Microwave Magnetics

Graduate Course

Electrical Engineering (Communications)

2nd Semester, 1394-1395

Sharif University of Technology

General information

- **Contents of lecture 2:**
  - Magnetic phenomena and alignment of dipoles in magnetic materials
    - *Classical paramagnetism*
    - *Ferromagnetism*
    - *Antiferromagnetism and ferrimagnetism*
    - *Ferromagnets in equilibrium*

Magnetism and magnetic phenomena

- We discussed magnetic field generated by a dipole. We generalized that to a collection of dipoles through the concept of magnetization.
- Most solids in nature consist of atoms with a nonzero total angular momentum and, thus, a magnetic moment.
- But in most materials no net magnetization is observed. Why some materials have magnetic properties and some do not?

Magnetism and magnetic phenomena

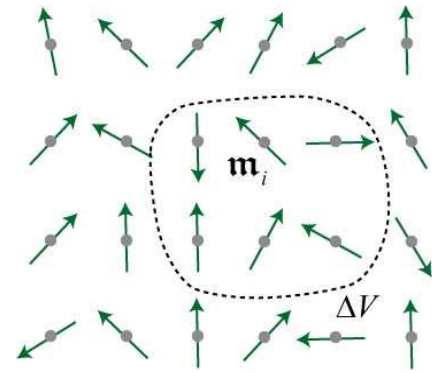
- Also, we described the magnetization dynamics by the equation

$$\frac{d\boldsymbol{M}}{dt} = -\gamma \boldsymbol{M} \times \boldsymbol{H}$$

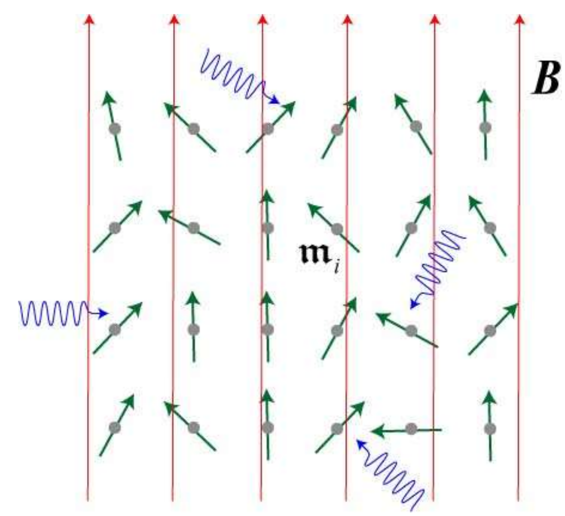
- But this equation describes the evolution of magnetization from an initial state. What is nature of this state?
- Are magnetic forces the only forces acting on dipoles?
- What about thermal “forces” and fluctuations?
- Answering these questions requires knowledge about microscopic forces between dipoles, and their relative alignment in magnetic materials

(i) *Classical paramagnetism*

- How are the magnetic moments of atoms aligned in solids?
- For the moment, *neglect* all the interactions between the dipoles
- If no magnetic field is present, no net magnetization survives due to thermal processes
- If a field exists: competition between order and thermal disorder

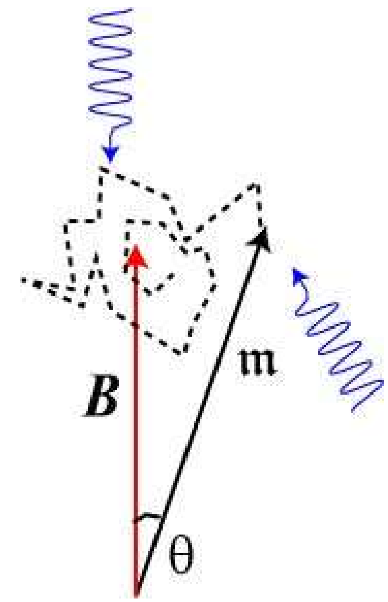


$$\boldsymbol{M}(\boldsymbol{r}) = \frac{1}{\Delta V}\sum_{i=1}^{N} \mathbf{m}_i$$



(i) *Classical paramagnetism*

- Motion of dipoles: combination of precession in a magnetic field, and random motion due to thermal collisions with phonons, electrons, etc
- Random thermal motion much faster than other time scales: one can apply averaging and find the average dipole direction (magnetization direction)
- (Time-) average state of a system found by statistical equilibrium



$$U = U(\theta) = -\mathfrak{m} \cdot \boldsymbol{B} = -\mathfrak{m} B_z \cos\theta$$

(i) *Classical paramagnetism*

- At room temperature, in realistic materials $mB_z << k_B T$
- Since thermal energy is much larger than magnetic energy, no net magnetization survives the when applied magnetic field is removed
- Even in presence of the field paramagnetism is a relatively small effect because of thermal fluctuations
- But, in nature, we observe materials with
  - Remnant magnetization even when there is no applied field
  - Large magnetic response (eg: high permeability)
- What is the physical mechanism for these phenomena?

