Microwave Magnetics

Graduate Course

Electrical Engineering (Communications)

2^{nd} Semester, 1389-1390

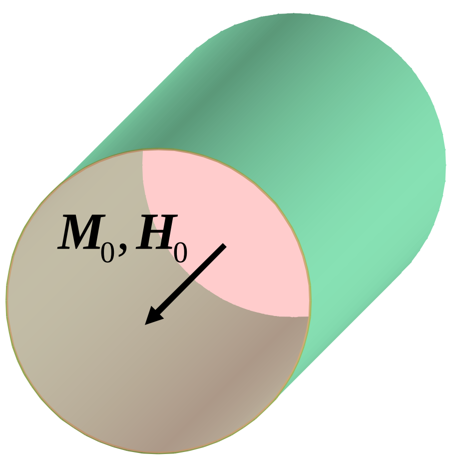
Sharif University of Technology

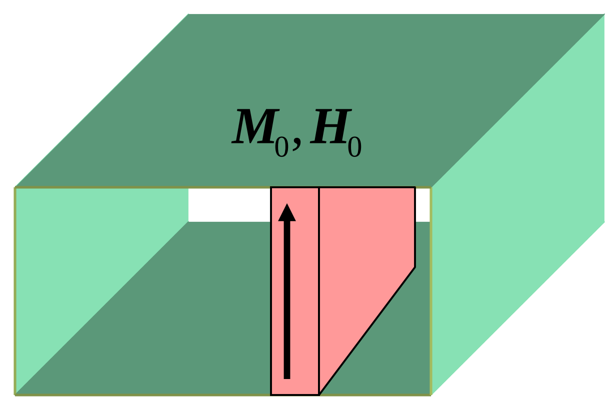
General information

- **Contents of lecture 8:**
  - Planar magnetic devices
    - Microstrip devices on a magnetic (ferrite) substrate
    - Field analysis
    - Edge-guided modes
    - Reciprocal phase shifters
    - Nonreciprocal phase shifters
    - Field displacement isolators
    - Mixed boundary conditions

Planar magnetic (ferrite) devices

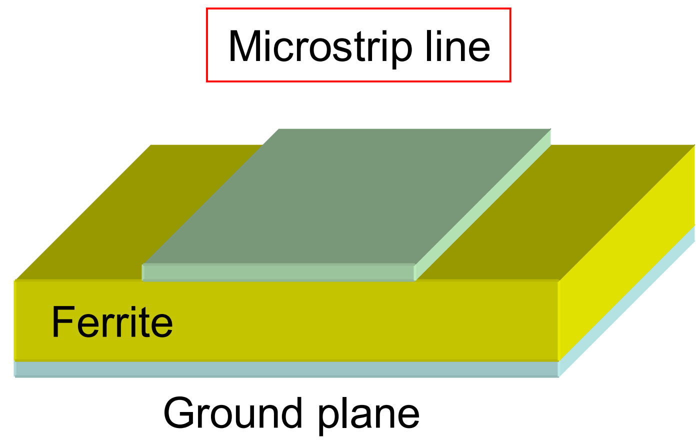
- The devices we have discussed so far utilize metallic waveguides (partially) loaded by magnetic materials
- But waveguides are difficult to build in integrated microwave circuits where devices are flat, and are implemented on a horizontal substrate

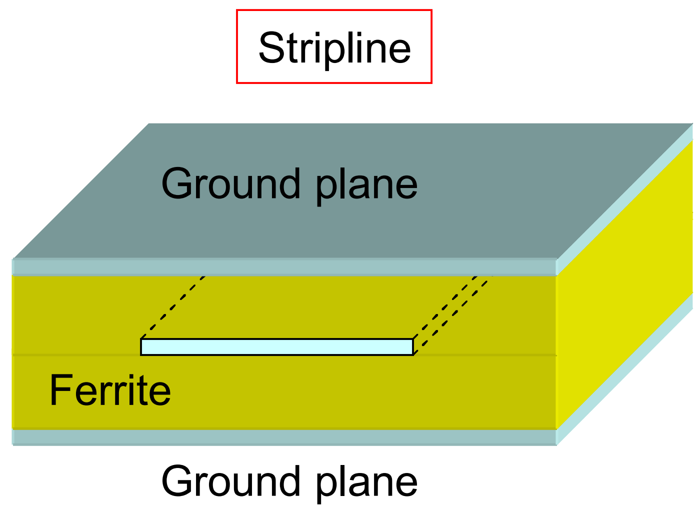




Planar magnetic (ferrite) devices

- Late 1960's: attention turned to devices which could be built on a ferrite substrate
- In particular, waveguides became less popular, and attention was shifted to devices based on planar transmission lines like microstrip and stripline
- Those were easily realized in microwave integrated circuits

