = 1$ for PID PD
- Relative intensity noise PSD: $\sigma^2_{RIN} = M^2 F_M 2\mathcal{R}^2 P_1^2 RIN$
    - Relative intensity noise of the laser source: RIN
    - Photodiode responsivity: $\mathcal{R}$
    - Signal 1 power level: P_1
    - Photodiode noise figure: $F_M > 1$ for avalanche PD and $F_M = 1$ for PID PD
    - Photodiode gain: $M > 1$ for avalanche PD and $M = 1$ for PID PD

RIN

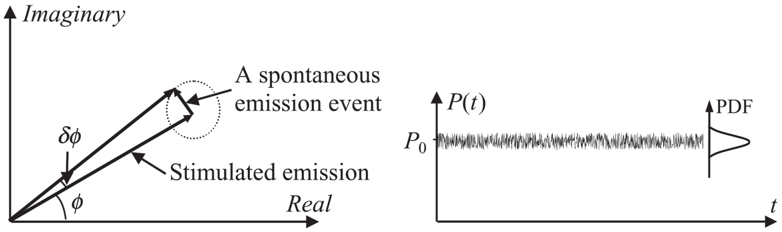

Figure: Optical intensity and optical phase experience random noisy variations due to spontaneous emission events.

- Relative intensity noise PSD: $\sigma^2_{RIN} = M^2 F_M 2\mathcal{R}^2 P_1 RIN$

ASE Noise in Electrical Domain

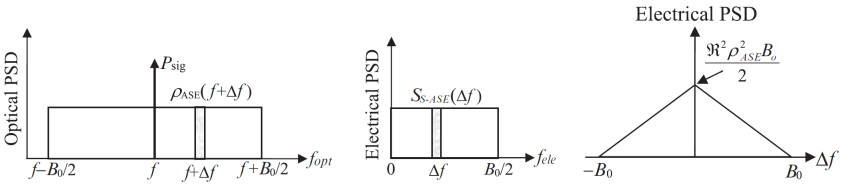

Figure: One-sided PSDs in the process of S-ASE and ASE-ASE beat noise generation.

- Photodiode square-law detection:
  $I = \mathcal{R}P = \mathcal{R}|E_{sig} + E_{noise}|^2 = \mathcal{R}|E_{sig}|^2 + \mathcal{R}2\,\mathrm{Re}\{E_{sig}E^*_{noise}\} + \mathcal{R}|E_{noise}|^2$
- Photodiode responsivity: $\mathcal{R} = \frac{\eta q \lambda}{hc}$
- Shot noise: $\sigma^2_{sh} = 2q\mathcal{R}(P_{sig} + P_{noise}) \approx 2q\mathcal{R}P_{sig}$
- ASE noise power: $P_{noise} \approx \rho_{ASE}B_o = 2n_{sp}h\nu(G-1)B_o$
- Signal-ASE beat noise PSD: $\sigma^2_{S-ASE} \approx 2\mathcal{R}^2 P_{sig}\rho_{ASE}$
- ASE-ASE beat noise PSD: $\sigma^2_{ASE-ASE} \approx \mathcal{R}^2\rho^2_{ASE}B_o/2$

Distortion

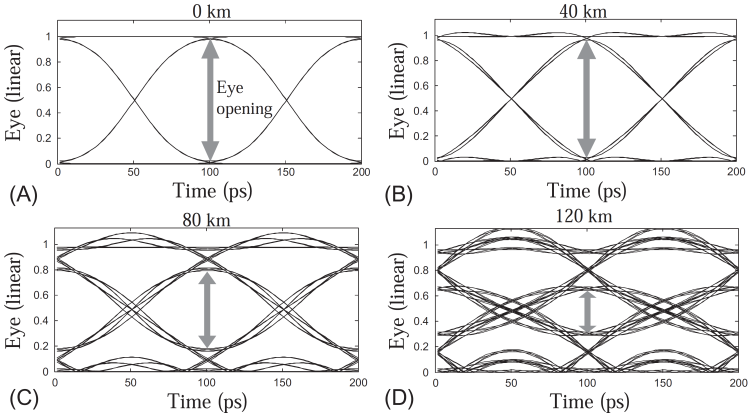

Figure: Eye diagrams of received 10 Gb/s binary optical signal propagating through single mode fiber with lengths of 0 km (A), 40 km (B), 80 km (C), and 120 km (D). The eye diagram is distorted due to linear impairments.