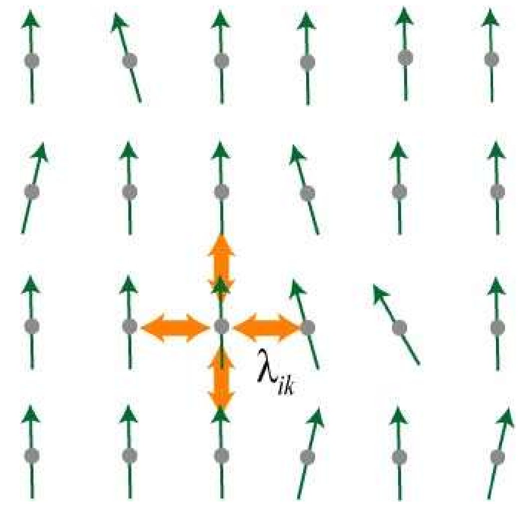
(ii) *Ferromagnetism*

- Remnant magnetization implies that there is a force which can resist the thermal processes, and align the microscopic dipoles, but which force?
- Magnetic force between dipoles cannot be the answer: energy far too small compared to thermal energy
- Solution provided by quantum mechanics in the early 20's century: in magnetic materials another type of dipole interaction exists, which is of electrical origin
- This interaction, called exchange interaction, is related to the Pauli exclusion principle, combined with Coulomb forces between electrons in adjacent atoms

(ii) *Ferromagnetism*

- Exchange interaction expressed as

$$H_{ex} = -\frac{1}{2}\sum_{i}\sum_{k(\neq i)} \lambda_{ik} \boldsymbol{J}_i \cdot \boldsymbol{J}_k$$



- If exchange coefficients are positive the exchange interaction will tend to align the dipoles in parallel
- Since exchange energy is large (much larger than thermal energy), it will resist the thermal fluctuations; random thermal motion will be less effective

(ii) *Ferromagnetism*

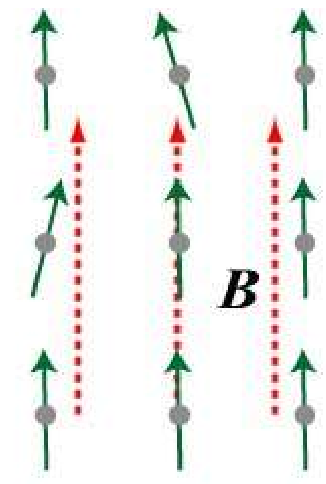
- But thermal effects are not unimportant: above a certain temperature (Curie temperature) they will destroy the ferromagnetic order
- Well below Curie temperature: dipoles more or less aligned everywhere
- At any point inside the ferromagnet the magnitude of $\boldsymbol{M}$ is given by the saturation magnetization

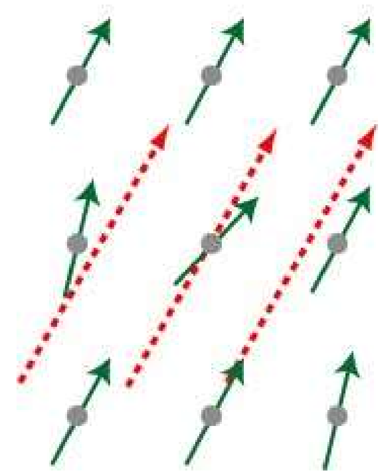
$$\left|\boldsymbol{M}(\boldsymbol{r})\right| = M_s \approx N_m \mathfrak{m} = N_m \gamma J$$

