Optical Fiber

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Overview

Preliminaries

Polarized Plane Wave

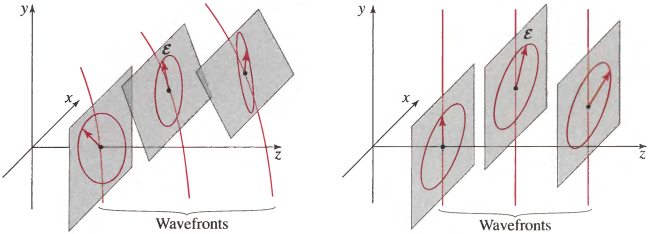


Figure: Time course of the electric field vector for a monochromatic arbitrary wave and for a monochromatic plane wave or a monochromatic paraxial wave traveling in the z direction.

- Polarized monochromatic plane wave:
  $\boldsymbol{\mathcal{E}}(z,t) = \mathrm{Re}\{\boldsymbol{A} e^{-j\frac{\omega z}{c}} e^{j\omega t}\} = \mathcal{E}_x \hat{\mathbf{x}} + \mathcal{E}_y \hat{\mathbf{y}}$
- Complex envelope: $\boldsymbol{A} = A_x \hat{\mathbf{x}} + A_y \hat{\mathbf{y}} = a_x e^{j\phi_x} \hat{\mathbf{x}} + a_y e^{j\phi_y} \hat{\mathbf{y}}$
- Intensity: $I = (|A_x|^2 + |A_y|^2)/(2\eta) \propto |A_x|^2 + |A_y|^2$
- x component: $\mathcal{E}_x = a_x \cos\big(\omega(t - \frac{z}{c}) + \phi_x\big)$
- y component: $\mathcal{E}_y = a_y \cos\big(\omega(t - \frac{z}{c}) + \phi_y\big)$
- Polarization elliptic: $(\frac{\mathcal{E}_x}{a_x})^2 + (\frac{\mathcal{E}_y}{a_y})^2 - 2\frac{\mathcal{E}_x \mathcal{E}_y}{a_x a_y}\cos(\phi_y - \phi_x) = \sin^2(\phi_y - \phi_x)$

Polarized Plane Wave

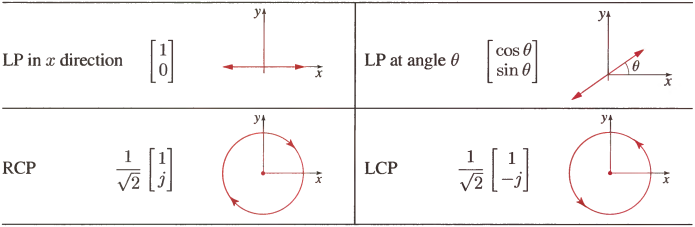


Figure: Jones vectors of linearly polarized (LP) and right- and left-circularly polarized (RCP,LCP) light.

- Jones vector representation: $\boldsymbol{J} = \begin{bmatrix}A_x\\A_y\end{bmatrix}$
- Orthogonal polarization: $(\boldsymbol{J}_1, \boldsymbol{J}_2) = A_{1x}A_{2x}^* + A_{1y}A_{2y}^* = 0$
- Superposition of two orthogonal polarizations: $\boldsymbol{J} = \alpha_1\boldsymbol{J}_1 + \alpha_2\boldsymbol{J}_2$

Polarized Plane Wave

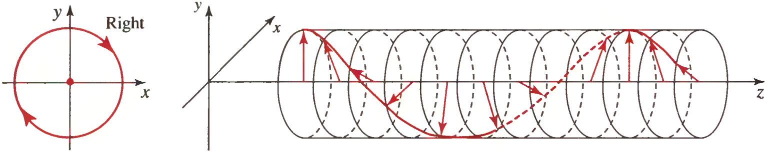
Example (Jones representation for RCP)

Jones representation can fully describe a plane wave with RCP.

$$A_x = a, \quad A_y = ae^{j\frac{\pi}{2}}, \quad \boldsymbol{J} = \begin{bmatrix} a \\ ae^{j\frac{\pi}{2}} \end{bmatrix}$$

$$\boldsymbol{\mathcal{E}}(z,t) = a\cos\left(\omega(t - \frac{z}{c})\right)\hat{\boldsymbol{x}} + a\cos\left(\omega(t - \frac{z}{c}) + \frac{\pi}{2}\right)\hat{\boldsymbol{y}}$$

$$(\frac{\mathcal{E}_x}{a})^2 + (\frac{\mathcal{E}_y}{a})^2 = 1$$



Polarized Plane Wave

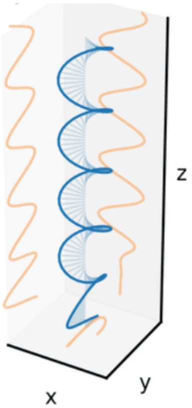
Example (Orthogonality of horizontal and vertical LPs)

Horizontal and vertical LPs are orthogonal and can be used to represent other polarization.

$$\boldsymbol{J}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{J}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\boldsymbol{J}_1, \boldsymbol{J}_2) = A_{1x}A_{2x}^* + A_{1y}A_{2y}^* = 0 + 0 = 0$$

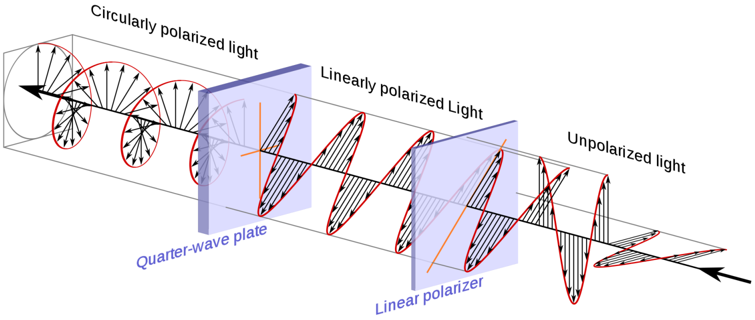
$$\boldsymbol{J} = \begin{bmatrix} a \\ ae^{j\frac{\pi}{2}} \end{bmatrix} = a\boldsymbol{J}_1 + ae^{j\frac{\pi}{2}}\boldsymbol{J}_2$$



Polarized Plane Wave

Example (Polarizer)

A polarizer can chnage the polarization of a polarized wave.



Reflection and Refraction

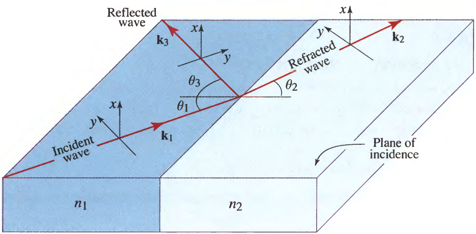


Figure: Reflection and refraction at the boundary between two linear, homogeneous, isotropic, nonmagnetic, and lossless die1ectric media.

- Reflection angle: $\theta_3 = \theta_1$
- Snell's equation: $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$
- TE polarization reflectivity: $r_x = \frac{E_{3x}}{E_{1x}} = \frac{n_1 \cos(\theta_1) - n_2 \cos(\theta_2)}{n_1 \cos(\theta_1) + n_2 \cos(\theta_2)}$
- TE polarization transmitivity: $t_x = \frac{E_{2x}}{E_{1x}} = 1 + r_x$
- TM polarization reflectivity: $r_y = \frac{E_{3y}}{E_{1y}} = \frac{n_1 \sec(\theta_1) - n_2 \sec(\theta_2)}{n_1 \sec(\theta_1) + n_2 \sec(\theta_2)}$
- TM polarization transmitivity: $t_y = \frac{E_{2y}}{E_{1y}} = (1 + r_y)\frac{\cos(\theta_1)}{\cos(\theta_2)}$

Reflection and Refraction

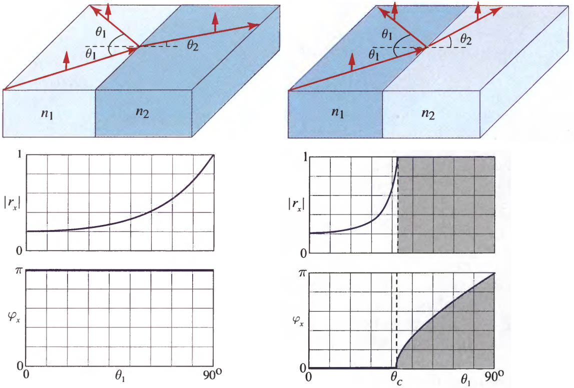


Figure: External and internal reflection for TE polarization.