Distortion

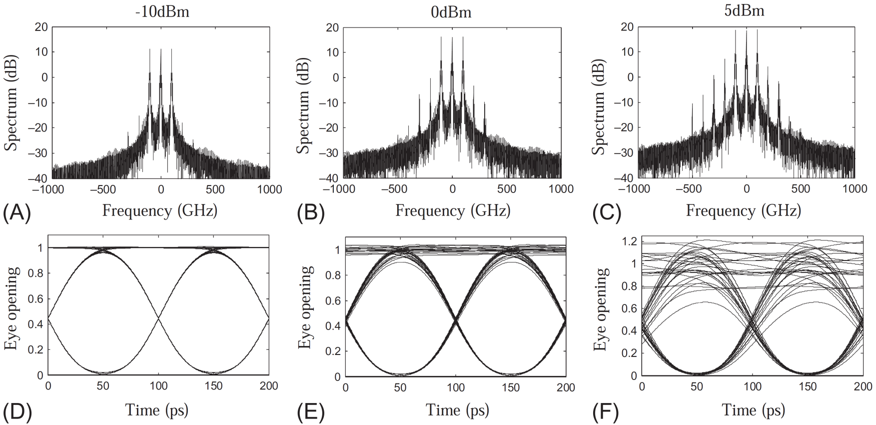

Figure: Optical spectra (top row) and eye diagrams (bottom row) of a three-channel WDM system with per-channel average powers of -10 dBm [(A) and (D)], 0 dBm [(B) and (E)], and 5 dBm [(C) and (F)] at the input of 80 km standard single-mode fiber. The eye diagram is distorted due to nonlinear impairments.

Distortion

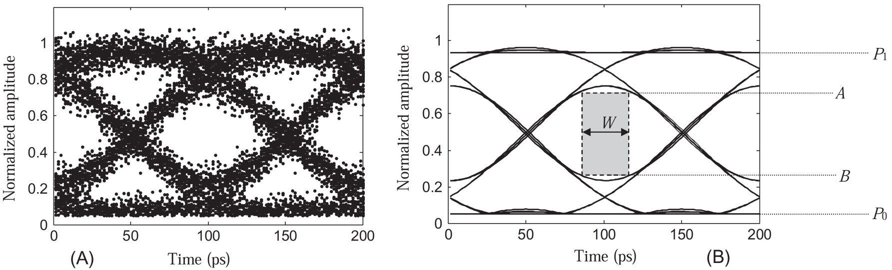

Figure: Normalized eye distortion mask mapped onto a measured eye diagram with (A) and without (B) the random noise. Sources of BER degradation can be categorized by deterministic distortion and stochastic noise. Waveform distortion can be caused by linear distortion, nonlinear impairments.

- Sampling instant phase uncertainty:

  $W \approx 7 \times$ standard deviation of sampling instant phase uncertainty
- Long 1 signal power level: P_1
- Long 0 signal power level: P_0
- Lowest inner upper eye: A
- Highest inner lower eye: B
- Average power: $P_{ave} = 0.5(P_0 + P_1)$

