Microwave Magnetics

Graduate Course

Electrical Engineering (Communications)

2nd Semester, 1389-1390

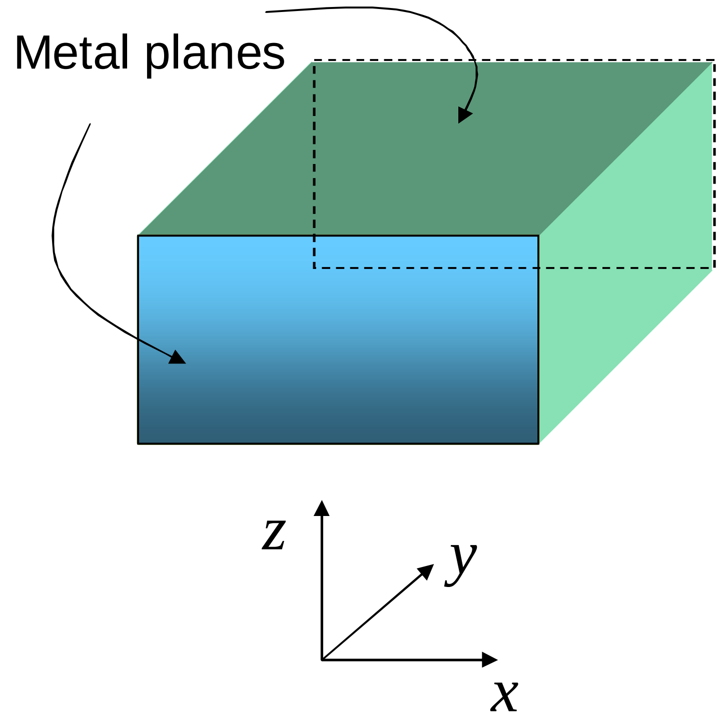
Sharif University of Technology

General information

- **Contents of lecture 8:**
  - Waveguide resonators
    - General principles
    - Completely filled rectangular cavity resonator
    - Partially filled rectangular cavity resonator

(i) Waveguides resonators

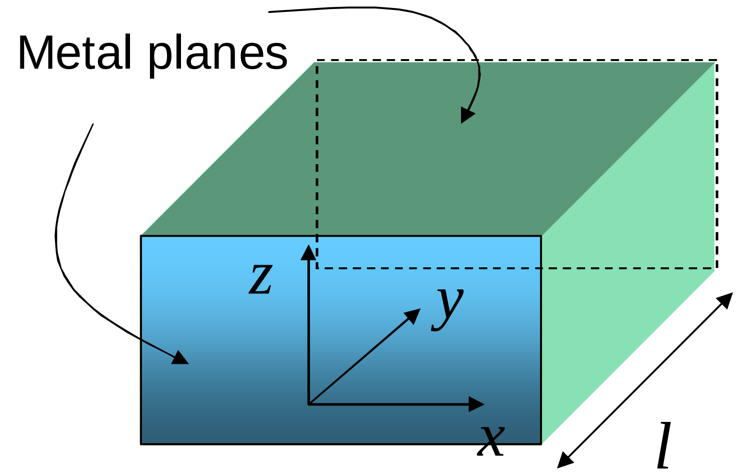
- Waveguide resonator: section of a waveguide bounded by two metal planes perpendicular to the waveguide axis
- For any particular mode *n*, the solution inside an infinite waveguide is



$$\mathbf{e}_n = A_n^+ \tilde{\mathbf{e}}^{n,+}(x,z)\exp(-j\beta_n^+ y) + A_n^- \tilde{\mathbf{e}}^{n,-}(x,z)\exp(j\beta_n^- y)$$
$$\mathbf{h}_n = A_n^+ \tilde{\mathbf{h}}^{n,+}(x,z)\exp(-j\beta_n^+ y) + A_n^- \tilde{\mathbf{h}}^{n,-}(x,z)\exp(j\beta_n^- y)$$

(i) Waveguides resonators

- On the metal planes, tangential electric field must be zero
- For a single-mode solution, we must have



$$A_n^+ \begin{bmatrix} \tilde{e}_x^{n,+}(x,z) \\ \tilde{e}_z^{n,+}(x,z) \end{bmatrix} + A_n^- \begin{bmatrix} \tilde{e}_x^{n,-}(x,z) \\ \tilde{e}_z^{n,-}(x,z) \end{bmatrix} = 0$$

$$A_n^+ \begin{bmatrix} \tilde{e}_x^{n,+}(x,z) \\ \tilde{e}_z^{n,+}(x,z) \end{bmatrix} \exp(-j\beta_n^+ l) + A_n^- \begin{bmatrix} \tilde{e}_x^{n,-}(x,z) \\ \tilde{e}_z^{n,-}(x,z) \end{bmatrix} \exp(j\beta_n^- l) = 0$$