- External reflection: $n_1 < n_2$
- Internal reflection: $n_1 > n_2$
- Critical angle: $\theta_c = \sin^{-1}(\frac{n_2}{n_1})$

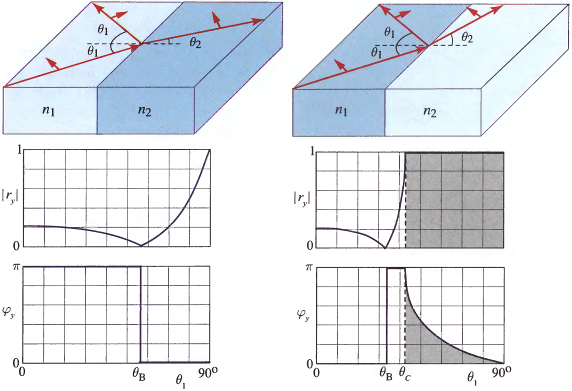


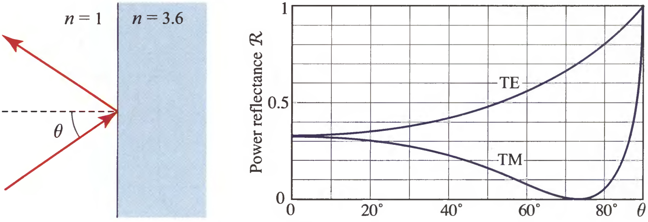
Figure: External and internal reflection for TM polarization.

- External reflection: $n_1 < n_2$
- Internal reflection: $n_1 > n_2$
- Brewster angle: $\theta_B = \tan^{-1}(\frac{n_2}{n_1})$
- Critical angle: $\theta_c = \sin^{-1}(\frac{n_2}{n_1})$

Reflection and Refraction

Example (Polarizer)

Power reflectance of TE- and TM-polarization plane waves, i.e., $|r_x|^2$ and $|r_y|^2$, at the boundary between air ($n = 1$) and GaAs ($n = 3.6$) is a function of the incidence angle θ



Sellmeier Equation

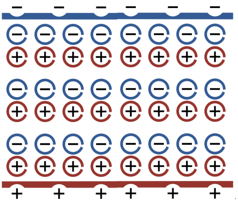


Figure: Electric waves passing through a linear homogeneous isotropic transparent dielectric plate.

- Electrical displacement: $D = \epsilon_0 E + P = \epsilon_0(1+\chi)E = \epsilon_0\epsilon_r E = \epsilon E$
- Polarization density: $P = \epsilon_0 \chi E = -NeX$
- Electrical permittivity and susceptivity: $\epsilon_r = 1 + \chi = 1 + \chi' + j\chi''$
- Hemlholtz equation: $\nabla^2 U + k^2 U = 0$
- z-traveling plane wave: $U = Ae^{-jkz} = Ae^{-0.5\alpha z}e^{-j\beta z}$
- Complex wave number: $k = \beta - 0.5j\alpha = \frac{\omega}{c_0}\sqrt{\epsilon_r \mu_r} = k_0\sqrt{1+\chi} \in \mathbb{C}$
- Propagation constant: $\beta = \mathrm{Re}\{k\} \in \mathbb{R}$
- Attenuation constant: $\alpha = -2\,\mathrm{Im}\{k\} \in \mathbb{R}$
- Characteristic impedance: $\eta = \sqrt{\frac{\mu_0}{\epsilon}} = \frac{\eta_0}{\sqrt{1+\chi}} \in \mathbb{C}$

Sellmeier Equation

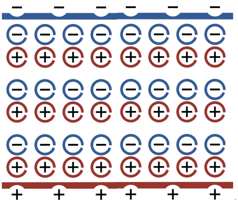


Figure: Electric waves passing through a linear homogeneous isotropic transparent dielectric plate.

- Refractive index: $n = \frac{c_0}{c} = \frac{\beta}{k_0} = j0.5\frac{\alpha}{k_0} + \sqrt{1 + \chi' + j\chi''} \in \mathbb{R}$
- Weakly absorbing medium: $n \approx \sqrt{1 + \chi'}, \alpha \approx -\frac{k_0}{n}\chi'', \chi'' \ll 1 + \chi'$
- Strongly absorbing medium: $n \approx \sqrt{-0.5\chi''}, \alpha \approx 2k_0 n, |\chi''| \gg |1 + \chi'|$

Sellmeier Equation

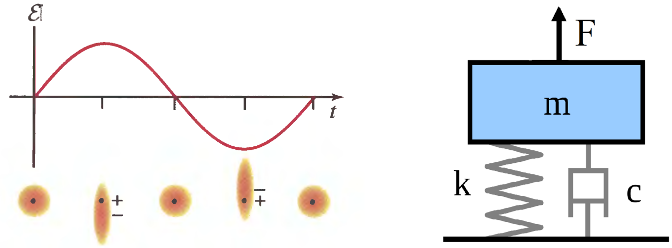


Figure: A time-varying electric field, applied to a Lorentz-oscillator atom induces a time-varying dipole moment.

- Lorentz oscillator model: $\frac{d^2x(t)}{dt^2} + \sigma \frac{dx(t)}{dt} + \omega_0^2 x(t) = -\frac{e}{m}\mathcal{E}(t)$
- Resonant dielectric medium:
  $\frac{d^2\mathcal{P}(t)}{dt^2} + \sigma \frac{d\mathcal{P}(t)}{dt} + \omega_0^2 \mathcal{P}(t) = \frac{Ne^2\epsilon_0\omega_0^2}{m\epsilon_0\omega_0^2}\mathcal{E}(t) = \chi_0\epsilon_0\omega_0^2\mathcal{E}(t)$
- Applied electric field: $\mathcal{E}(t) = \text{Re}\{E\exp(j\omega t)\}$
- Induced polarization density:
  $\mathcal{P}(t) = -Nex(t) = \text{Re}\{P\exp(j\omega t)\} = \text{Re}\{\frac{\chi_0\epsilon_0\omega_0^2}{\omega_0^2-\omega^2+j\sigma\omega}E\exp(j\omega t)\}$

Sellmeier Equation

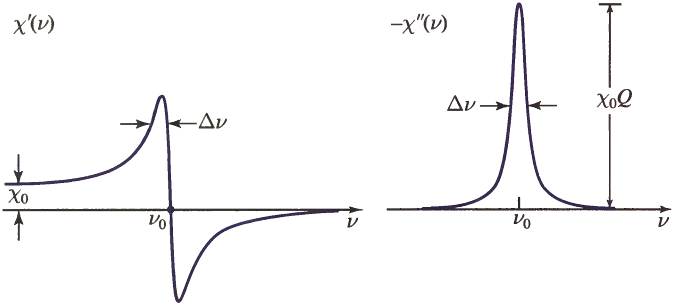


Figure: Real and imaginary parts of the susceptibility of a resonant dielectric medium, where $Q = \nu_0/\Delta\nu$.

- Electrical susceptivity: $\chi(\nu) = \chi'(\nu) + j\chi''(\nu) = \chi_0 \frac{\nu_0^2}{\nu_0^2 - \nu^2 + j\nu\Delta\nu}$
- Resonance vicinity behavior: $\chi(\nu) \approx \chi_0 \frac{\nu_0}{2(\nu_0 - \nu) + j\Delta\nu}, \nu \sim \nu_0$
- Electrical susceptivity imaginary part: $\chi''(\nu) \approx -\chi_0 \frac{\nu_0 \Delta\nu}{4(\nu_0 - \nu)^2 + (\Delta\nu)^2}, \nu \sim \nu_0$
- Electrical susceptivity real part: $\chi'(\nu) \approx 2\frac{\nu - \nu_0}{\Delta\nu}\chi''(\nu), \nu \sim \nu_0$
- Far from resonance susceptivity: $\chi(\nu) \approx \chi'(\nu) \approx \chi_0 \frac{\nu_0^2}{\nu_0^2 - \nu^2}, |\nu - \nu_0| \gg \Delta\nu$

Sellmeier Equation

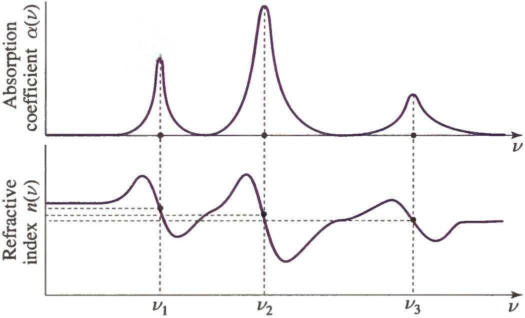
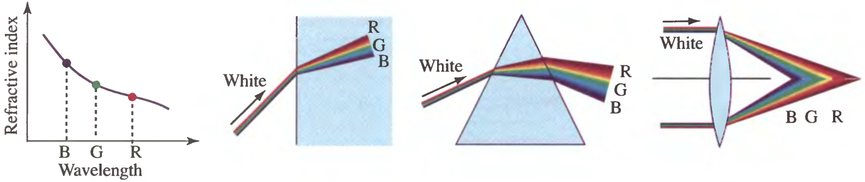


Figure: Frequency dependence of absorption coefficient and refractive index for a medium with multiple resonances.