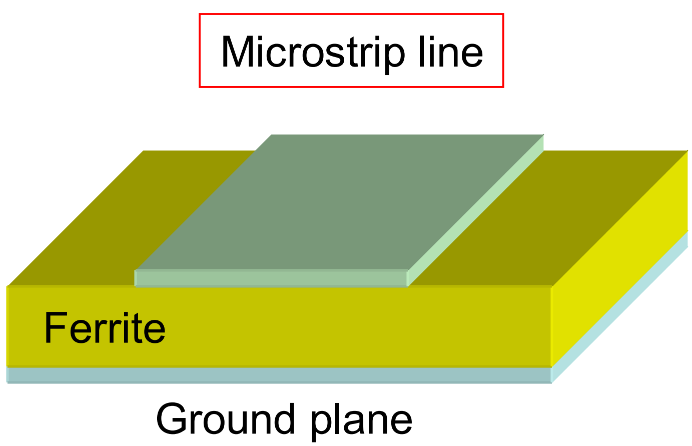




(i) *Microstrip magnetic (ferrite) devices*

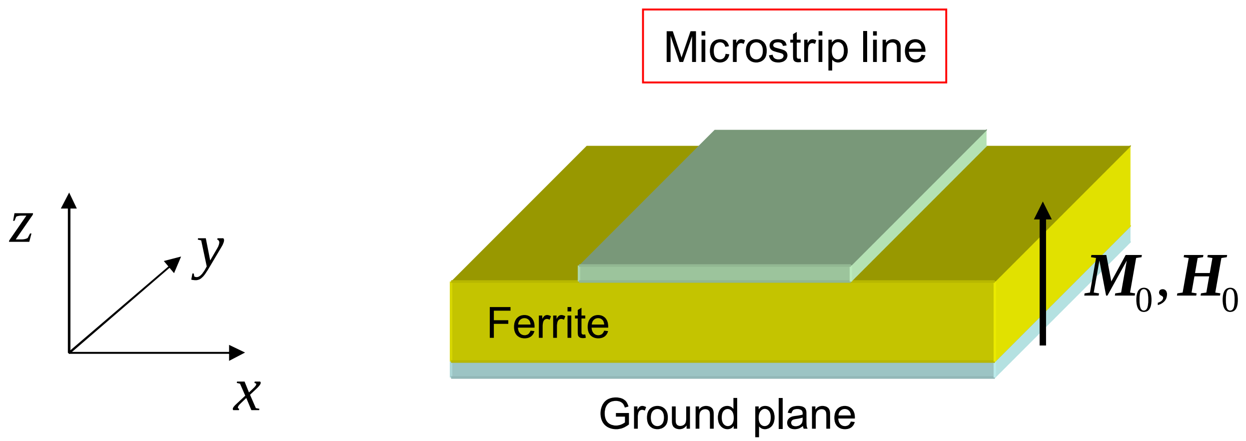
- Here we shall focus on microstrip magnetic devices which are more popular due to the ease of fabrication
- Analysis will be based the paper:

M.E. Hines, "Reciprocal and Nonreciprocal Modes of Propagation in Ferrite Stripline and Microstrip Devices", IEEE Transactions on Microwave Theory and Techniques, Vol. 19, Issue 5, 1971, pp. 442-451



