MATHEMATICAL QUESTIONS

Question 1

Prove the following statements for the limiting forms of the Lagurre distribution.

(a) Lagurre$(0, b, 0) \equiv$ *Bose*(b).

(b) Lagurre$(a, 0, 0) \equiv$ *Poisson*(a).

(c) Lagurre$(a, b \to 0, c \to \infty) \equiv$ *Poisson*$(a + bc)$.

Question 2

M **independent noisy quasi-monochromatic optical fields around central frequency** ν**, each having a signal part with the distinct intensity** I_i **polluted by an independent identical zero-mean narrow-band stationary Gaussian random noise part with the mean square intensity** σ^2**, impinge on a photo-detector with quantum efficiency** η **and area** A **over a short time interval** T**.**

(a) Find the characteristic function $\Psi_{m_v}(\omega)$ *of the overall carrier generation parameter* $m_v = \sum_{i=1}^{M} m_{v_i}$.

(b) Find the probability density function $P_{m_v}(m)$ *of the overall carrier generation parameter* $m_v = \sum_{i=1}^{M} m_{v_i}$.

(c) Find the probability mass function $P(k)$ *of the overall output carrier count.*

Question 2 continued on next page...

(d) Find the mean, variance, and SNR of the overall output carrier count.

Question 3

A photo-detector is located at the focal plane of a circular lens with diameter D and focal length f, as shown in Fig. 1. The deterministic plane wave $a(t)e^{-jkz}$ passes through the lens. Find the statistics of the output carrier count over a short time interval width T. Assume that the photo-detector has a quantum efficiency of η and its area A is large enough to collect the main lobes of the focused beam.

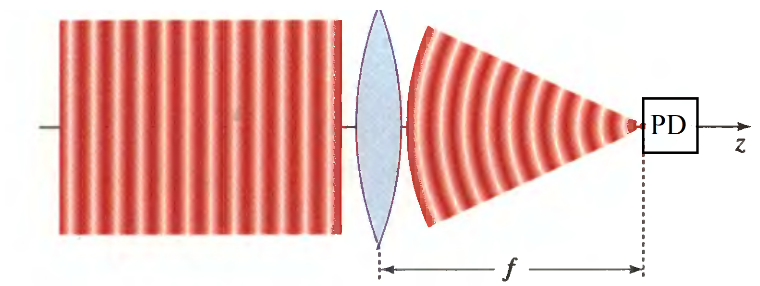

Figure 1: A photo-detector and the focal place of a circular lens.

Question 4

Let $x(t)$ be a random process with the semi-invariants χ_q. Define the normalized $x(t)$ as

$$\hat{x}(t) = \frac{x(t) - \chi_1}{\sqrt{\chi_2}}$$

with semi-invariants $\hat{\chi}_q$.

(a) Show that if $x(t)$ is a Gaussian random process, then $\hat{\chi}_1 = 0$, $\hat{\chi}_2 = 1$, and $\hat{\chi}_q = 0, q > 2$.

(b) For a Poisson shot noise process $i(t)$, $\chi_q = \int_{-\infty}^{\infty} h^q(t-z)n(z)dz$. Determine the conditions on $n(t)$ such that $i(t)$ approaches a Gaussian shot noise process.

Question 5

A plane wave field with intensity I impinges on the detector systems shown in Fig. 2. Assume that all detectors have ideal gain g and quantum efficiency η. Neglect background noise and dark current. Determine the difference, if any, in the spectral densities at the output of the two systems. The detectors in the left system have area A while the detector in the right system has area $3A$.

Question 5 continued on next page. . .

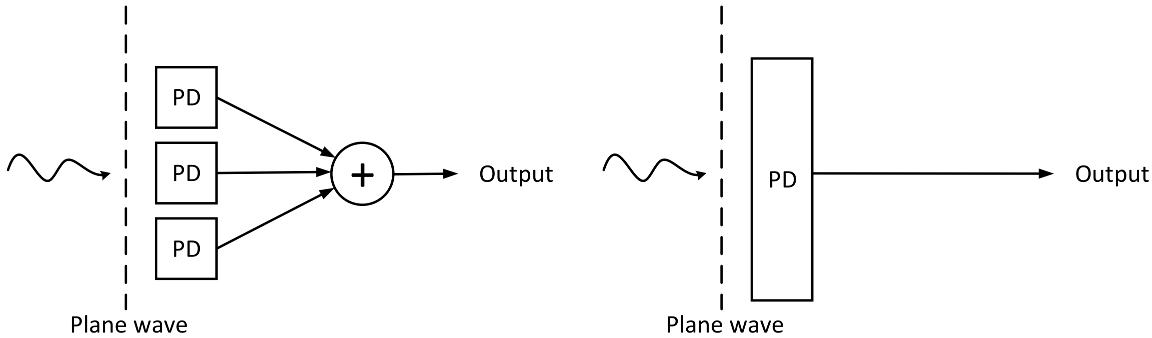

Figure 2: Two detecting systems.

SOFTWARE QUESTIONS

Question 6

A deterministic field with intensity $I(t)$ impinges on a photo-detector with area A, ideal gain g, and quantum efficiency η. Use Python or MATLAB to develop a function than produces random carrier generation times over an interval $[0, T]$.

BONUS QUESTIONS

Question 7

Extend the code in Question 6 to produce random carrier generation times when the impinging field is random with PDF $P_I(i)$.

Question 8

Return your answers by filling the LaTeXtemplate of the assignment.