(i) Waveguides resonators

- In order to satisfy these equations, the tangential electric fields in the two propagation directions should have a simple relationship
- For instance, it the tangential electric field is the same in both directions, then it is sufficient to have

$$A_n^+ + A_n^- = 0$$

$$A_n^+ \exp(-j\beta_n^+ l) + A_n^- \exp(j\beta_n^- l) = 0$$

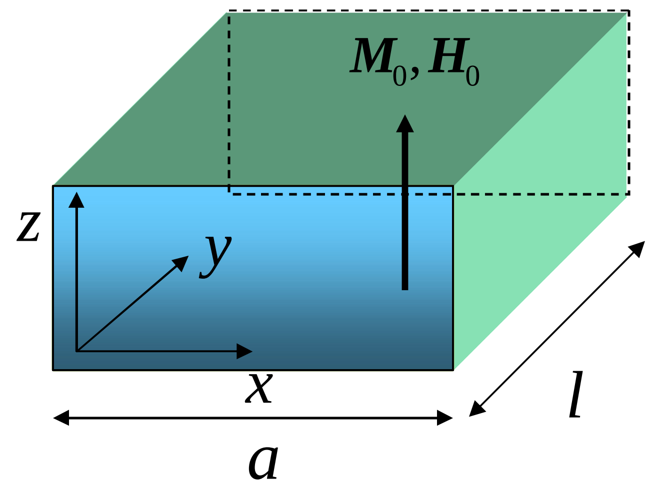
$$\Rightarrow \quad \frac{\beta_n^+ + \beta_n^-}{2} = \frac{p\pi}{l} \quad p = 1, 2, 3, \cdots$$

Condition of resonance

(ii) *Ferrite filled rectangular resonator*

- A simple example is a rectangular cavity resonator completely filled with a magnetic material magnetized along one axis (here z-axis)

- For a waveguide completely filled with magnetic material we can easily analyze this problem ***for solutions uniform in z-direction***



$$\tilde{e}_x^{n0,\pm}(x,y) = 0$$

$$\tilde{e}_z^{n0,\pm}(x,y) = \sin\left(\frac{n\pi x}{a}\right) \qquad \beta_{n0}^{\pm} = \sqrt{k_0^2 \varepsilon \mu_\perp - \left(\frac{n\pi}{a}\right)^2}$$

(ii) *Ferrite filled rectangular resonator*

- The resonance condition is

$$k_0^2 \varepsilon \mu_\perp = \left(\frac{n\pi}{a}\right)^2 + \left(\frac{p\pi}{l}\right)^2$$

$$k_0^2 \varepsilon = \omega^2 \varepsilon_0 \mu_0 \varepsilon$$

$$\mu_\perp = \frac{(\omega_M + \omega_H)^2 - \omega^2}{\omega_\perp^2 - \omega^2}$$

- Solution:

$$\omega_{np}^2 = \frac{1}{2}\left[\omega_{0,np}^2 + (\omega_H + \omega_M)^2 \pm \sqrt{\omega_{0,np}^4 + (\omega_H + \omega_M)^4 + 2\omega_{0,np}^2 \left(\omega_M^2 - \omega_H^2\right)}\right]$$

$$\omega_{0,np}^2 = \frac{1}{\varepsilon_0 \varepsilon \mu_0}\left[\left(\frac{n\pi}{a}\right)^2 + \left(\frac{p\pi}{l}\right)^2\right]$$