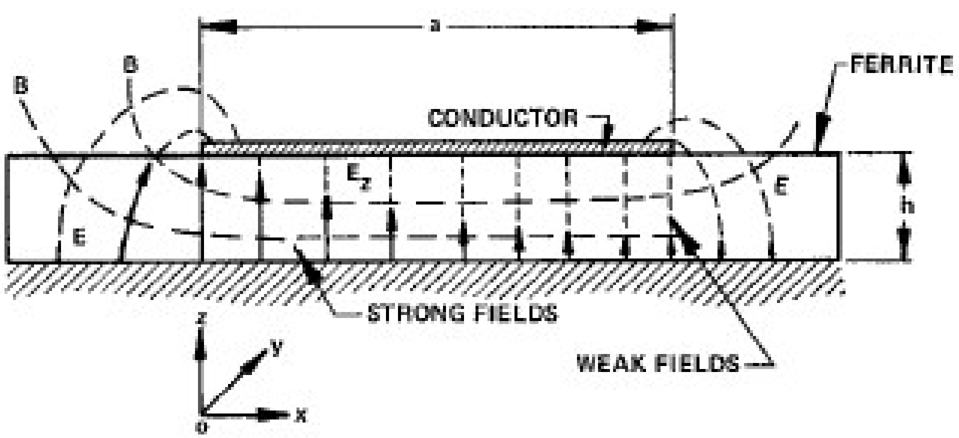
(i) *Microstrip magnetic (ferrite) devices*

- Consider a microstrip line on a grounded ferrite substrate, magnetized **perpendicular** to the substrate
- Other cases (in-plane magnetization along or transverse to the microstrip) have also been investigated, but their full electromagnetic analysis is difficult!



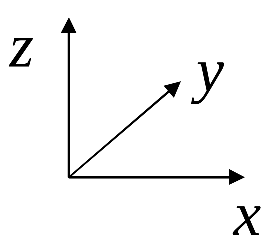
(i) *Microstrip magnetic (ferrite) devices*

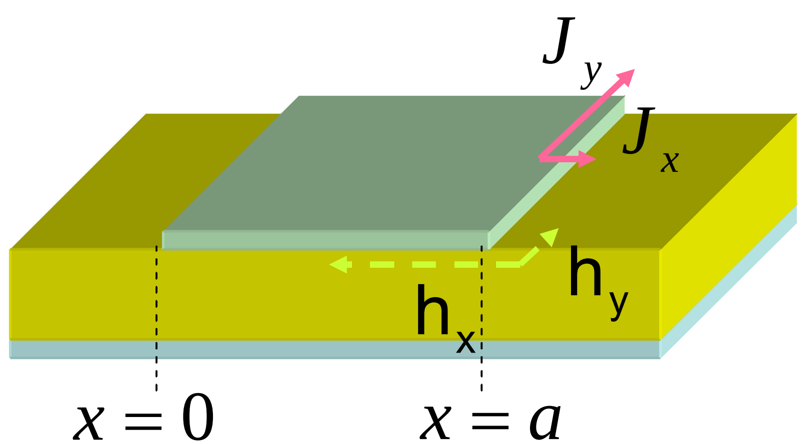
- We assume that the width of the microstrip is much larger than the thickness of the (ferrite) substrate
- Then most of the field is confined to the substrate region between the two metals.
- Fringing field becomes less important and can be replaced by boundary conditions



(i) *Microstrip magnetic (ferrite) devices*

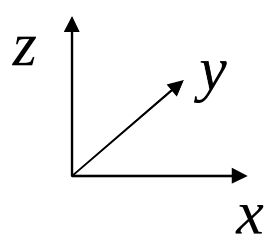
- A qualitative way to formulate approximate boundary conditions for microstrips (in general) is as follows:
- At the edges ($x = 0$, $x = a$) the transverse component (J_x) of current density is very small. The magnetic field h_y induced by current should also be small at the edges
- Also h_z is small (actually everywhere)