Distortion

- Q-value: $Q = \frac{(A-B)2\mathcal{R}P_{ave}}{\sqrt{(\sigma_{ind}^2+\zeta A2P_{ave})B_e}+\sqrt{(\sigma_{ind}^2+\zeta B2P_{ave})B_e}}$
- Independent noise PSD: $\sigma_{ind}^2 = \sigma_{th}^2 + \sigma_{dk}^2 + \sigma_{ASE-ASE}^2$
- Signal-dependent noise factor (negligible RIN): $\zeta = 2\mathcal{R}M^2F_M(q+\mathcal{R}\rho_{ASE})$
- Distortion-less Q-value: $Q_0 = \frac{2\mathcal{R}P_{ave}}{\sqrt{(\sigma_{ind}^2+\zeta 2P_{ave})B_e}+\sqrt{\sigma_{ind}^2 B_e}}$
- Q-value degradation: $D(A,B,x) = \frac{Q}{Q_0} = \frac{A-B}{Y_e(A,B,x)}, x = \frac{2\zeta P_{ave}}{\sigma_{ind}^2}$
- Degradation function: $Y_e(A,B,x) = \frac{\sqrt{1+xA}+\sqrt{1+xB}}{1+\sqrt{1+x}}$
- Worst-case Q-value degradation: $D_{wc} = \frac{A-B}{Y_{max}} = \begin{cases} \sqrt{A}-\sqrt{B}, & \sqrt{A}+\sqrt{B} \geq 1 \\ A-B, & \sqrt{A}+\sqrt{B} < 1 \end{cases}$
- Dominant signal-independent noise: $Y_e(A,B,0) = 1 \equiv D(A,B,0) = A-B$
- Dominant signal-dependent noise: $Y_e(A,B,\infty) = \sqrt{A}+\sqrt{B} \equiv D(A,B,\infty) = \sqrt{A}-\sqrt{B}$

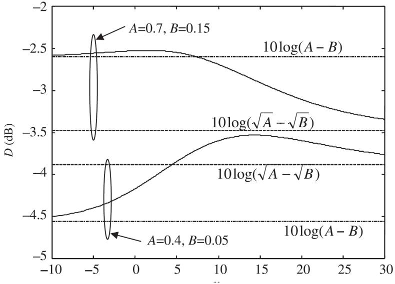

Figure: Q-degradation parameter D as a function of x.

- Q-value degradation: $D(A,B,x) = \frac{Q}{Q_0} = \frac{A-B}{Y_e(A,B,x)}, x = \frac{2\zeta P_{ave}}{\sigma^2_{ind}}$
- Dominant signal-independent noise: $Y_e(A,B,x) \approx 1 \equiv D(A,B,x) \approx A - B$
- Dominant signal-dependent noise: $Y_e(A,B,x) \approx \sqrt{A} + \sqrt{B} \equiv D(A,B,x) \approx \sqrt{A} - \sqrt{B}$
- Worst-case Q-value degradation: $D_{wc} = \frac{A-B}{Y_{max}} = \min\{\sqrt{A} - \sqrt{B}, A - B\}$

Receiver Sensitivity

DD Receiver

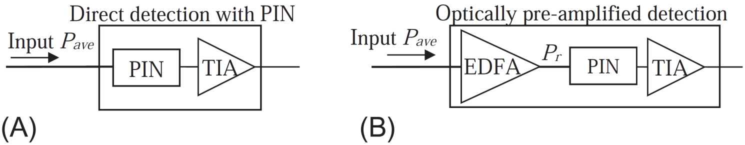

Figure: Direct detection receivers with (A) and without (B) optical preamplifier. In a relatively short distance optical fiber system without in-line optical amplifiers, transmission quality can be guaranteed as long as the signal optical power is high enough, and thus receiver sensitivity is the most relevant measure of the system performance.

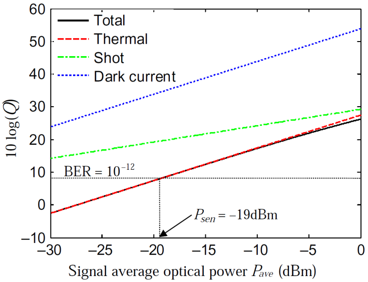

Figure: Receiver sensitivity plot (continuous line) for a 10-Gb/s system without preamplification having $B_e = 7.5$ GHz, $\mathcal{R} = 0.85$ mA/mW, $R_L = 50\Omega$, $I_d = 5$nA, and $T = 300$ K. Here, the sensitivity of the 10Gb/s receiver is -19dBm for a BER of 10^{-12}.