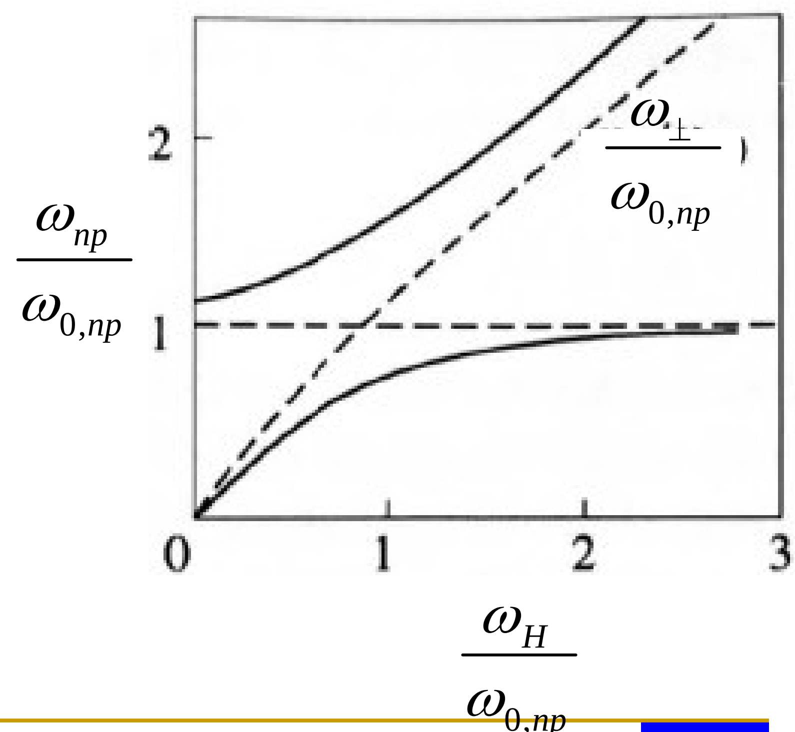
Resonance frequency of the same resonator but filled with a nonmagnetic dielectric with the same dielectric constant

(ii) *Ferrite filled rectangular resonator*

- Plotting resonance frequency as function of ω_H:

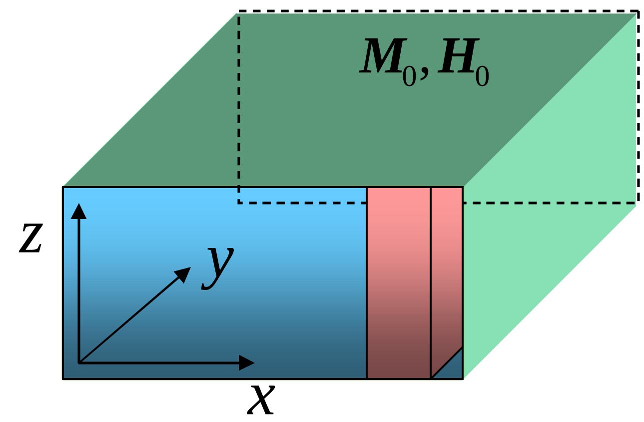
$$\omega_{np}^2 = \frac{1}{2}\left[\omega_{0,np}^2 + \left(\omega_H + \omega_M\right)^2 \pm \sqrt{\omega_{0,np}^4 + \left(\omega_H + \omega_M\right)^4 + 2\omega_{0,np}^2\left(\omega_M^2 - \omega_H^2\right)}\right]$$

- Behavior resembles that of coupled oscillators.
- Resonator may be viewed as consisting of two coupled subsystems: cavity resonator and the magnetic filling



(ii) *Ferrite filled rectangular resonator*

- Can we extend the analysis presented to partially filled waveguide?
- Actually we cannot. The analysis of a partially filled waveguide terminated by two metallic planes cannot be performed using a single propagating mode of a partially filled waveguide.
- Since the exact treatment is complicated, we once more try the perturbation theory



(iii) *Perturbation methods*

- Perturbation lemmas:
- Unperturbed (initial) system which is considered to have scalar permeability and dielectric constant. Assume the solution to be known and to satisfy Maxwell

$$\nabla \times \mathbf{e_0} = -j\omega_0\mu_0\mu_{ref}\mathbf{h_0} \qquad \nabla \times \mathbf{h_0} = j\omega_0\varepsilon_0\varepsilon_{ref}\mathbf{e_0}$$

- For the perturbed system:

$$\nabla \times \mathbf{e} = -j\omega\mu_0\bar{\bar{\mu}}\cdot\mathbf{h} \qquad \nabla \times \mathbf{h} = j\omega\varepsilon_0\varepsilon\mathbf{e}$$

- Note: frequencies not the same

(iii) *Perturbation methods*

- Perturbation lemma:

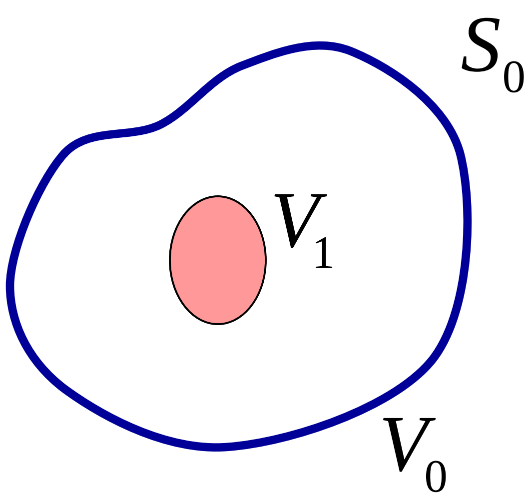
$$\nabla\cdot\left(\mathbf{e}\times\mathbf{h}_0^*+\mathbf{e}_0^*\times\mathbf{h}\right)=-j\mu_0\mathbf{h}_0^*\cdot\left(\omega\bar{\bar{\mu}}-\omega_0\mu_{ref}\bar{\bar{1}}\right)\cdot\mathbf{h}$$

$$-j\varepsilon_0\left(\omega\varepsilon-\omega_0\varepsilon_{ref}\right)\mathbf{e}_0^*\cdot\mathbf{e}$$

$$\oint_{S_0}\left(\mathbf{e}\times\mathbf{h}_0^*+\mathbf{e}_0^*\times\mathbf{h}\right)\cdot d\mathbf{s}=$$

$$-j\mu_0\omega\int_{V_1}\mathbf{h}_0^*\cdot\left(\bar{\bar{\mu}}-\mu_{ref}\bar{\bar{1}}\right)\cdot\mathbf{h}\,dV$$

$$-j\varepsilon_0\omega\int_{V_1}\left(\varepsilon-\varepsilon_{ref}\right)\mathbf{e}_0^*\cdot\mathbf{e}\,dV$$



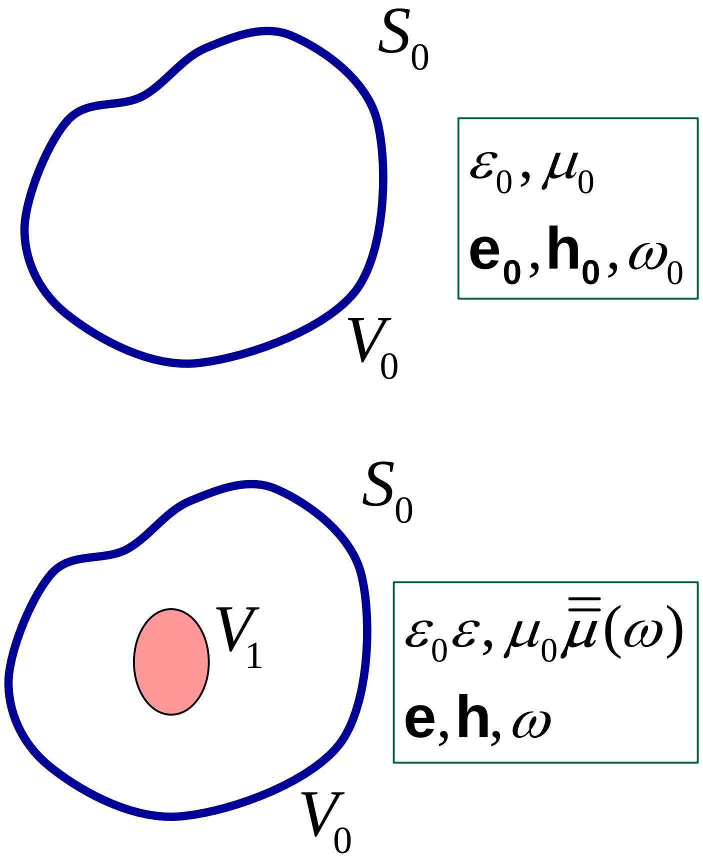
(iii) *Perturbation methods*

- Comparing two cases: cavity resonator with- and without magnetic material, we find the exact result:

$$\frac{\omega-\omega_0}{\omega}=-\frac{\int_{V_1}\left[\mu_0\mathbf{h}_0^*\cdot\delta\bar{\bar{\boldsymbol{\mu}}}\cdot\mathbf{h}+\varepsilon_0\delta\varepsilon\mathbf{e}_0^*\cdot\mathbf{e}\right]dV}{\int_{V_0}\left[\mu_0\mathbf{h}_0^*\cdot\mathbf{h}+\varepsilon_0\mathbf{e}_0^*\cdot\mathbf{e}\right]dV}$$