- Electrical susceptivity: $\chi(\nu) = \chi_0 \frac{\nu_0^2}{\nu_0^2 - \nu^2 + j\nu\Delta\nu}$
- Multi-resonance electrical susceptivity: $\chi(\nu) = \sum_k \chi_{0k} \frac{\nu_k^2}{\nu_k^2 - \nu^2 + j\nu\Delta\nu}$
- Sellmeier formula:
  $n^2(\nu) \approx 1 + \sum_k \chi_{0k} \frac{\nu_k^2}{\nu_k^2 - \nu^2} = 1 + \sum_k \chi_{0k} \frac{\lambda^2}{\lambda^2 - \lambda_k^2}, |\nu - \nu_k| \gg \Delta\nu$

Example (Prism)

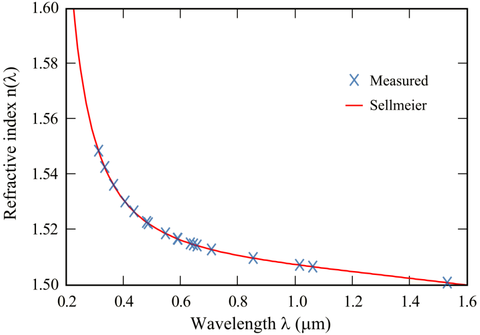
A prism decomposes the white light using different refractive indices of the different wavelengths.



Sellmeier Equation

Example (Sellmeier equation for silica)

The Sellmeier equation for the silica at room temperature has three resonance wavelengths.



$$n^2(\lambda) = 1 + \frac{0.6962\lambda^2}{\lambda^2 - 0.06840^2} + \frac{0.4079\lambda^2}{\lambda^2 - 0.1162^2} + \frac{0.8975\lambda^2}{\lambda^2 - 9.8962^2}$$

Optical Waveguide

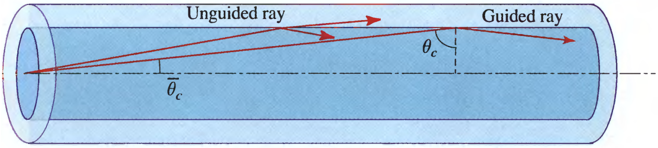


Figure: A cylinderical optical waveguide.

- Spherical Hemholtz equation: $\nabla^2 U + n^2(r)k_0^2 U = 0$
- Wave function: $U(r, \phi, z) = u(r)e^{-jl\phi}e^{-j\beta z}, \quad l = 0, \pm 1, \cdots$
- Radial profile equation: $\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} + (n^2(r)k_0 - \beta^2 - \frac{l^2}{r^2})u = 0$
- Step-index refractive index profile: $n(r) = \begin{cases} n_1, & r \leq a \\ n_2, & r > a \end{cases}$

Optical Waveguide

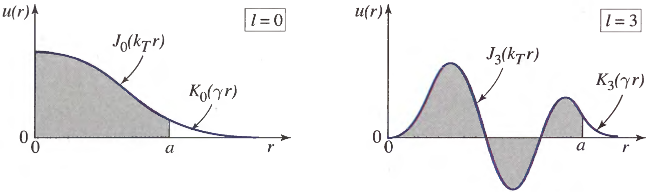


Figure: Examples of the radial profile $u(r)$ for $l = 0$ and $l = 3$.

- Fractional refractive index: $\Delta = (n_1 - n_2)/n_1$
- Numerical aperture: $NA = \sqrt{n_1^2 - n_2^2} \approx n_1\sqrt{2\Delta}$
- V parameter: $V = 2\pi \frac{a}{\lambda_0} NA$
- Propagation parameter: $k_T^2 = (\frac{X}{a})^2 = n_1^2 k_0^2 - \beta^2$
- Decay parameter: $\gamma^2 = (\frac{Y}{a})^2 = \beta^2 - n_2^2 k_o^2, \quad k_T^2 + \gamma^2 = (NA)^2 k_0^2$
- Boundary conditions: $\frac{XJ_{l\pm1}(X)}{J_l(X)} = \pm Y \frac{k_T a K_{l\pm1}(Y)}{K_l(Y)}, \quad Y = \sqrt{V^2 - X^2}$
- Radial profile: $u(r) \propto \begin{cases} J_l(X_{lm}\frac{r}{a}), & r \leq a \\ K_l(Y_{lm}\frac{r}{a}), & r > a \end{cases}, l = 0, \pm1, \cdots, m = 1, 2, \cdots, M_l$

Optical Waveguide

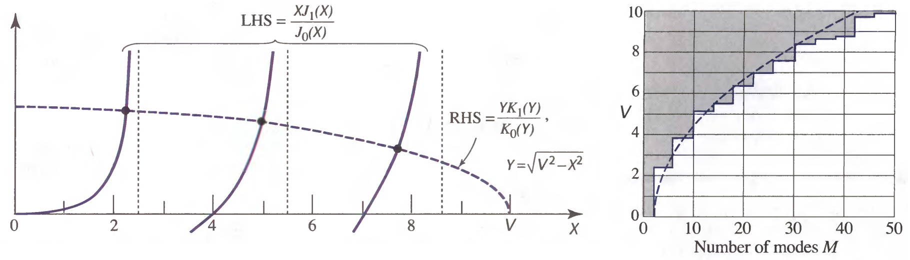


Figure: Total number of modes M versus the fiber parameter V.

l, m	1	2	3
0	0	3.832	7.016
1	2.405	5.520	8.654

Table: Cutoff V parameter for low-order modes.

- Approximated number of modes: $M \approx 4\frac{V^2}{\pi^2} + 2 \approx 4\frac{V^2}{\pi^2}, V \gg 1$
- Single mode condition: $V < 2.405$

Nonlinear Schrodinger Equation

- Nonlinear dispersive wave equation: $\nabla^2\mathcal{E} - \frac{1}{c_0^2}\frac{\partial^2\mathcal{E}}{\partial t^2} = \mu_0\frac{\partial^2\mathcal{P}}{\partial t^2}$
- Polarization density: $\mathcal{P} = \epsilon_0\chi\mathcal{E} + \mathcal{P}_{NL} \approx \epsilon_0\chi\mathcal{E} + 4\chi^{(3)}\mathcal{E}^3$
- Quasi-monochromatic plane wave: $\mathcal{E} = \mathrm{Re}\{\mathcal{A}(t,z)e^{j(\omega_0 t - \beta_0 z)}\}$
- Nonlinear Schrodinger equation (NSE): $\frac{\partial\mathcal{A}}{\partial z} + \frac{j\beta_2}{2}\frac{\partial^2\mathcal{A}}{\partial t^2} + \frac{\alpha}{2}\mathcal{A} - j\gamma|\mathcal{A}|^2\mathcal{A} = 0$
- Group delay dispersion parameter: β_2
- Attenuation coefficient: α
- Nonlinear parameter: γ

Physical Description of Optical Fiber

Physical Structure

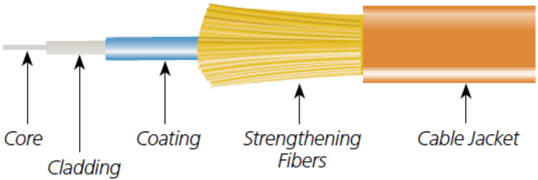


Figure: An optical fiber cable consists of core, cladding, coating, strengthening fibers, and cable jacket.

Physical Structure

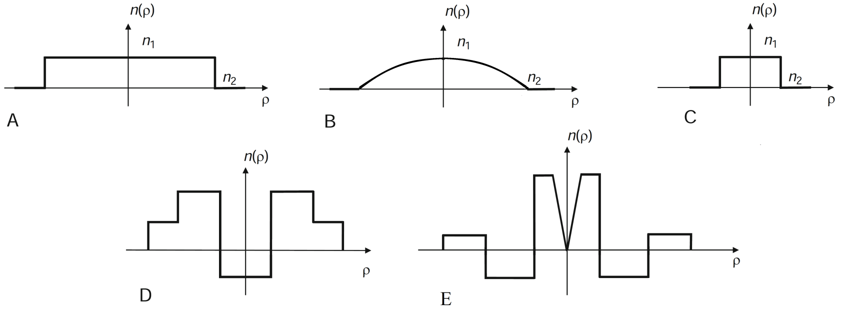


Figure: Refractive index profiles for (A) multi-mode fiber (MMF), (B) graded-index fiber (GRIN), (C) single-mode fiber (SMF), (D) Non-zero dispersion-shifted fiber (NZDSF), and (D) dispersion compensating fiber (DCF).

Physical Structure

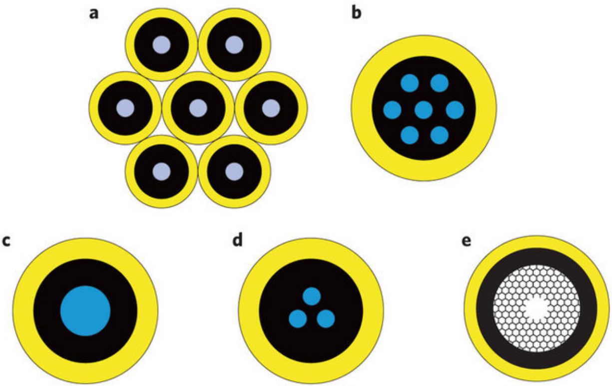


Figure: Space division multiplexed (SDM) optical fibers (a) SMF bundle, (b) multi-core fiber (MCF), (C) few-mode fiber (FMF), (D) multi-core few-mode fiber (MCFMF), and (D) photonic bandgap fibre (FBF).