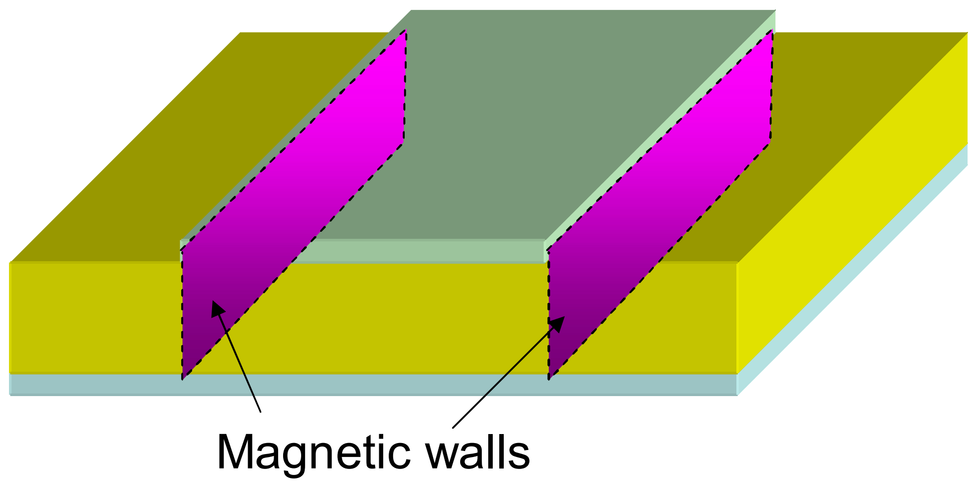




(i) *Microstrip magnetic (ferrite) devices*

- Magnetic wall approximation: assume the magnetic field in the y and z directions vanishes on the vertical walls between the microstrip edges and the ground plane
- These walls are magnetic walls: tangential magnetic field vanishes on these walls



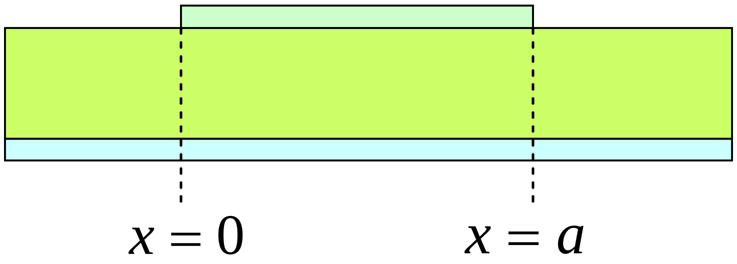


(i) *Microstrip magnetic (ferrite) devices*

- Resulting boundary conditions:

$$h_y = h_z = 0$$

when $x = 0$ and $x = a$



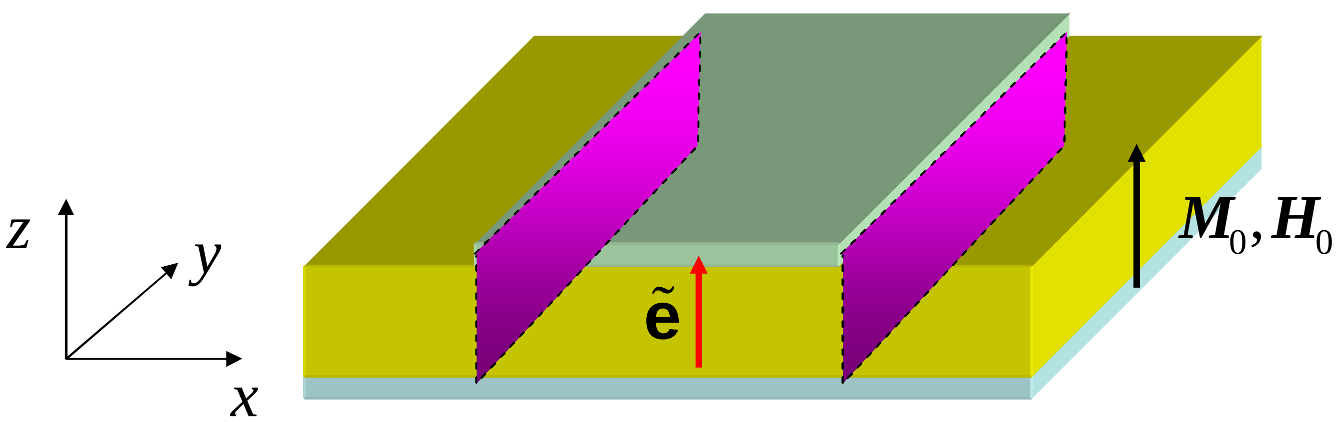
- We only have to solve the field equations in the region enclosed by the microstrip, the ground plane, and the two magnetic walls
- This problem is similar to that of a rectangular waveguide completely filled with magnetic material, only the lateral boundary conditions are different