$$\delta\bar{\bar{\boldsymbol{\mu}}}=\bar{\bar{\mu}}(\omega)-\bar{\bar{\mathbf{1}}}$$

$$\delta\varepsilon=\varepsilon-\mathbf{1}$$



(iv) *Partially filled rectangular resonator*

- We next adopt the same approximation as before:
  - In the denominator fields are replaced by fields in an empty resonator
  - The magnetic slab is modeled as a uniformly magnetized ellipsoid described by demagnetization factors along the 3 axes
  - We use the external susceptibility tensor

$$\frac{\omega-\omega_0}{\omega} \approx -\frac{\int_{V_1}\left[\mu_0 \mathbf{h}_0^* \cdot \overline{\overline{\chi}}^e \cdot \mathbf{h}_0 + \varepsilon_0 \delta\varepsilon \mathbf{e}_0^* \cdot \mathbf{e}\right] dV}{\int_{V_0}\left[\mu_0 \mathbf{h}_0^* \cdot \mathbf{h}_0 + \varepsilon_0 \mathbf{e}_0^* \cdot \mathbf{e}_0\right] dV}$$

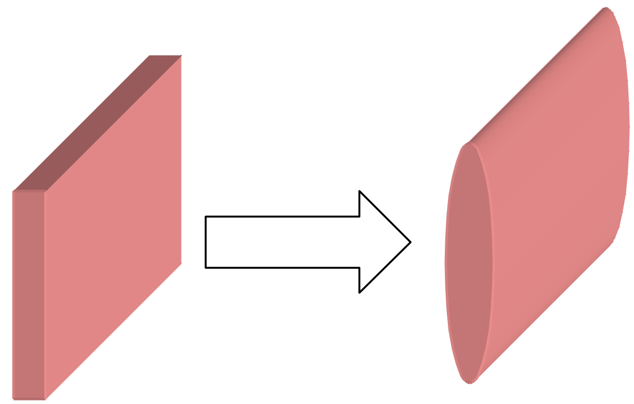
(iv) *Partially filled rectangular resonator*

- We can disregard the electric term if the dielectric constant of the magnetic material is close to one.
- If this is not the case then we have to define the reference system as one with a dielectric slab ***with*** the same dielectric constant as the magnetic slab, but ***without*** magnetic properties
- We could have done the same for waveguides. But we did not do so to keep the reference system simple.
- Again, for simplicity, we assume the dielectric constant of the material to be nearly one.

(iv) *Partially filled rectangular resonator*

- Therefore, we have

$$\frac{\omega-\omega_0}{\omega} \approx -\frac{\int_{V_1}\left[\mu_0 \mathbf{h}_0^* \cdot \bar{\bar{\chi}}^e \cdot \mathbf{h}_0\right] dV}{\int_{V_0}\left[\mu_0 \mathbf{h}_0^* \cdot \mathbf{h}_0 + \varepsilon_0 \mathbf{e}_0^* \cdot \mathbf{e}_0\right] dV}$$



$$\bar{\bar{\chi}}^e = \frac{1}{\Delta}\begin{bmatrix} \left(\omega_H + N_y\omega_M\right)\omega_M & j\omega_M\omega & 0 \\ -j\omega_M\omega & \left(\omega_H + N_x\omega_M\right)\omega_M & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\bar{\bar{N}} = \begin{bmatrix} N_x & 0 & 0 \\ 0 & N_y & 0 \\ 0 & 0 & N_z \end{bmatrix}$$

$$\Delta = \left(\omega_H + N_x\omega_M\right)\left(\omega_H + N_y\omega_M\right) - \omega^2$$

(iv) *Partially filled rectangular resonator*

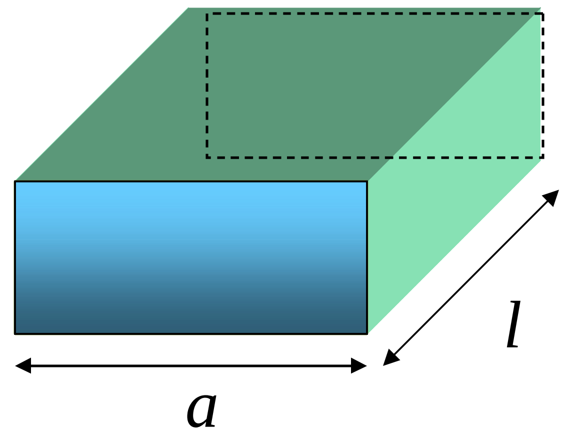
- Example: np0 mode of an empty resonator (fields uniform in the z-direction, no magnetic field along the z-direction)