Electromagnetic Description

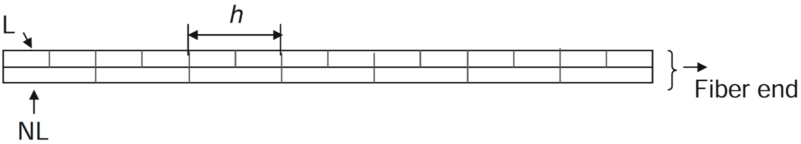


Figure: Illustration of Fourier split-step algorithm.

- Nonlinear dispersive wave equation: $\nabla^2\mathcal{E} - \frac{1}{c_0^2}\frac{\partial^2\mathcal{E}}{\partial t^2} = \mu_0\frac{\partial^2\mathcal{P}}{\partial t^2}$
- Quasi-monochromatic plane wave: $\mathcal{E} = \text{Re}\{\mathcal{A}(t,z)e^{j(\omega_0 t - \beta_0 z)}\}$
- Nonlinear Schrodinger equation: $\frac{D_\nu}{4\pi}\frac{\partial^2\mathcal{A}(t,z)}{\partial t^2} + \gamma|\mathcal{A}|^2\mathcal{A} + j\frac{\partial\mathcal{A}}{\partial z} + \frac{j}{v_g}\frac{\partial\mathcal{A}}{\partial t} = 0$

Ray Description

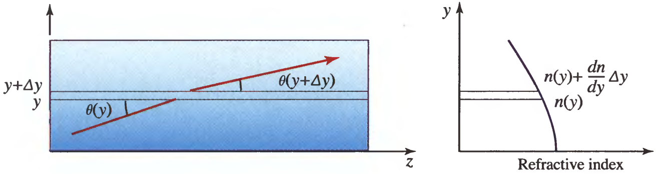
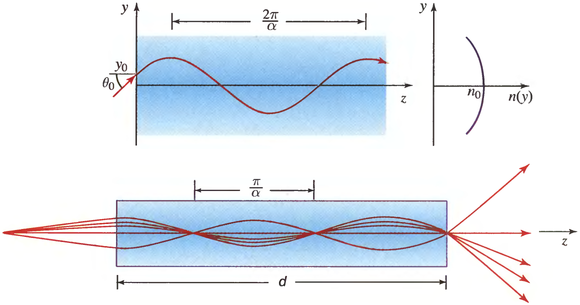


Figure: Refraction in a graded-index slab.

- Snell's law: $n(y)\cos(\theta(y)) = n(y+\Delta y)\cos(\theta(y+\Delta y))$
- Taylor series: $n(y+\Delta y)\cos(\theta(y+\Delta y)) = [n(y) + \frac{dn}{dy}\Delta y][\cos(\theta(y)) - \frac{d\theta}{dy}\Delta y\sin(\theta(y))]$
- Limit form: $\frac{dn}{dy} = n\frac{d\theta}{dy}\tan(\theta)$
- Paraxial approximation: $\frac{dn}{dy} \approx n\frac{d\theta}{dy}\theta = n\frac{d\theta}{dy}\frac{dy}{dz} = n\frac{d\theta}{dz} = n\frac{d^2y}{dz^2}$
- Paraxial ray equation: $\frac{d^2y}{dz^2} = \frac{1}{n(y)}\frac{dn(y)}{dy}$
- Paraxial ray equation: $\frac{d}{dz}(n\frac{dx}{dz}) \approx \frac{\partial n}{\partial x}, \quad \frac{d}{dz}(n\frac{dy}{dz}) \approx \frac{\partial n}{\partial y}$

Example (Slab with parabolic index profile)

The ray trajectory for a glass slab with index profile $n(y) = n_0\sqrt{1-\alpha^2 y^2} \approx n_0(1-0.5\alpha^2 y^2)$ can be found using the paraxial ray equation.



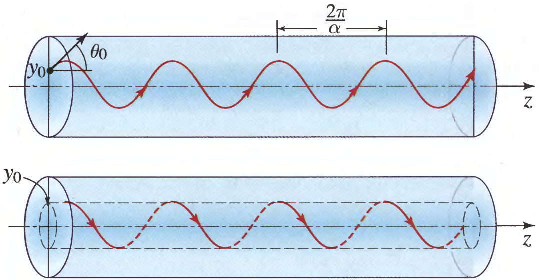
$$\frac{d^2y}{dz^2} = \frac{1}{n(y)}\frac{dn(y)}{dy} \Rightarrow \frac{d^2y}{dz^2} \approx -\alpha^2 y$$

$$y(z) = y_0\cos(\alpha z) + \frac{\theta_0}{\alpha}\sin(\alpha z), \quad y_0 = y(0), \theta_0 = \frac{dy}{dz}|_{z=0}$$

Ray Description

Example (Optical fiber with parabolic index profile)

The ray trajectory for an optical fiber with index profile $n(r) = n_0\sqrt{1-\alpha^2 r^2} \approx n_0(1-0.5\alpha^2 r^2)$ can be found using the paraxial ray equation.

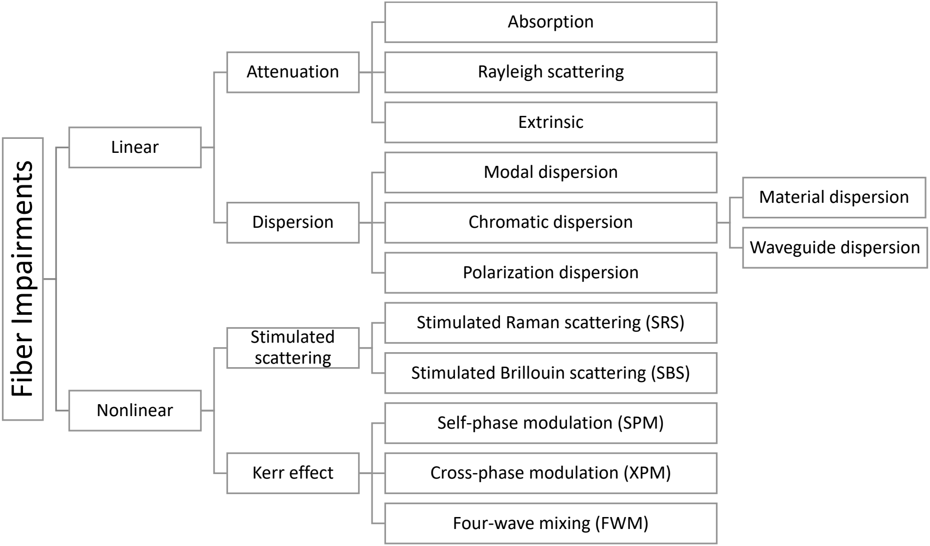


$$\frac{d}{dz}(n\frac{dx}{dz}) \approx \frac{\partial n}{\partial x}, \quad \frac{d}{dz}(n\frac{dy}{dz}) \approx \frac{\partial n}{\partial y} \Rightarrow \frac{d^2x}{dz^2} \approx -\alpha^2 x, \quad \frac{d^2y}{dz^2} \approx -\alpha^2 y$$

$$x(z) = \frac{\theta_{x0}}{\alpha}\sin(\alpha z) + x_0\cos(\alpha z), \quad y(z) = \frac{\theta_{y0}}{\alpha}\sin(\alpha z) + y_0\cos(\alpha z)$$

Analytical Description of Optical Fiber

Impairments



Acceptance Cone

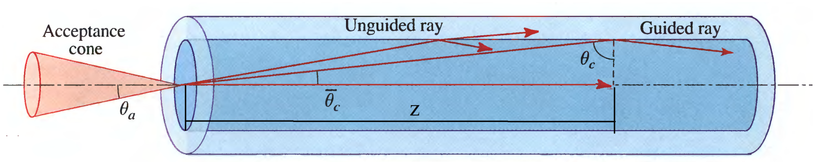


Figure: Rays within the acceptance cone are guided by total internal reflection.

- Fractional refractive index: $\Delta = (n_1 - n_2)/n_1$
- Numerical aperture: $NA = \sqrt{n_1^2 - n_2^2} \approx n_1\sqrt{2\Delta}$
- Acceptance cone: $1.\sin(\theta_a) = n_1 \sin\left(\frac{\pi}{2} - \theta_c\right) = n_1\sqrt{1 - (\frac{n_2}{n_1})^2} = NA$
- V parameter: $V = 2\pi \frac{a}{\lambda_0} NA$
- Approximated number of modes: $M \approx 4\frac{V^2}{\pi^2}, V \gg 1$

Group Velocity

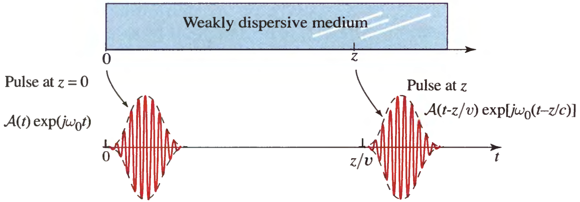


Figure: An optical pulse traveling in a dispersive medium. The envelope travels with group velocity v_g while the underlying wave travels with phase velocity c.