Volume density of dipoles

(ii) *Ferromagnetism*

- Even without a magnetic field, the dipoles are more or less aligned (on average)
- When there is a field, the dipoles collectively turn and become parallel to the field (through damped precession) in order to minimize the magnetic energy
- The field is the sum of the “external” and the “internal” fields. It may vary in space
- As a result, $\boldsymbol{M}$ follows the magnetic field, changing its direction in space





(ii) *Ferromagnetism*

- Remarks:
  - The picture presented above is oversimplified
  - If magnetization follows a non-uniform field to minimize magnetic energy, then its direction will change in space
  - Since the dipoles are not aligned anymore, exchange force acts on dipoles and tries to restore them to the parallel case (especially when changes occur over short distances < 1um)
  - The final magnetization landscape is the result of the competition between exchange energy and magnetic energy, with the latter caused by internal as well as external magnetic fields
  - This leads to the formation of magnetic domains which are not considered in these lectures

Ferromagnetic materials

- Ferromagnets are often metals or metallic alloys

Material	Fe	Ni	$Ni_{80}Fe_{20}$	Co	MnBi
M_s x 10^5 A/m	~16	~5	8-9	14-15	~6

- There are also some insulating ferromagnets

Material	CrO_2	$MnOFe_2O_3$	$NiOFe_2O_3$	$CuOFe_2O_3$
M_s x 10^5 A/m	~5	~4	~2.7	~1.4

Ferromagnetic materials

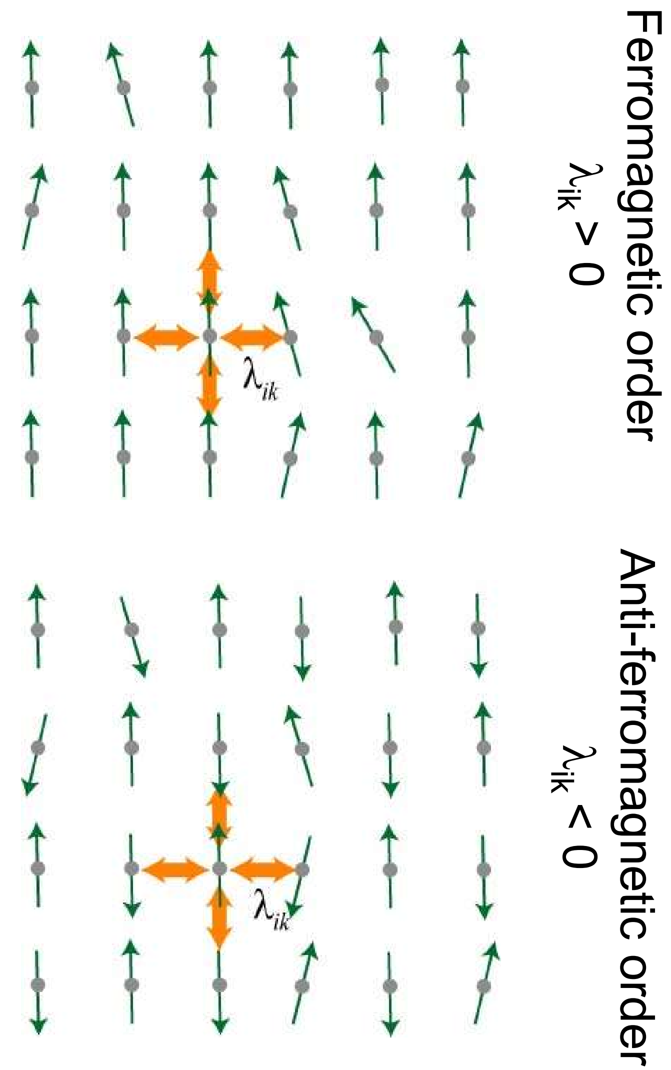
- But metallic ferromagnets are very lossy at microwave frequencies due to their high electric conductance
- Insulating ferromagnets are also lossy because of a high damping (friction) constant
- In the 1950's huge advances were made in fabricating insulating magnetic materials with very low losses
- They were called "ferrites" or ferrimagnets. To understand them, it is necessary to return to the theory of ordering of magnetic dipoles.
- First, let us consider "anti-ferromagnetic" ordering