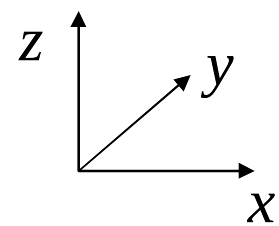
$$\mathrm{e}_z^{\mathrm{np0}}(x,y) = A_{np0}\sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{p\pi y}{l}\right)$$

$$\mathrm{h}_x^{\mathrm{np0}}(x,y) = -\frac{A_{np0}}{j\omega_{0,np}\mu_0}\left(\frac{p\pi}{l}\right)\sin\left(\frac{n\pi x}{a}\right)\cos\left(\frac{p\pi y}{l}\right)$$

$$\mathrm{h}_y^{\mathrm{np0}}(x,y) = \frac{A_{np0}}{j\omega_{0,np}\mu_0}\left(\frac{n\pi}{a}\right)\cos\left(\frac{n\pi x}{a}\right)\sin\left(\frac{p\pi y}{l}\right)$$

$$\omega_{0,np}^2 = \frac{1}{\varepsilon_0\mu_0}\left[\left(\frac{n\pi}{a}\right)^2 + \left(\frac{p\pi}{l}\right)^2\right]$$





(iv) *Partially filled rectangular resonator*

- Denominator:

$$\int_V \left[\mu_0 \mathbf{h}_0^* \cdot \mathbf{h}_0 + \varepsilon_0 \mathbf{e}_0^* \cdot \mathbf{e}_0\right] dV = \int_V \left[\mu_0 \left(\left|\mathrm{h}_x^{np0}\right|^2 + \left|\mathrm{h}_y^{np0}\right|^2\right) + \varepsilon_0 \left|\mathrm{e}_z^{np0}\right|^2\right] dV$$

$$= \frac{\varepsilon_0 \left|A_{np0}\right|^2 \int_V \left[\left(\frac{p\pi}{l}\right)^2 \sin\left(\frac{n\pi x}{a}\right)^2 + \left(\frac{n\pi}{a}\right)^2 \sin^2\left(\frac{p\pi y}{l}\right)\right] dV}{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{p\pi}{l}\right)^2}$$

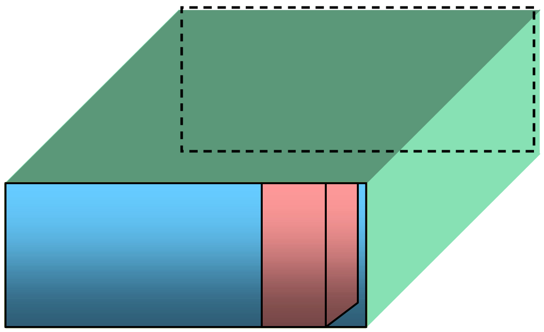
$$= \frac{1}{2} \varepsilon_0 \left|A_{np0}\right|^2 V$$

(iv) *Partially filled rectangular resonator*

- Nominator:

$$\left(\tilde{\mathbf{h}}_0^{np0}\right)^* \cdot \bar{\bar{\chi}}^e \cdot \tilde{\mathbf{h}}_0^{np0} = \bar{\bar{\chi}}^e_{xx} \left|\tilde{h}_x^{np0}\right|^2 + \bar{\bar{\chi}}^e_{xy} \left(\tilde{h}_x^{np0}\right)^* \tilde{h}_y^{np0} + \bar{\bar{\chi}}^e_{yx} \left(\tilde{h}_y^{np0}\right)^* \tilde{h}_x^{np0}$$

$$+\bar{\bar{\chi}}^e_{yy} \left|\tilde{h}_y^{np0}\right|^2 = \bar{\bar{\chi}}^e_{xx} \left|\tilde{h}_x^{np0}\right|^2 + \bar{\bar{\chi}}^e_{yy} \left|\tilde{h}_y^{np0}\right|^2$$



$$\mu_0 \int_{V_1} \left[\left(\tilde{\mathbf{h}}_0^{np0}\right)^* \cdot \bar{\bar{\chi}}^e \cdot \tilde{\mathbf{h}}_0^{np0}\right] dV = \frac{1}{2} \frac{\varepsilon_0 \left|A_{np0}\right|^2 V}{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{p\pi}{l}\right)^2} \times$$

$$\frac{1}{a} \int_{x_1}^{x_2} \left[\bar{\bar{\chi}}^e_{xx} \left(\frac{p\pi}{l}\right)^2 \sin\left(\frac{n\pi x}{a}\right)^2 + \bar{\bar{\chi}}^e_{yy} \left(\frac{n\pi}{a}\right)^2 \cos^2\left(\frac{n\pi x}{a}\right)\right] dx$$