- Wavelength dependent propagation constant: $\beta(\omega) = \frac{\omega n(\omega)}{c_0}$
- Weakly dispersive media: $\beta(\omega_0 + \Omega) \approx \beta(\omega_0) + \Omega \frac{d\beta}{d\omega} = \frac{\omega_0}{c} + \frac{\Omega}{v_g}$
- Initial complex wavefunction: $\mathcal{A}(t)\exp(j\omega_0 t)$
- Wavefunction component: $A(\Omega)e^{j(\omega_0+\Omega)t}e^{-j\beta(\omega_0+\Omega)z}$
- Approximated wavefunction component: $A(\Omega)e^{j\omega_0(t-z/c)}e^{j\Omega(t-z/v_g)}$
- Traveled complex wavefunction: $\mathcal{A}(t - z/v_g)\exp(j\omega_0(t - z/c))$
- Group index: $N = \frac{d\beta(\omega)}{d\omega}|_{\omega=\omega_0} = n(\omega_0) + \omega_0 \frac{dn(\omega)}{d\omega}|_{\omega=\omega_0} = n(\lambda_0) - \lambda_0 \frac{dn(\lambda)}{d\lambda}|_{\lambda=\lambda_0}$
- Group velocity: $v_g = \frac{1}{d\beta(\omega)/d\omega|_{\omega=\omega_0}} = \frac{c_0}{N}$
- Phace velocity: $c = \frac{\omega_0}{\beta(\omega_0)} = \frac{c_0}{n(\omega_0)}$

Attenuation

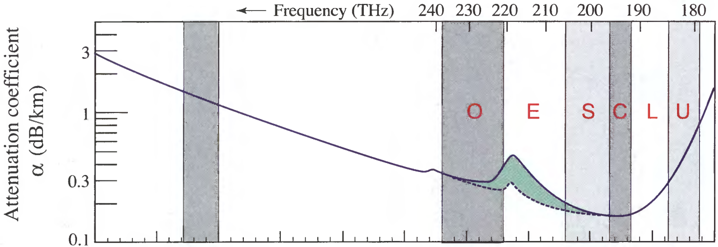


Figure: Wavelength dependence of the attenuation coefficient of silica-glass fiber.

- z-traveling plane wave: $U(z) = Ae^{-jkz} = Ae^{-0.5\alpha z}e^{-j\beta z}$
- Power attenuation: $P(z) = |U(z)|^2 = A^2e^{-\alpha z} = P(0)e^{-\alpha z}$
- Power attenuation: $P(z)_{dB} = P(0)_{dB} - [10\log_{10} e]\alpha z$
- Attenuation coefficient: $\alpha_{dB} = 10\alpha\log_{10} e = 4.3478\alpha$

Attenuation

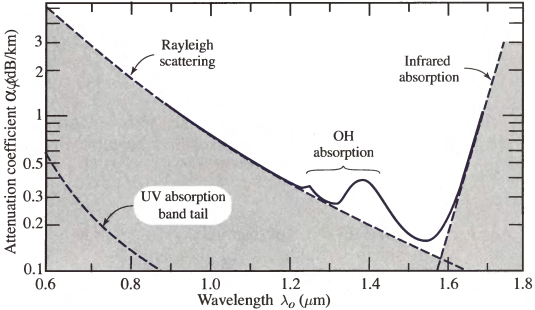


Figure: Attenuation coefficient α_{dB} of silica glass versus wavelength λ. There is a local minimum at 1.3 μm ($\alpha_{dB} \approx 0.3$ dB/km) and an absolute minimum at 1.55 μm ($\alpha_{dB} \approx 0.15$ dB/km).

- Absorption: Infrared and altraviolet absorption due to vibrational and electronic transitions
- Rayleigh scattering: random localized variations of the molecular position, proportional to $1/\lambda^4$
- Extrinsic effects: random impurities such as OH, random variation in geometry by bend, mode-dependent attenuation

Attenuation

Example (1.3 μm optical communication)

An optical transmitter injects 0 dBm power to an optical fiber which connects to an optical receiver with power sensitivity of -19 dBm. So, the optical fiber length should be less than $\frac{19}{0.3} = 63.3$ km if the operating wavelength is 1.3 μm.

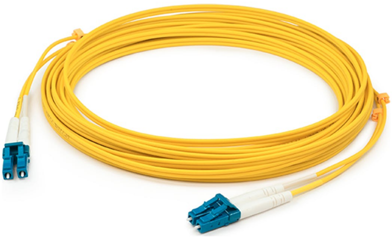
Example (1.55 μm optical communication)

An optical transmitter injects 0 dBm power to an optical fiber which connects to an optical receiver with power sensitivity of -19 dBm. So, the optical fiber length should be less than $\frac{19}{0.15} = 126.6$ km if the operating wavelength is 1.55 μm.

Attenuation

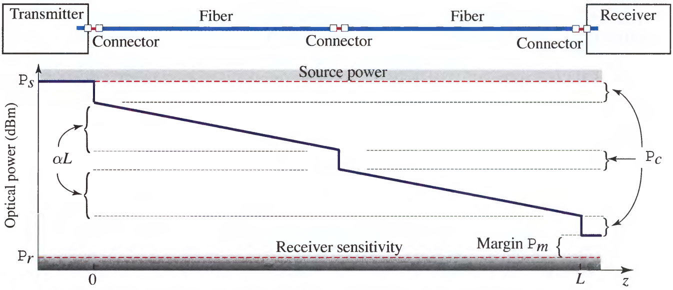
Example (Power budgeting)

An optical transmitter injects 0 dBm power to an optical fiber which connects to an optical receiver with power sensitivity of -19 dBm. The fiber connects to the transmitter and receiver using LC connectors with 0.3 dB loss. So, the optical fiber length should be less than $\frac{19-0.3-0.3}{0.15} = 122.6$ km if the operating wavelength is 1.55 μm.



Example (Power budgeting)

An optical transmitter injects 0 dBm power to a two-segment optical fiber which connects to an optical receiver with power sensitivity of -19 dBm. The fiber segments connect to the transmitter, receiver, and each other using using LC connectors with 0.3 dB loss. So, the optical fiber length should be less than $\frac{19-0.3-0.3-0.3}{0.15} = 120.6$ km if the operating wavelength is 1.55 μm.



Modal Dispersion

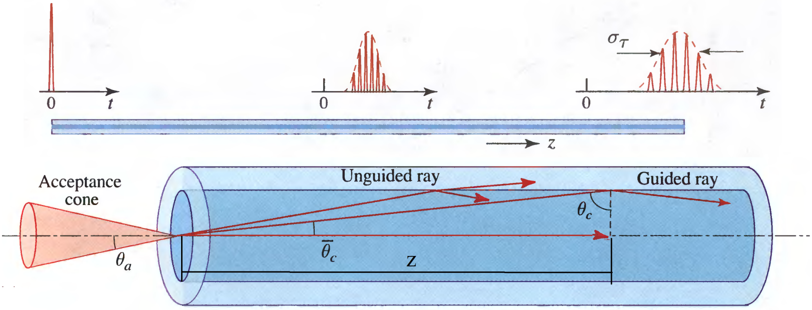


Figure: Modal dispersion in step-index multi-mode fiber.

- Modal dispersion delay: $\sigma_\tau = \left(\frac{z}{\sin(\theta_c)} - z\right)\frac{n_1}{c} = z\frac{n_1}{c}\frac{n_1}{n_2}\Delta \approx z\frac{n_1}{c}\Delta$
- Rate-distance product: $\sigma_\tau < T_b = \frac{1}{R_b} \Rightarrow R_b z \lesssim \frac{c}{n_1 \Delta}$

Modal Dispersion

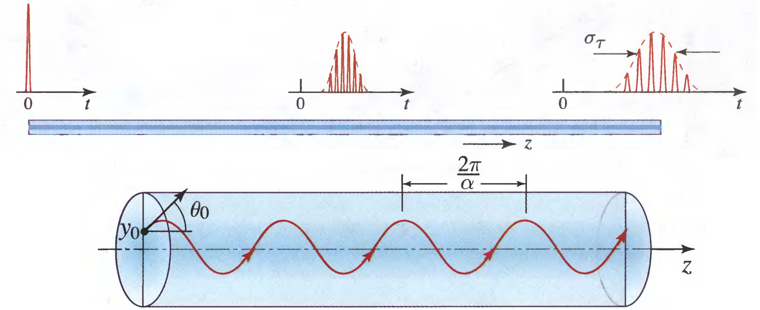


Figure: Modal dispersion in graded-index multi-mode fiber.