Anti-ferromagnetic materials

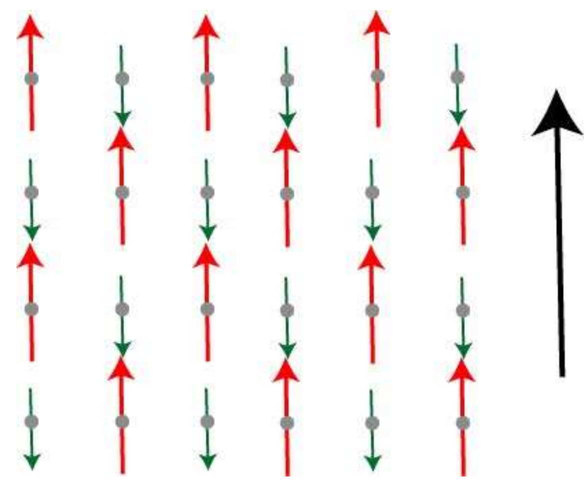
- In anti-ferromagnets the coefficients of exchange interaction have a negative sign
- The exchange force tends to give neighboring dipoles an anti-parallel orientation

$$H_{ex} = -\frac{1}{2}\sum_{i}\sum_{k(\neq i)}\lambda_{ik}\boldsymbol{J}_i\cdot\boldsymbol{J}_k$$

- Not very interesting for microwaves since net magnetization is zero!



Ferromagnetic materials (ferrites)

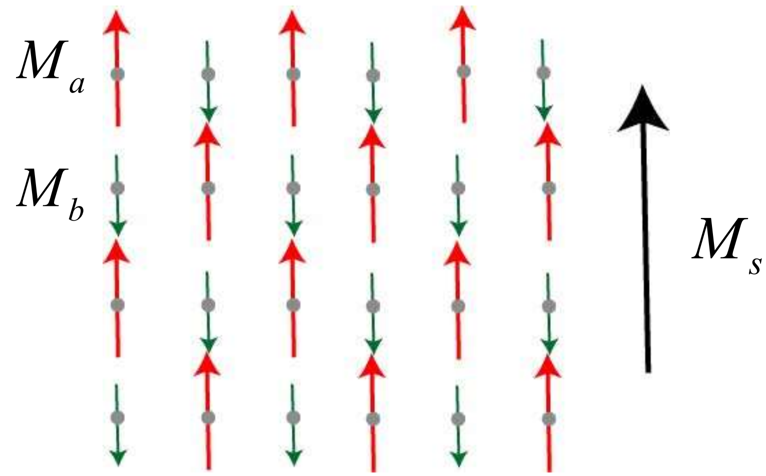


- But if neighboring atoms are different, and have a different magnetic moment, a net magnetization arises
- Such materials are called ferrimagnets or ferrites
- In a magnetic field the dipoles will again turn together
- Average magnetization behaves like in ferromagnets under dc and ac fields
- A too strong field will overcome the exchange force and destroy this order (making dipoles parallel), but this is out of the scope of this course

Ferromagnetic materials (ferrites)

- Disadvantage: saturation magnetization smaller than ferromagnets

$$M_s = M_a - M_b$$



- But they are insulating with low magnetic losses

Material	$Y_3Fe_5O_{12}$ (YIG)	$(Ni,Zn)Fe_2O_4$	$BaFe_{12}O_{19}$
M_s x 10^5 A/m	~1.5	~4	~4

Ferromagnetic materials (ferrites)

- In problems involving ferrites, we use the same formalism as for ferromagnets (LLG equation, susceptibility, permeability, anisotropy, …)
- This is allowed if the applied fields are not too large and the frequency is not too high

*Free energy in equilibrium

- Understanding the behavior of a magnetic medium with microscopic exchange interactions is quite complicated
- Common method: Landau's theory of phase transitions. Here, the free energy of the system is expressed in terms of the **macroscopic** magnetization ***M***

$$U = U_{ex,0} + U_{ex,1} + U_M^{ext} + U_M^{dip}$$

$$\begin{cases} U_{ex,0} = -\frac{1}{2}\int_V u_{ex,0}\left[M^2(\boldsymbol{r})\right]dV \\ U_{ex,1} = \frac{1}{2}q\int_V \nabla\boldsymbol{M}(\boldsymbol{r})\cdot\nabla\boldsymbol{M}(\boldsymbol{r})dV = \frac{1}{2}q\sum_{\nu=1}^{3}\frac{\partial\boldsymbol{M}(\boldsymbol{r})}{\partial r_\nu}\cdot\frac{\partial\boldsymbol{M}(\boldsymbol{r})}{\partial r_\nu}dV \end{cases}$$