(ii) *Field analysis*

- As for waveguides, neglect the variation along z (for simplicity), and look for solutions of the type

$$\mathbf{e} = \tilde{\mathbf{e}}(x)\exp(-j\beta y) \quad \mathbf{h} = \tilde{\mathbf{h}}(x)\exp(-j\beta y)$$

- Again, only solutions with the electric field in the z-direction will be acceptable

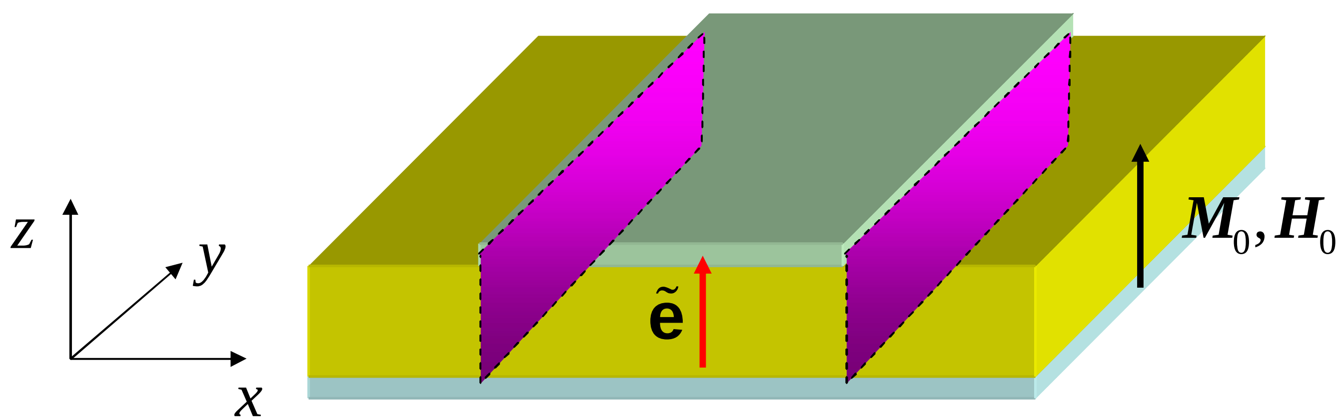


(ii) *Field analysis*

- Field equations

$$\frac{\partial^2 \tilde{e}_z}{\partial x^2} + \left(k_0^2 \varepsilon \mu_\perp - \beta^2\right)\tilde{e}_z = 0$$

$$\begin{bmatrix} \tilde{h}_x \\ \tilde{h}_y \end{bmatrix} = \frac{1}{j\omega\mu_0(\mu^2 - \mu_a^2)} \begin{bmatrix} \mu & -j\mu_a \\ j\mu_a & \mu \end{bmatrix} \begin{bmatrix} j\beta\tilde{e}_z \\ d\tilde{e}_z / dx \end{bmatrix}$$



(ii) *Field analysis*

- General solution

$$\tilde{e}_z = A\sin(k_x x) + B\cos(k_x x)$$

$$k_x^2 = k_0^2 \varepsilon \mu_\perp - \beta^2$$

$$\gamma = \frac{\mu_a}{\mu}\beta$$

$$\tilde{h}_y = \frac{1}{j\omega\mu_0\mu_\perp}\left(\frac{d\tilde{e}_z}{dx} - \frac{\mu_a}{\mu}\beta\tilde{e}_z\right) =$$

$$\frac{A}{\omega\mu_0\mu_\perp}\left[k_x\cos(k_x x) - \gamma\sin(k_x x)\right]$$

$$-\frac{B}{\omega\mu_0\mu_\perp}\left[k_x\sin(k_x x) + \gamma\cos(k_x x)\right]$$