- Modal dispersion delay: $\sigma_\tau \approx z\frac{n_1}{c}\Delta\frac{\Delta}{2}$
- Rate-distance product: $\sigma_\tau < T_b = \frac{1}{R_b} \Rightarrow R_b z \lesssim \frac{2c}{n_1\Delta^2}$

Modal Dispersion

Example (Unclad step-index MMF)

Rate-distance product of an unclad step-index MMF with $n_1 = 1.5$, $n_2 = 1$, and $\Delta = 0.33$ is 0.6 Mbps.km.

Example (Cladded step-index MMF)

Rate-distance product of an cladded step-index MMF with $n_1 = 1.5$, $n_2 = 1.497$, and $\Delta = 0.002$ is 100 Mbps.km.

Example (Cladded graded-index MMF)

Rate-distance product of an cladded graded-index MMF with $n_1 = 1.5$, $n_2 = 1.497$, and $\Delta = 0.002$ is 10^5 Mbps.km.

Material Dispersion

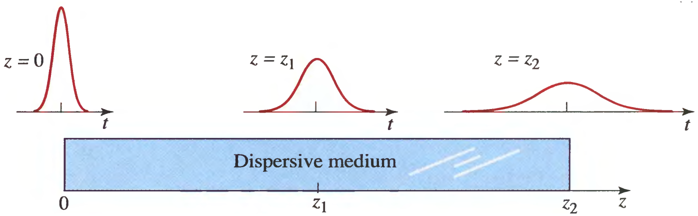


Figure: An optical pulse traveling in a SMF is broadened due to chromatic dispersion at a rate proportional to the product of the dispersion coefficient D_v (ps/km.GHz), the spectral width σ_ν (GHz), and the distance traveled z (km).

- Differential group delay of two identical pulses at frequencies ν and $\nu + \delta\nu$: $\sigma_\tau = \frac{d\tau_d}{d\nu}\delta\nu = \frac{d}{d\nu}(\frac{z}{v_g})\delta\nu = D_\nu z \delta\nu$
- Dispersion coefficient: $D_\nu = \frac{d}{d\nu}(\frac{1}{v_g}) = 2\pi\beta''(\omega_0) = \frac{\lambda_0^3}{c_0^2}\frac{d^2 n(\lambda)}{d\lambda^2}|_{\lambda=\lambda_0}$
- Dispersion coefficient: $D_\lambda = -\frac{\lambda_0}{c_0}\frac{d^2 n(\lambda)}{d\lambda^2}|_{\lambda=\lambda_0}$
- Pulse spread: $\sigma_\tau = |D_\nu|\sigma_\nu z = |D_\lambda|\sigma_\lambda z$
- Rate-distance product: $\sigma_\tau < T_b = \frac{1}{R_b} \Rightarrow R_b z \lesssim \frac{1}{|D_\lambda|\sigma_\lambda} = \frac{1}{|D_\nu|\sigma_\nu}$

Material Dispersion

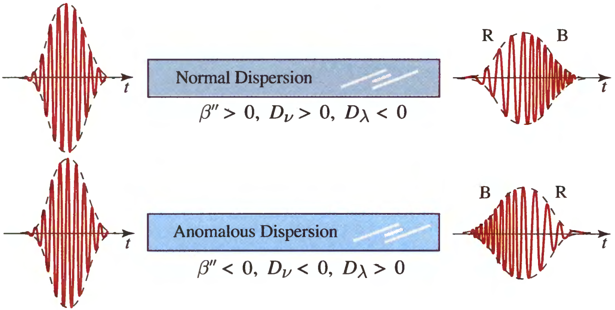


Figure: Propagation of an optical pulse through media with normal and anomalous dispersion.

- Normal dispersion: $D_\nu > 0 \equiv D_\lambda < 0$
- Anomalous dispersion: $D_\nu < 0 \equiv D_\lambda > 0$

Material Dispersion

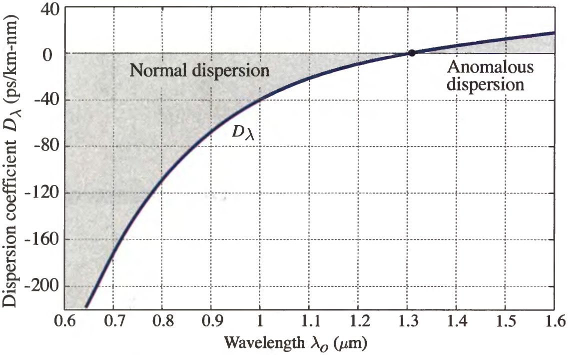


Figure: Dispersion coefficient D_λ for a silica-glass fiber as a function of wavelength λ. At $\lambda = 1.312\ \mu m$, the dispersion coefficient vanishes.

Example (SMF at 0.87 μm)

The dispersion coefficient D_λ for a silica-glass fiber is approximately -80 ps/km-nm at $\lambda = 0.87$ μm. For an LED source of spectral linewidth $\sigma_\lambda = 50$ nm, the pulse-spread rate in a SMF with no other sources of dispersion is $|D_\lambda|\sigma_\lambda = 4$ ns/km. So, the rate-distance product is 250 Mbps.km.

Example (SMF at 1.3 μm)

The dispersion coefficient D_λ for a silica-glass fiber is approximately -1 ps/km-nm at $\lambda = 1.3$ μm. For a LASER source of spectral linewidth $\sigma_\lambda = 2$ nm, the pulse-spread rate in a SMF with no other sources of dispersion is $|D_\lambda|\sigma_\lambda = 2$ ps/km. So, the rate-distance product is 5×10^5 Mbps.km.

Waveguide Dispersion

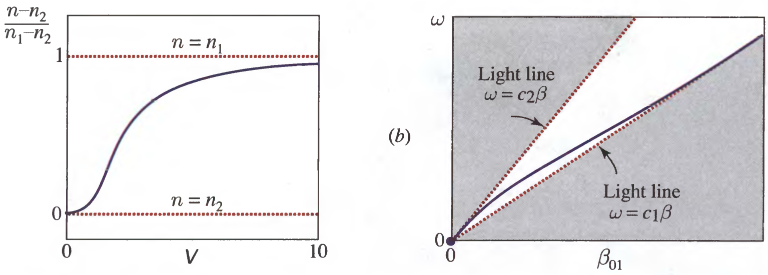


Figure: Dependency of the propagation constant β_{01} of the fundamental mode of SMF on frequency ω leads to waveguide dispersion. Waveguide dispersion may be controlled by altering the radius of the core or, for graded-index fibers, the index grading profile.

- V parameter: $V = 2\pi \frac{a}{\lambda_0} NA = \frac{aNA}{c_0}\omega$
- Group velocity at zero modal and material dispersion: $\frac{1}{v_g} = \frac{d\beta}{d\omega} = \frac{d\beta}{dV}\frac{dV}{d\omega} = \frac{aNA}{c_0}\frac{d\beta}{dV}$
- Waveguide dispersion coefficient: $D_w = \frac{d}{d\lambda}\left(\frac{1}{v_g}\right) = -\frac{1}{2\pi c_0} V^2 \frac{d^2\beta}{dV^2}$
- Pulse spread: $\sigma_\tau = |D_w|\sigma_\lambda z$
- Rate-distance product: $\sigma_\tau < T_b = \frac{1}{R_b} \Rightarrow R_b z \lesssim \frac{1}{|D_w|\sigma_\lambda}$

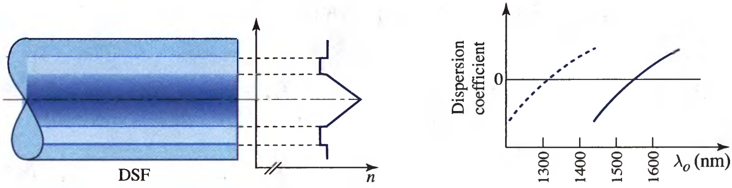


Figure: Chromatic dispersion in Dispersion Shifted Fiber (DSF).

- Chromatic dispersion: Combined effects of material and waveguide dispersions
- DSF: Zero dispersion at 1.55 μm

Chromatic Dispersion

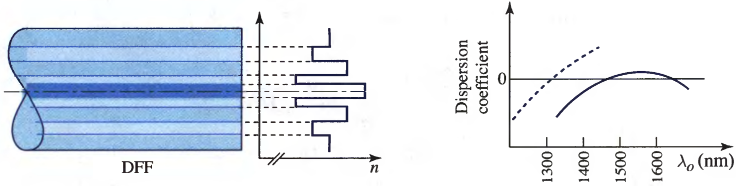


Figure: Chromatic dispersion in Dispersion Flattened Fiber (DFF).