- Distortion-less Q-value: $Q = \frac{2\mathcal{R}P_{ave}}{\sqrt{(4kT/R_L + 4q\mathcal{R}P_{ave} + 2qI_{dk})B_e} + \sqrt{(4kT/R_L + 2qI_{dk})B_e}}$
- Thermal noise-dominant Q-value: $Q = \sqrt{\frac{R_L}{4kTB_e}}\mathcal{R}(A-B)P_{ave}$
- Thermal noise-dominant sensitivity: $P_{sen} = \frac{Q}{\mathcal{R}(A-B)}\sqrt{\frac{4kTB_e}{R_L}}$

DD Receiver with Preamplifier

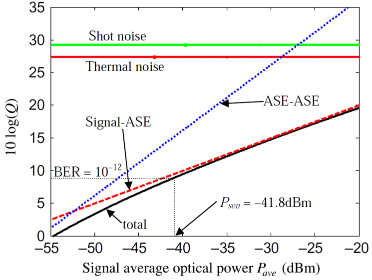

Figure: Receiver sensitivity plot (continuous line) for a 10-Gb/s system with preamplification having $B_e = 7.5$ GHz, $\mathcal{R} = 0.85$ mA/mW, $R_L = 50\Omega$, $I_d = 5$nA, $T = 300$ K, $P_r = 0$ dBm, $B_o = 25$ GHz, and $\lambda = 1550$ nm. Here, the sensitivity of the 10Gb/s receiver is -41.8 dBm for a BER of 10^{-12}.

- Amplified signal average optical power: $P_r = GP_{ave}$
- Distortion-less Q-value:

$$Q = \frac{2\mathcal{R}P_r}{\sqrt{(4kT/R_L + 2q(2\mathcal{R}P_r + I_{dk}) + 4\rho_{ASE}\mathcal{R}^2P_r + \rho_{ASE}^2\mathcal{R}^2B_o/2)B_e} + \sqrt{(4kT/R_L + 2qI_{dk} + \rho_{ASE}^2\mathcal{R}^2B_o/2)B_e}}$$

- Signal-ASE noise-dominant Q-value: $Q \approx \sqrt{\frac{P_{ave}}{2n_{sp}\frac{hc}{\lambda}B_e}}(\sqrt{A} - \sqrt{B})$
- Signal-ASE noise-dominant sensitivity: $P_{sen} = \frac{Q^2 2n_{sp}\frac{hc}{\lambda}B_e}{(\sqrt{A} - \sqrt{B})^2}$

Required OSNR

IMDD with In-line Optical Amplification

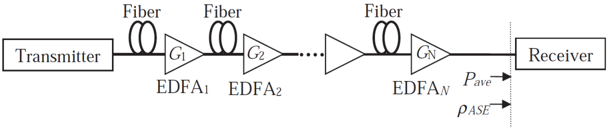

Figure: Fiber-optic transmission system with N optically amplified fiber spans. In a long distance optical fiber system with in-line optical amplifiers, the performance is no longer limited by the signal optical power that reaches the receiver; rather, it is limited by the OSNR.

1. Accumulated ASE noise PSD: $\rho_{ASE} = \sum_{i=1}^{N} \rho_{ASE,i} = \sum_{i=1}^{N} 2n_{2p}(hc/\lambda)(G_i - 1)$
2. Optical SNR: $OSNR = \frac{P_{ave}}{\rho_{ASE}}$

DD Receiver with Preamplifier

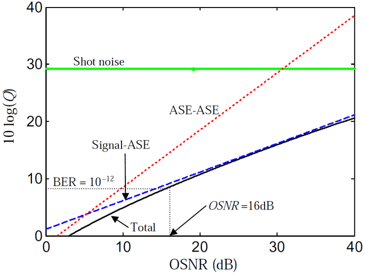

Figure: Q-value as the function of signal OSNR (curve marked with total) considering contributions from shot noise, ASE-ASE beat noise, and signal-ASE beat noise. Dark current and thermal noises are negligible since the received power is high. Here, $B_o = 25$ GHz, $B_e = 7.5$ GHz, $\lambda = 1550$ nm, $P_{ave} = 0$ dBm, $\mathcal{R} = 0.85$ mA/mW.