(ii) *Field analysis*

- Magnetic wall boundary conditions→

$$k_x A - \gamma B = 0$$

$$\left[k_x \cos(k_x a) - \gamma \sin(k_x a)\right] A \\ - \left[k_x \sin(k_x a) + \gamma \cos(k_x a)\right] B = 0$$

$$\Rightarrow \quad \boxed{(\gamma A + k_x B) \sin(k_x a) = 0}$$

(ii) *Field analysis*

- Conventional solutions:

$$\sin(k_x a) = 0 \rightarrow k_x = \frac{n\pi}{a} \rightarrow \beta = \pm\sqrt{k_0^2 \varepsilon \mu_\perp - \left(\frac{n\pi}{a}\right)^2}$$

$$n = 1, 2, 3, \cdots$$

- Note that $n = 0$ is ***not*** a solution (why?)
- These modes are identical (in propagation constant) to those of a transversely magnetized, completely filled rectangular waveguide
- They have a cutoff frequency, and they cannot propagate when $\mu_\perp < 0$

(iii) *Edge-guided modes*

- The 'other' solution:

$$\gamma A + k_x B = 0$$
$$k_x A - \gamma B = 0$$

$$\Rightarrow \quad k_x^2 + \gamma^2 = 0 \quad \Rightarrow \quad \beta = \pm k_0 \sqrt{\varepsilon \mu}$$

- This mode has no cutoff
- It's propagation constant is determined by μ instead of $\mu_\perp$
- It propagates if $\mu > 0$:

$$\omega < \omega_H \quad \text{or} \quad \omega > \omega_\perp = \sqrt{\omega_H(\omega_H + \omega_M)}$$

(iii) *Edge-guided modes*

- It is the dominant mode at frequencies where the conventional modes cannot propagate due to negative $\mu_\perp$ or cutoff (e.g. due to small microstrip width)
- In particular, if the internal dc field is very small, so that both ω_H and $\omega_\perp$ are small compared to operation frequency then

$$\mu = \frac{\omega_\perp^2 - \omega^2}{\omega_H^2 - \omega^2} \approx 1$$

- We will have dispersionless propagation of this mode

(iii) *Edge-guided modes*

- Electric field profile:

$$\frac{B}{A} = -\frac{\gamma}{k_x} \;,\; k_x = j\gamma \rightarrow$$

$$\tilde{e}_z(x) = A\left[\sin(k_x x) + j\cos(k_x x)\right]$$

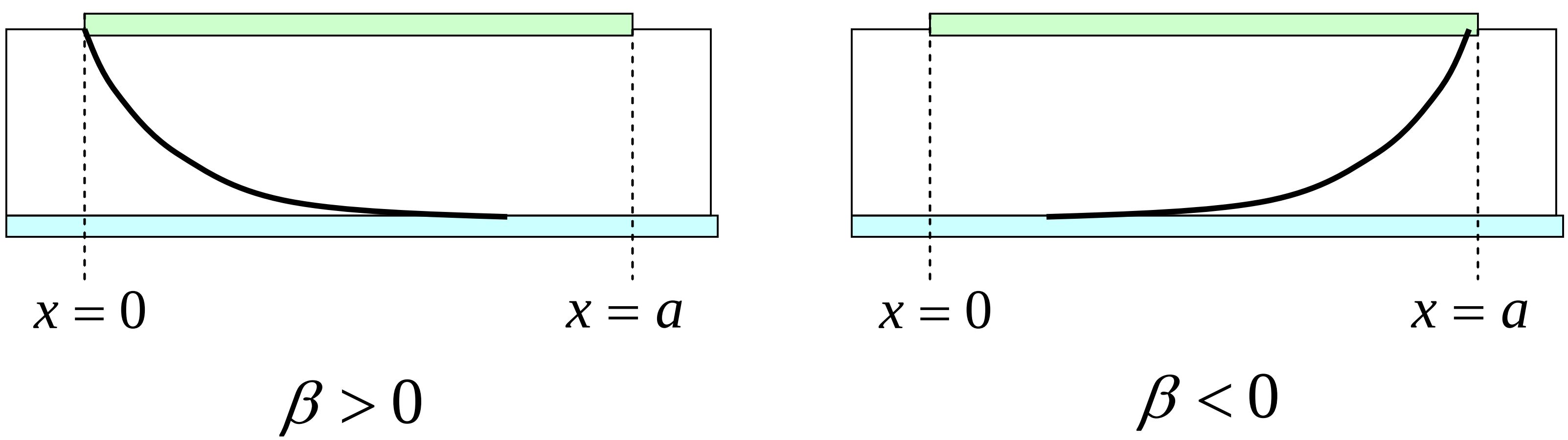
$$= jA\exp\left(\gamma x\right) = jA\exp\left(\frac{\mu_a}{\mu}\beta x\right)$$

$$\frac{\mu_a}{\mu} = \frac{\omega_M \omega}{\omega_\perp^2 - \omega^2}$$

- Electric field shows exponential behavior along x

(iii) *Edge-guided modes*

- In the often used range of operation where $\omega > \omega_{\perp}$ we will have the following situation:
- For propagation in +y direction, electric field is concentrated near the left edge, whereas for waves moving in –y direction the field is near the right edge