- Chromatic dispersion: Combined effects of material and waveguide dispersions
- DSF: Flattened dispersion around 1.55 μm

Chromatic Dispersion

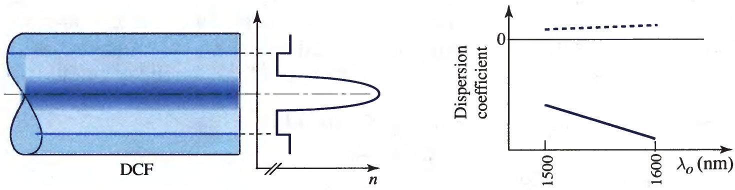


Figure: Chromatic dispersion in Dispersion Flattened Fiber (DCF).

- Chromatic dispersion: Combined effects of material and waveguide dispersions
- DCF: Compensating compensation for conventional fibers

Polarization Dispersion

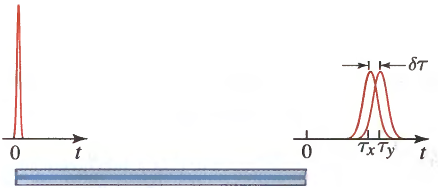


Figure: Differential group delay associated with polarization mode dispersion (PMD). PMD appears since the fiber is not perfectly circular and isotropic.

- x polarization group index: N_x
- y polarization group index: N_y
- x polarization group delay: $\tau_x = z\frac{N_x}{c_0}$
- y polarization group delay: $\tau_y = z\frac{N_y}{c_0}$
- Differential group delay: $\sigma_\tau = |N_x - N_y|\frac{z}{c_0}$
- Rate-distance product: $\sigma_\tau < T_b = \frac{1}{R_b} \Rightarrow R_b z \lesssim \frac{c_0}{|N_x - N_y|}$

Kerr Effects

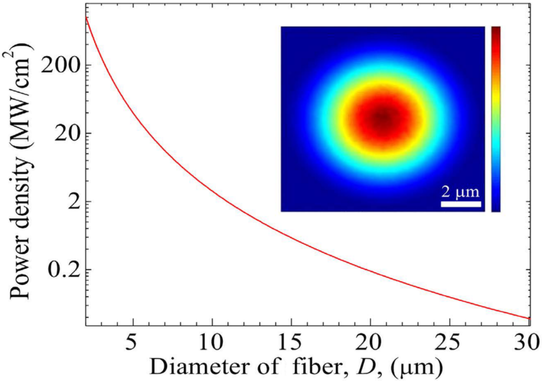


Figure: Optical power density versus core diameter in an optical fiber.

- SMF cross section: $a = 5\ \mu$ m $\Rightarrow 80\mu$ m^2
- SMF power density in core: $P(0) = 10$ mW $\Rightarrow 12.5\text{kW/cm}^2$
- Power-dependent refractive index: $n = n_0 + n_2 \frac{P}{A_{eff}} = n_0 + n_2 I$
- Fiber cross-section: A_{eff}

Kerr Effects

- Quasi-monochromatic plane wave: $\mathcal{E} = \text{Re}\{\mathcal{A}(t,z)e^{j(\omega_0 t - \beta_0 z)}\}, \beta_0 = \beta(\omega_0)$
- Complex envelope: $\mathcal{A}(t,z)$
- Nonlinear Schrodinger equation (NSE): $\frac{\partial \mathcal{A}}{\partial z} + \frac{j\beta_2}{2}\frac{\partial^2 \mathcal{A}}{\partial t^2} + \frac{\alpha}{2}\mathcal{A} - j\gamma|\mathcal{A}|^2\mathcal{A} = 0$
- Group delay dispersion parameter: $\beta_2 = \beta''(\omega_0) = -\frac{\lambda_0^2}{2\pi c_0}D_\lambda$
- Attenuation coefficient: α
- Nonlinear parameter: $\gamma = \frac{n_2\omega_0}{c_0 A_{eff}}$

Example (NSE for $\beta_2 = 0$, $\alpha = 0$, and $\gamma = 0$)

The NSE can be solved for $\beta_2 = 0$, $\alpha = 0$, and $\gamma = 0$.

$$
\begin{aligned}
&\frac{\partial \mathcal{A}}{\partial z} + \frac{j\beta_2}{2}\frac{\partial^2 \mathcal{A}}{\partial t^2} + \frac{\alpha}{2}\mathcal{A} - j\gamma|\mathcal{A}|^2\mathcal{A} = 0 \\
&\frac{\partial \mathcal{A}}{\partial z} = 0 \\
&\mathcal{A}(z, t) = \mathcal{A}(0, t) \\
&\mathcal{P}(z, t) = \mathcal{P}(0, t)
\end{aligned}
$$

Example (NSE for $\beta_2 = 0$, $\alpha \neq 0$, and $\gamma = 0$)

The NSE can be solved for $\beta_2 = 0$, $\alpha \neq 0$, and $\gamma = 0$.

$$
\begin{aligned}
&\frac{\partial \mathcal{A}}{\partial z} + \frac{j\beta_2}{2}\frac{\partial^2 \mathcal{A}}{\partial t^2} + \frac{\alpha}{2}\mathcal{A} - j\gamma|\mathcal{A}|^2\mathcal{A} = 0 \\
&\frac{\partial \mathcal{A}}{\partial z} + \frac{\alpha}{2}\mathcal{A} = 0 \\
&\mathcal{A}(z,t) = \mathcal{A}(0,t)e^{-0.5\alpha z} \\
&\mathcal{P}(z,t) = \mathcal{P}(0,t)e^{-\alpha z}
\end{aligned}
$$

Example (NSE for $\beta_2 \neq 0$, $\alpha = 0$, and $\gamma = 0$)

The NSE can be solved for $\beta_2 \neq 0$, $\alpha = 0$, and $\gamma = 0$.

$$
\begin{aligned}
&\frac{\partial \mathcal{A}}{\partial z} + \frac{j\beta_2}{2}\frac{\partial^2 \mathcal{A}}{\partial t^2} + \frac{\alpha}{2}\mathcal{A} - j\gamma|\mathcal{A}|^2\mathcal{A} = 0 \\
&\frac{\partial \mathcal{A}}{\partial z} + \frac{j\beta_2}{2}\frac{\partial^2 \mathcal{A}}{\partial t^2} = 0 \\
&\mathcal{A}(z,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} A(z,\omega)e^{j\omega t}d\omega \\
&A(z,\omega) = A(0,\omega)e^{j0.5\omega^2\beta_2 z} \\
&\mathcal{A}(z,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} A(0,\omega)e^{j0.5\omega^2\beta_2 z}e^{j\omega t}d\omega \\
&\mathcal{P}(z,t) = |\mathcal{A}(z,t)|^2 \\
&\delta\Phi = \omega_0\delta t = 0.5(\omega^2 - \omega_0^2)\beta_2 z \approx \omega_0(\omega - \omega_0)\beta_2 z \\
&\delta t \approx \beta_2 z(\omega - \omega_0) = \beta_2 z\delta\omega \equiv D_\lambda z\delta\lambda \\
&|\delta t| \approx |D_\lambda| z\delta\lambda
\end{aligned}
$$

Kerr Effects

Example (NSE for $\beta_2 = 0$, $\alpha \neq 0$, and $\gamma \neq 0$)

The NSE can be solved for $\beta_2 = 0$, $\alpha \neq 0$, and $\gamma \neq 0$.

$$\frac{\partial \mathcal{A}}{\partial z} + \frac{j\beta_2}{2}\frac{\partial^2 \mathcal{A}}{\partial t^2} + \frac{\alpha}{2}\mathcal{A} - j\gamma|\mathcal{A}|^2\mathcal{A} = 0$$

$$\frac{\partial \mathcal{A}}{\partial z} + \frac{\alpha}{2}\mathcal{A} = j\gamma|\mathcal{A}|^2\mathcal{A}$$

$$\mathcal{P}(z,t) = |\mathcal{A}(z,t)|^2 = \mathcal{P}(0,t)e^{-\alpha z}$$

$$\mathcal{A}(z,t) = \sqrt{\mathcal{P}(0,t)}e^{-0.5\alpha z}\mathcal{E}(z,t), |\mathcal{E}(z,t)| = 1$$

$$\frac{\partial \mathcal{E}}{\partial z} = j\gamma\mathcal{P}(0,t)e^{-\alpha z}\mathcal{E}$$

$$\mathcal{E}(z,t) = \mathcal{E}(0,t)e^{j\Phi_{NL}} = \mathcal{E}(0,t)e^{j\gamma P(0,t)L_{eff}}, L_{eff} = \frac{1-e^{-\alpha z}}{\alpha} \approx \frac{1}{\alpha}$$

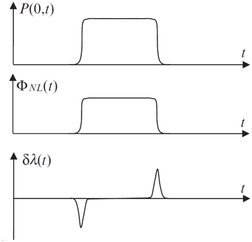
Example (SPM)

Self-phase modulation occurs since the nonlinear phase shift follows the time-dependent change of the optical power.

$$\mathcal{E}(z,t) = \mathcal{E}(0,t)e^{j\gamma P(0,t)L_{eff}}$$

$$\delta f(t) = \frac{1}{2\pi}\gamma L_{eff}\frac{\partial \mathcal{P}(0,t)}{\partial t}$$

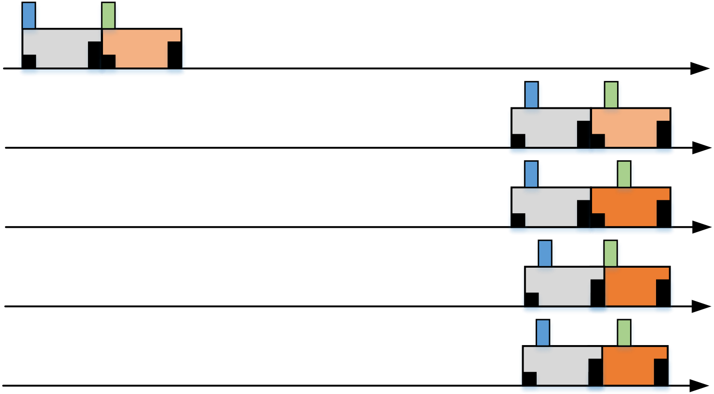
$$\delta\lambda(t) = \frac{\lambda_0^2}{2\pi c_0}\gamma L_{eff}\frac{\partial \mathcal{P}(0,t)}{\partial t}$$



SPM

Example (Soliton transmission)

Self-phase modulation and linear dispersion can compensate each other.