- Distortion-less Q-value: $Q = \frac{2\mathcal{R}P_{ave}}{\sqrt{(4\mathcal{R}(q+\rho_{ASE}\mathcal{R})P_{ave}+\rho_{ASE}^2\mathcal{R}^2B_o/2)B_e}+\sqrt{\rho_{ASE}^2\mathcal{R}^2B_o/2B_e}}$
- Distortion-less Q-value: $Q = \frac{2\mathcal{R}.OSNR}{\sqrt{(4\mathcal{R}q.OSNR^2/P_{ave}+4\mathcal{R}^2.OSNR+\mathcal{R}^2B_o/2)B_e}+\sqrt{\mathcal{R}^2B_o/2B_e}}$
- Signal-ASE noise-dominant Q-value: $Q \approx \sqrt{\frac{P_{ave}}{\rho_{ASE}B_e}}(\sqrt{A}-\sqrt{B}) = \frac{\sqrt{A}-\sqrt{B}}{\sqrt{B_e}}\sqrt{OSNR}$
- Required OSNR: $ROSNR = \frac{Q^2B_e}{(\sqrt{A}-\sqrt{B})^2}$

Inline Amplification Distance

Accumulated ASE Noise

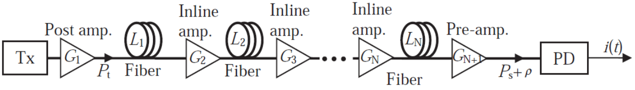

Figure: Fiber-optic transmission system with multiple optical amplifiers.

1. Span length: l_i
2. Span attenuation: L_i
3. Amplifier gain: G_i
4. Noise figure parameter: $n_{sp,i}$
5. Accumulated ASE noise PSD:
   $\rho_{ASE} = 2h\nu \sum_{i=1}^{N} \left[n_{sp,i}(G_i - 1) \prod_{m=i}^{N} G_{m+1} L_m \right] + 2h\nu n_{sp,N+1}(G_{N+1} - 1)$
6. Same span and amplifiers: $\rho_{ASE} \approx h\nu n_{sp}(N+1)G,\ G = L = e^{\alpha l}$

Optimal Amplifier Distance

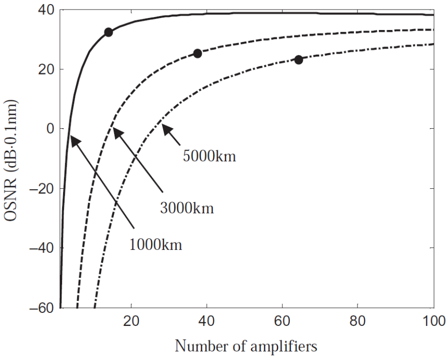

Figure: OSNR as the function of the number of fiber spans for the total system length of 1000km (solid line), 3000km (dashed line), and 5000km (dash-dotted line). The solid dot on each curve indicates where the length of each fiber span is 80km. Here, $n_{sp} = 1.58$, $\alpha = 0.25$ dB/km, and $P_t = 1$ mW.

1. OSNR: $OSNR = \frac{P_t}{2h\nu n_{sp}(N+1)G} = \frac{P_t}{2h\nu n_{sp}(N+1)10^{\frac{L_{tot}}{10(N+1)}}}$

Modulation Level

M-ary IMDD

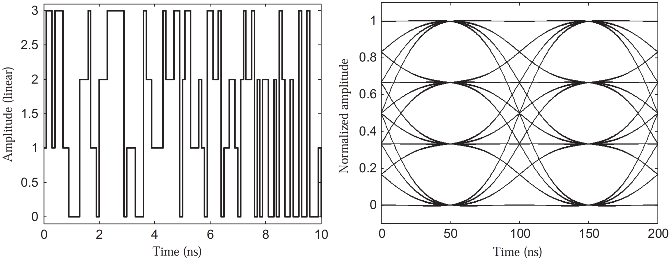

Figure: (A) ideal waveform of a four-level amplitude modulated signal at 10Gbaud/s rate and (B) the corresponding typical eye diagram.

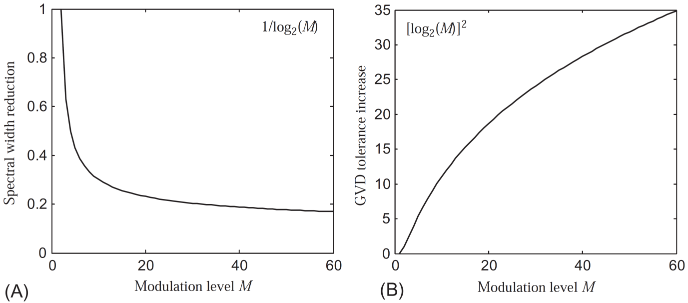

Figure: (A) reduction of spectral bandwidth, and (B) increase of group-velocity-dispersion (GVD) tolerance as the function of modulation level M.