(iii) *Edge-guided modes*

- Magnetic field profile:

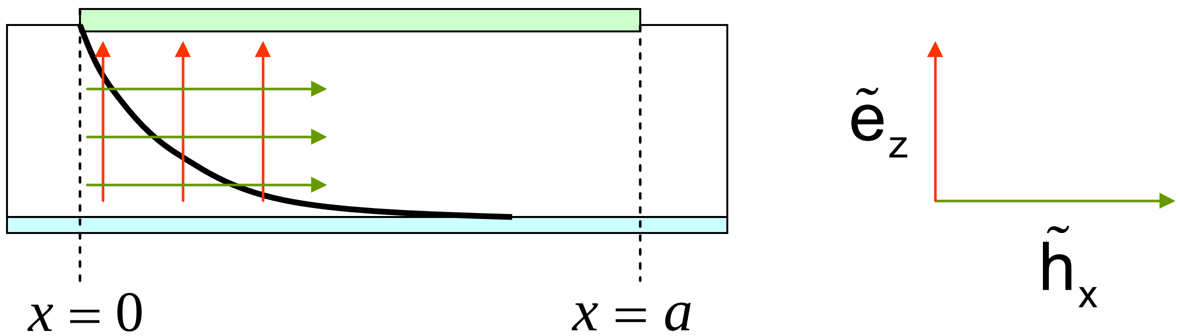
$$\tilde{h}_y = 0$$

$$\tilde{h}_x = \frac{A}{\omega\mu_0\mu_\perp}\left[\beta\sin(k_x x) - \frac{\mu_a}{\mu}k_x\cos(k_x x)\right] + \frac{B}{\omega\mu_0\mu_\perp}\left[\beta\cos(k_x x) + \frac{\mu_a}{\mu}k_x\sin(k_x x)\right]$$

$$= \frac{\beta}{\omega\mu_0\mu} jA\exp(\gamma x) = \frac{\beta}{\omega\mu_0\mu} jA\exp\left(\frac{\mu_a}{\mu}\beta x\right)$$

(iii) *Edge-guided modes*

- This is a TEM wave (both electric and magnetic fields perpendicular to the direction of propagation)

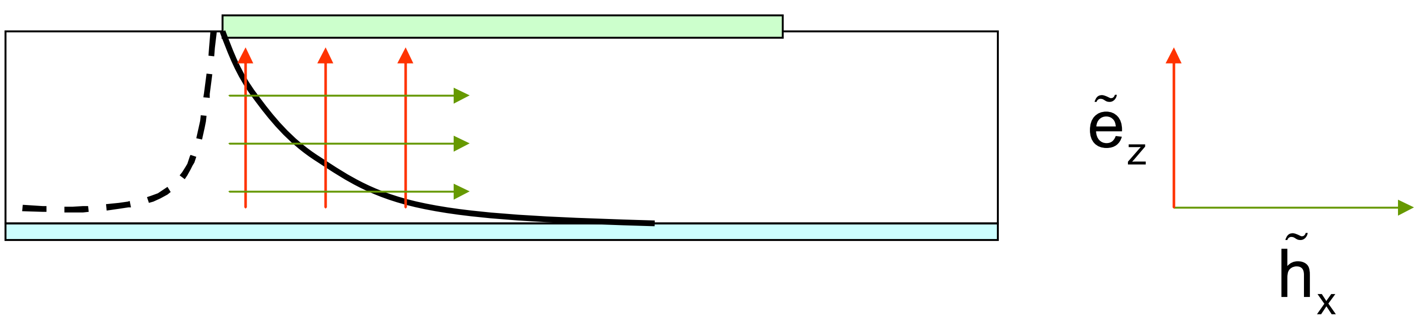


- The relation between electric and magnetic field for each wave is

$$\tilde{h}_x = \pm \tilde{e}_z \sqrt{\frac{\varepsilon_0 \varepsilon}{\mu_0 \mu}}$$