Example (SPM/XPM/FWM)

Self-phase and cross-phase modulations and four-wave mixing affect fiber transmission when several wavelengths are used for transmission.

$$\mathcal{A}(z,t) = \mathcal{A}_1(z,t)e^{-j\theta_1} + \mathcal{A}_2(z,t)e^{-j\theta_2}, \quad \theta_i = n\omega_i/c$$

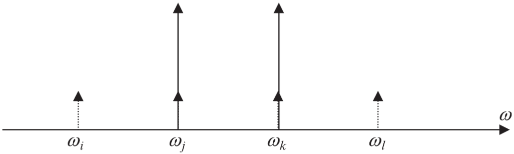
$$\begin{cases} \frac{\partial \mathcal{A}_1}{\partial z} + \frac{j\beta_2}{2}\frac{\partial^2 \mathcal{A}_1}{\partial t^2} + \frac{\alpha}{2}\mathcal{A}_1 = j\gamma|\mathcal{A}_1|^2\mathcal{A}_1 + j\gamma|\mathcal{A}_2|^2\mathcal{A}_2 + j\gamma\mathcal{A}_1^2\mathcal{A}_2^* e^{j(\theta_1-\theta_2)} \\ \frac{\partial \mathcal{A}_2}{\partial z} + \frac{j\beta_2}{2}\frac{\partial^2 \mathcal{A}_2}{\partial t^2} + \frac{\alpha}{2}\mathcal{A}_2 = j\gamma|\mathcal{A}_2|^2\mathcal{A}_2 + j\gamma|\mathcal{A}_1|^2\mathcal{A}_1 + j\gamma\mathcal{A}_2^2\mathcal{A}_1^* e^{j(\theta_2-\theta_1)} \end{cases}$$

FWM

Example (FWM)

In four-wave mixing, the refractive index is modulated at the frequency $\Delta\omega_{jk} = \omega_j - \omega_k$, which in turn phase-modulates a third carrier ω_l and creates extra modulation sidebands $\omega_{jkl} = \omega_l \pm \Delta\omega_{jk}$.

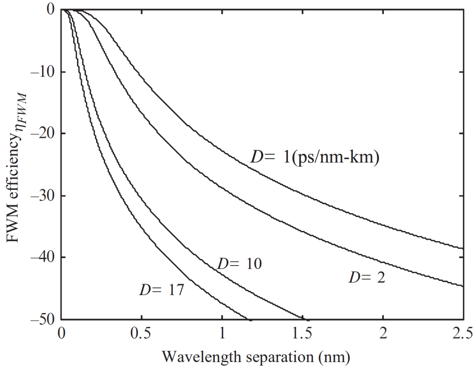
$$\omega_{jkl} = \omega_l \pm \omega_k \mp \omega_j$$
$$\omega_l = \omega_k \Rightarrow \omega_j, \omega_l = 2\omega_k - \omega_j$$
$$\omega_l = \omega_j \Rightarrow \omega_k, \omega_i = 2\omega_j - \omega_k$$



Example (FWM)

FWM can be mitigating by increasing the channel spaces.

$$P_{jkl}(z) = \eta_{FWM}\gamma^2 L_{eff}^2 P_j(0)P_k(0)P_l(0), \quad \eta_{FWM} = \frac{\alpha^2}{\Delta\beta_{jkl}^2 + \alpha^2}, \Delta\beta_{jkl} = \frac{2\pi c D_\lambda}{\lambda_0^2}(\lambda_j - \lambda_l)(\lambda_k - \lambda_l)$$



Scattering

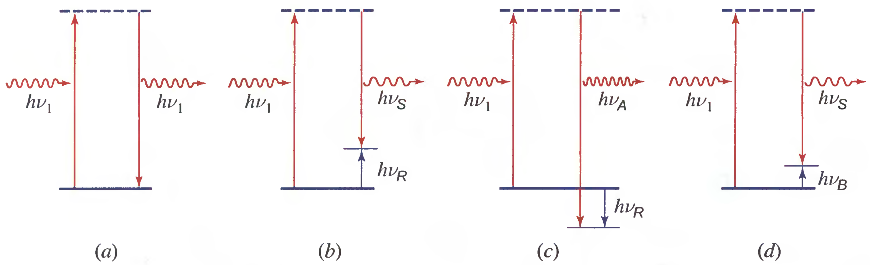


Figure: Several forms of light scattering: (a) Rayleigh, (b) Raman (Stokes), (c) Raman (anti-Stokes), and (d) Brillouin.

SBS

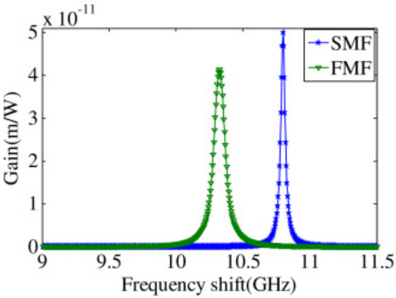


Figure: SBS can be mitigated by increasing source linewidth.

- Origination: Interaction of signal photons and acoustic (electrostriction)
- Shifted Stokes photon frequency: $\Delta f \approx 11$ GHz at 1550 nm
- Features: Threshold effect, narrow band (~ 20 MHz), and directional

SRS

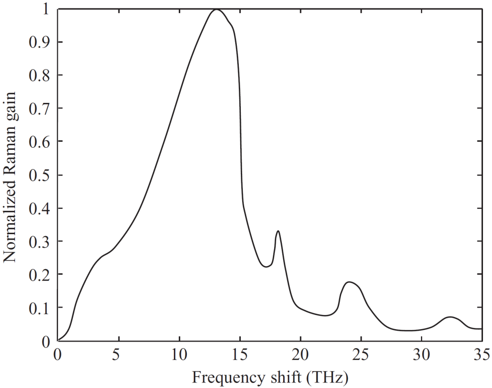
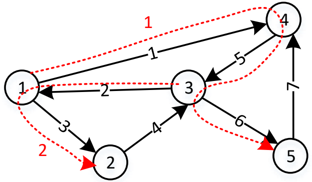


Figure: SRS can be mitigated by reducing injected power.

- Origination: Interaction of signal photons and molecular-level vibrations.
- Shifted Stokes photon frequency: $\Delta f \approx 13.2$ GHz at 1550 nm
- Features: Threshold effect, wide band (10 THz), and bidirectional

Example (Dispersion-aware resource allocation)

Assume that a network topology is described by directional graph $G(N, L)$, where each link $l = (b, e) \in L$ begins at node $b \in N$, ends at node $e \in N$, and has length W_l. There are R requests, where request $r = (s, d) \in R$ originates from source node $S(r) = s \in N$, terminates at destination node $D(r) = d \in N$, and requires transmission rate B_r. The requests can be routed by the following simple dispersion-aware resource allocation optimization process, where $x_{l,r} = 1$ if the request r passes through link l, 0 otherwise.



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$$\min_{x_{l,r}} \quad \sum_{l,r} x_{l,r} \qquad \text{s.t}$$

$$\sum_{l\in L:b=n} x_{l,r} = 1, \sum_{l\in L:e=n} x_{l,r} = 0, \quad \forall r \in R, \forall n \in N : n = S(r)$$

$$\sum_{l\in L:e=n} x_{l,r} = 1, \sum_{l\in L:b=n} x_{l,r} = 0, \quad \forall r \in R, \forall n \in N : n = D(r)$$

$$\sum_{l\in L:e=n} x_{l,r} = \sum_{l\in L:b=n} x_{l,r}, \quad \forall r \in R, \forall n \in N : n \neq S(r), n \neq D(r)$$

$$B_r \sum_{l\in L} x_{l,r} W_l \leq \frac{1}{|D_\lambda|\sigma_\lambda}, \quad \forall r \in R$$

The End