- Required rate: R_b
- Binary transmission required bandwidth: R_b
- Binary transmission rate distance product: $R_b z \lesssim \frac{1}{|D_\nu|\sigma_\nu} \equiv z \lesssim \frac{1}{|D_\nu|R_b^2}$
- M-ary transmission required bandwidth: $\frac{R_b}{\log_2(M)}$
- M-ary transmission rate distance product: $z \lesssim \frac{1}{|D_\nu|(\frac{R_b}{\log_2(M)})^2} = \frac{1}{|D_\nu|R_b^2}(\log_2(M))^2$

M-ary IMDD

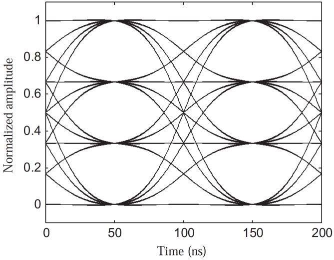

Figure: Typical eye diagram of a four-level amplitude modulated signal.

- Top eyelid level: $A = \frac{M-n}{M-1}, n = 1, \cdots, M-1$
- Down eyelid level: $B = \frac{M-n-1}{M-1}, n = 1, \cdots, M-1$

M-ary IMDD

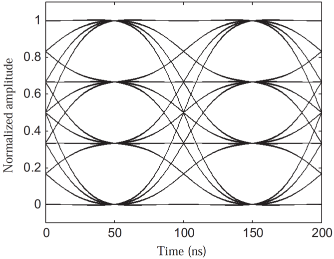

Figure: Typical eye diagram of a four-level amplitude modulated signal.

- Signal-independent noise-dominant sensitivity: $P_{sen} \propto \frac{\sqrt{B_e}}{A-B}$
- Signal-independent noise-dominant sensitivity penalty: $P_{pen} = \frac{\sqrt{\frac{1}{\log_2(M)}}}{\frac{M-n}{M-1} - \frac{M-n-1}{M-1}} = \frac{M-1}{\sqrt{\log_2(M)}}$

M-ary IMDD

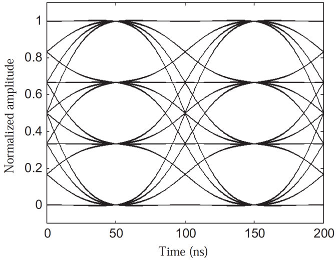

Figure: Typical eye diagram of a four-level amplitude modulated signal.

- Signal-dependent noise-dominant sensitivity: $P_{sen} \propto \frac{B_e}{(\sqrt{A}-\sqrt{B})^2}$
- Signal-dependent noise-dominant sensitivity penalty: $p_{pen} = \frac{\frac{1}{\log_2(M)}}{(\sqrt{\frac{M-n}{M-1}}-\sqrt{\frac{M-n-1}{M-1}})^2}$
- Worst-case signal-dependent noise-dominant sensitivity penalty: $p_{pen} \leq \frac{1}{\log_2(M)(1-\sqrt{\frac{M-2}{M-1}})^2}$

M-ary IMDD

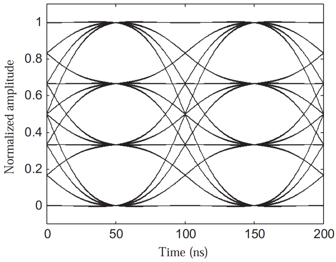

Figure: Typical eye diagram of a four-level amplitude modulated signal.

- Required OSNR: $ROSNR \propto \frac{B_e}{(\sqrt{A}-\sqrt{B})^2}$
- Required OSNR penalty: $ROSNR_{pen} = \frac{\frac{1}{\log_2(M)}}{(\sqrt{\frac{M-n}{M-1}}-\sqrt{\frac{M-n-1}{M-1}})^2}$
- Required OSNR penalty: $ROSNR_{pen} \leq \frac{1}{\log_2(M)(1-\sqrt{\frac{M-2}{M-1}})^2}$

The End