Question 1

Fig. 1 shows an analog optical communication system, where a Gaussian beam with the waist radius W_0, focus depth z_0, and beam divergence θ_0 is intensity modulated. At the location z, a large thin lens having the transmittance $\exp(jk\frac{\rho^2}{2f})$ focuses the beam on a circular photo-detector with the area A_d and efficiency η placed at distance $z'-l$ from the lens. Meanwhile, the lens collects the colored Gaussian background noise at the temperature T and optical bandwidth B_o centered at the frequency ν_0.

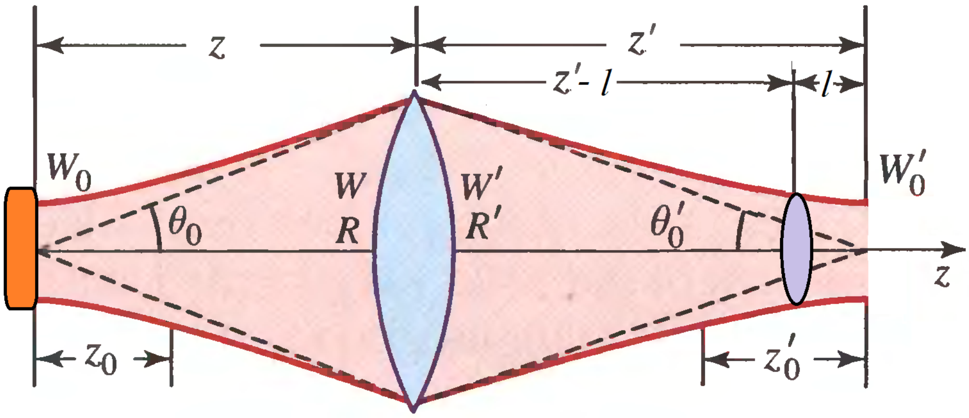

Figure 1: An analog communication system.

(a) *Assume that the beam width $W(z)$ and radius of curvature $R(z)$ are given at the location z. Determine the location of the waist z and waist radius W_0 in terms of $W(z)$, $R(z)$, and λ.*

(b) *Show that the focused beam is a Gaussian beam with appropriate wait radius W_0' and waist location z'. Find the field pattern $U(\rho)$ at the photo-detector surface.*

(c) *Find the intensity pattern $I(\rho)$ at the photo-detector surface.*

(d) *Find the signal power P_s impinged on the photo-detector.*

(e) *Find the noise power P_b impinged on the photo-detector*

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(f) Find the mean, variance, and SNR of the photo-detector carrier count if the background noise is weak and the number of received spatial and temporal modes over the interval T is high.

(g) Find the shot noise-limited SNR. Assume that the message bandwidth, message power, modulation index, and deterministic photo-multiplication gain are B_m, P_m, β, and $g = 1$, respectively.

(h) Find the optimal location $0 \leq l \leq z'$ of the photo-detector resulting in the maximum shot noise-limited SNR. Assume that the photo-detector area is very small.

(i) Find the optimal area A_d of the photo-detector resulting in the maximum shot noise-limited SNR. Assume that the photo-detector area is very small.

Question 2

An incoherent Mary digital optical communication system with the count intensities $n_i(t) = in_s, 0 \leq t \leq T_b, i = 0, 1, \cdots, M-1$ is given. Assume that

1. **N_0 is the background noise spectral density.**
2. **B_0 is the optical bandwidth.**
3. **D_s is the number of noise spatial modes.**
4. **T_b is the symbol interval.**
5. **τ_s is the sampling period.**
6. **α is the Fermi's proportionality constant.**

(a) Assuming the Poission detection condition, find the optimal ML decoder structure.

(b) Find the union bound for the word error probability. Merge the error terms involving random selection of a symbol among some possible choices to other terms.

(c) Find the approximated bit error probability.