Due to exchange

*Free energy

$$U_M^{ext} = -\mu_0 \int_V \boldsymbol{H}_{ext}(\boldsymbol{r}) \cdot \boldsymbol{M}(\boldsymbol{r}) dV$$

Due to external magnetic field

$$U_M^{dip} = \frac{\mu_0}{2} \int_V \int_V \boldsymbol{M}(\boldsymbol{r}) \cdot \nabla \nabla' \left(\frac{\mu_0}{4\pi |\boldsymbol{r} - \boldsymbol{r}'|} \right) \cdot \boldsymbol{M}(\boldsymbol{r}') dV dV'$$

$$= -\frac{\mu_0}{2} \int_V \boldsymbol{M}(\boldsymbol{r}) \cdot \boldsymbol{H}_M(\boldsymbol{r}) dV$$

Due to internal magnetic field

- First term is huge compared to the rest: its minimum gives a good starting point for the magnitude of ***M***. It is minimized according to

$$\min U_{ex,0} = \min \left\{ -\frac{1}{2} \int_V u_{ex,0} \left[M^2(\boldsymbol{r}) \right] dV \right\} = -\frac{V}{2} \min u_{ex,0} \left[M^2 \right]$$

*Free energy

- Let's call this minimum $M = |\boldsymbol{M}| = M_s$
- The rest of the free energy $U' = U_{ex,1} + U_M^{ext} + U_M^{dip}$ is next minimized under the condition $|\boldsymbol{M}| = M_s$
- Minimization of the functional U' under a constraint needs the use of Lagrange multipliers:

$$\frac{\delta U'}{\delta \boldsymbol{M}(\boldsymbol{r})} = \lambda(\boldsymbol{r})\boldsymbol{M}(\boldsymbol{r})$$

- An effective magnetic field is now defined as

$$\boldsymbol{H}_{eff}(\boldsymbol{r}) = -\frac{\delta U'}{\mu_0 \delta \boldsymbol{M}(\boldsymbol{r})} \rightarrow -\mu_0 \boldsymbol{H}_{eff}(\boldsymbol{r}) = \lambda(\boldsymbol{r})\boldsymbol{M}(\boldsymbol{r})$$

*Free energy

- In most engineering applications $U_{ex,1}$ is small (not in all cases like the study of magnetic domains)

$$U' \approx U_M^{ext} + U_M^{dip} = -\mu_0 \int_V \boldsymbol{H}_{ext}(\boldsymbol{r}) \cdot \boldsymbol{M}(\boldsymbol{r}) dV - \frac{\mu_0}{2} \int_V \boldsymbol{H}_M(\boldsymbol{r}) \cdot \boldsymbol{M}(\boldsymbol{r}) dV$$

$$\boldsymbol{H}_M(\boldsymbol{r}) = -\nabla \int_V \nabla' G_0(\boldsymbol{r} - \boldsymbol{r}') \cdot \boldsymbol{M}(\boldsymbol{r}') dV'$$

$$\boldsymbol{H}_{eff}(\boldsymbol{r}) = -\frac{\delta U'}{\mu_0 \delta \boldsymbol{M}(\boldsymbol{r})} = \boldsymbol{H}_{ext}(\boldsymbol{r}) + \boldsymbol{H}_M(\boldsymbol{r})$$

(ii) *Ferromagnets in equilibrium*

- So, to find the distribution of the magnetization in the equilibrium (static) state we have to:
  - Assume a magnetization with a constant magnitude depending on material (and temperature)
  - Calculate the extrinsic and intrinsic (demagnetization) field
  - Require the magnetization to be parallel to total field everywhere

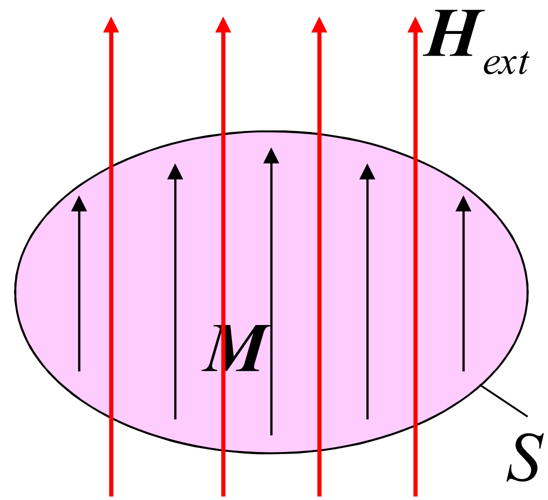
$$\left|\boldsymbol{M}(\boldsymbol{r})\right| = M_s$$