(iv) *Partially filled rectangular resonator*

- Resonance frequency:

$$\omega_{np} \approx \frac{\omega_{0,np}}{1+c_{np}}$$

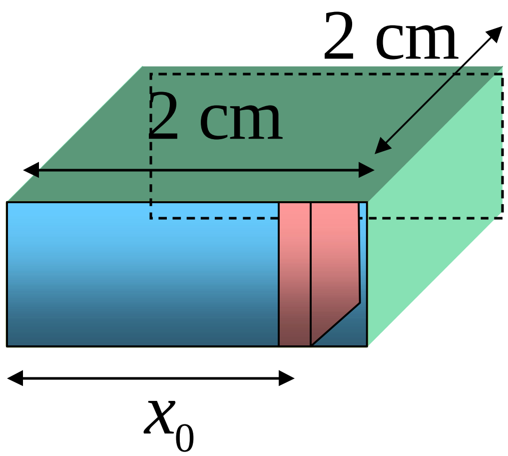
$$c_{np} = \frac{\int_{x_1}^{x_2} \left[\bar{\bar{\chi}}^e_{xx} \left(\frac{p\pi}{l}\right)^2 \sin\left(\frac{n\pi x}{a}\right)^2 + \bar{\bar{\chi}}^e_{yy} \left(\frac{n\pi}{a}\right)^2 \cos^2\left(\frac{n\pi x}{a}\right) \right] dx}{a\left[\left(\frac{n\pi}{a}\right)^2 + \left(\frac{p\pi}{l}\right)^2 \right]}$$

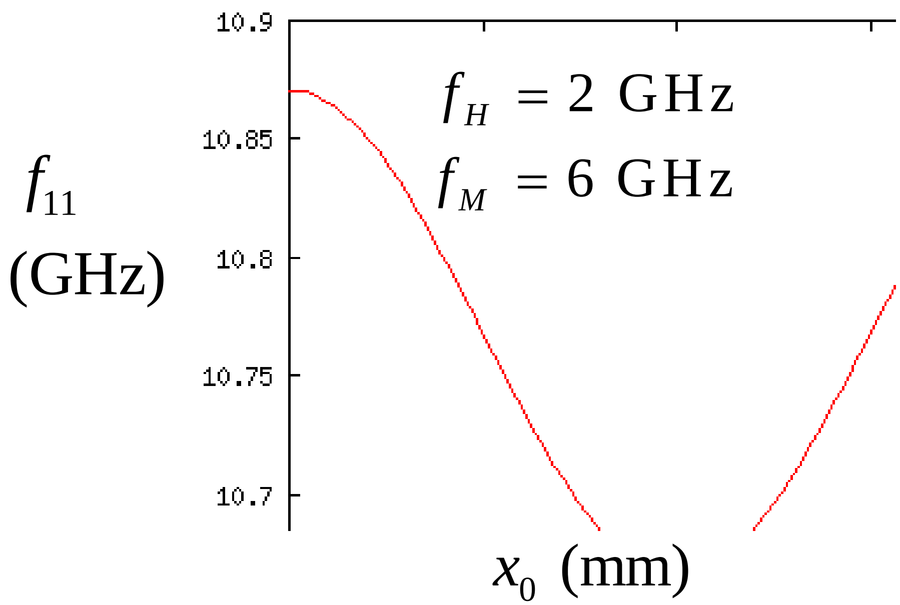
- Because susceptibilities depend on frequency, the above equation is an implicit one. But if the difference with the empty cavity is small, one can evaluate susceptibilities at the known resonance frequency of the empty cavity

(iv) *Partially filled rectangular resonator*

- Numerical example: $l=a=2$ cm, $n=p=1$ (110 mode), 2 mm-thick magnetic layer

$$f_{0,11} = 10.6\ \text{GHz}$$





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