$$\boldsymbol{H}(\boldsymbol{r}) = \boldsymbol{H}_{ext}(\boldsymbol{r}) + \boldsymbol{H}_M(\boldsymbol{r})$$

$$\boldsymbol{H}(\boldsymbol{r}) \,||\, \boldsymbol{M}(\boldsymbol{r}) \rightarrow \boldsymbol{H}(\boldsymbol{r}) \times \boldsymbol{M}(\boldsymbol{r}) = 0$$

(ii) *Example: uniformly magnetized ellipsoids*

- It was shown before that a magnetic ellipsoid subject to a uniform external field has a uniform magnetization



- Magnetic field *induced by magnetization (demagnetization field)*:

$$\boldsymbol{H}_M = -\nabla \int_V \nabla' G_0(\boldsymbol{r}-\boldsymbol{r}') \cdot \boldsymbol{M} dV' = -\begin{bmatrix} N_x & 0 & 0 \\ 0 & N_y & 0 \\ 0 & 0 & N_z \end{bmatrix} \cdot \boldsymbol{M}$$

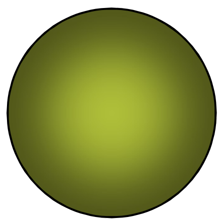
$$N_x + N_y + N_z = 1$$

Demagnetization factors

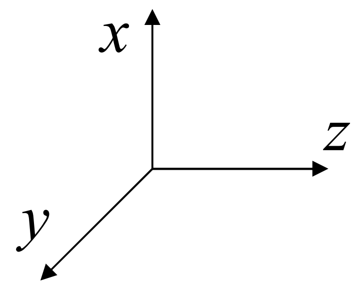
(ii) *Example: uniformly magnetized ellipsoids*

- Examples: some limiting cases of ellipsoids

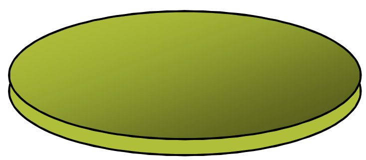
sphere



$$N_x = N_y = N_z = \frac{1}{3}$$



Infinitely thin disk



$$N_x = 1 \quad N_y = N_z = 0$$

Infinitely long rod

$$N_x = N_y = \frac{1}{2} \quad N_z = 0$$

(ii) *Example: uniformly magnetized ellipsoids*

- Hence, the total field is

$$\boldsymbol{H} = \boldsymbol{H}_{ext} + \boldsymbol{H}_{M} = \boldsymbol{H}_{ext} - \bar{\bar{\boldsymbol{N}}} \cdot \boldsymbol{M}$$

$$\bar{\bar{\boldsymbol{N}}} = \begin{bmatrix} N_x & 0 & 0 \\ 0 & N_y & 0 \\ 0 & 0 & N_z \end{bmatrix}$$

Demagnetization tensor

- Static direction of magnetization found by

$$\boldsymbol{H} \parallel \boldsymbol{M} \rightarrow \boldsymbol{H} \times \boldsymbol{M} = \left(\boldsymbol{H}_{ext} - \bar{\bar{\boldsymbol{N}}} \cdot \boldsymbol{M} \right) \times \boldsymbol{M} = 0$$

(ii) *Example: uniformly magnetized ellipsoids*

- If solution is not unique, or in cases where needed, one can look for the solution which minimizes the energy (in equilibrium)

$$U' = U_M^{ext} + U_M^{dip} = -\mu_0 \int_V \boldsymbol{H}_{ext}(\boldsymbol{r}) \cdot \boldsymbol{M}(\boldsymbol{r}) dV - \frac{\mu_0}{2} \int_V \boldsymbol{H}_M(\boldsymbol{r}) \cdot \boldsymbol{M}(\boldsymbol{r}) dV$$

- In case of a uniformly magnetized ellipsoid

$$U' = -\mu_0 V \boldsymbol{H}_{ext} \cdot \boldsymbol{M} + \frac{\mu_0 V}{2} \boldsymbol{M} \cdot \overline{\overline{\boldsymbol{N}}} \cdot \boldsymbol{M}$$