(iii) *Edge-guided modes*

- Note that although we neglected the fringe field by using the magnetic wall boundary conditions, it is not true that there are no fields beyond the edges of the microstrip
- But they rapidly drop away from the edges
- Fringe fields are difficult to calculate: better approximations or numerical methods are required

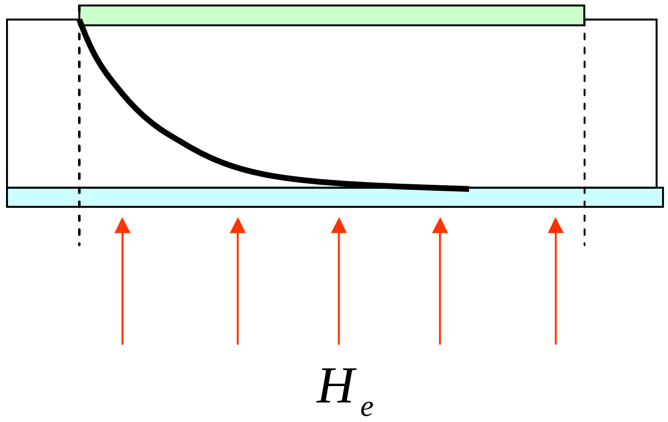


(iii) *Edge-guided modes*

- These modes are called edge-guided modes
- The exponential drop of fields away from the edges is similar to surface waves in a partially filled waveguide.
- But there, surface waves were formed at the magnetic slab-air interface. Here there is no such interface.
- The advantage of these modes is their zero cutoff.
- Besides, the concentration of fields near the edges and the field displacement effect allows the realization of a number of devices

(iv) *Reciprocal Phase shifters*

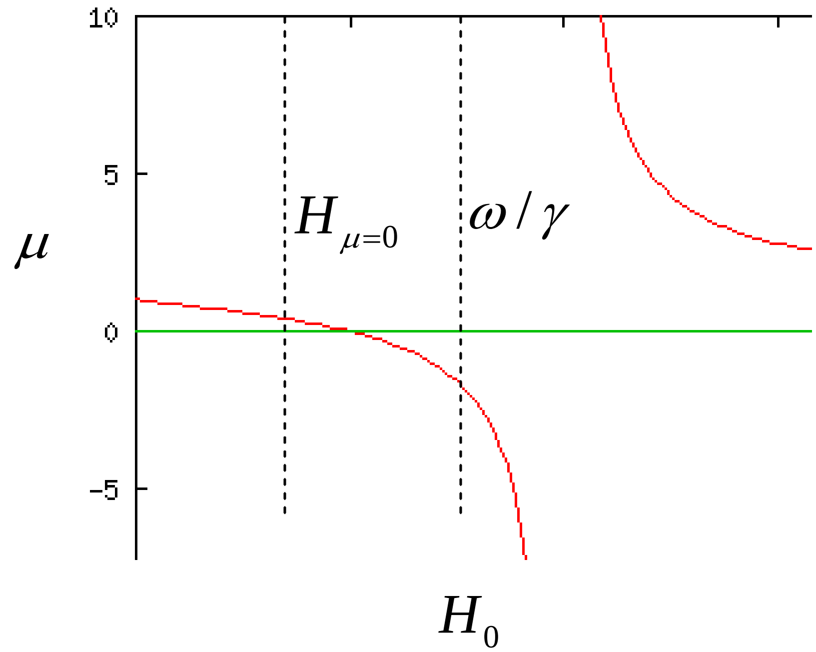
- The edge-guided mode can be used to design a reciprocal phase shifter simply by the application of an external dc field



$$\beta = \pm k_0 \sqrt{\varepsilon\mu}$$

$$\mu = \frac{\omega_\perp^2 - \omega^2}{\omega_H^2 - \omega^2}$$

$$H_{\mu=0} = \frac{\sqrt{\omega_M^2 + 4\omega^2} - \omega_M}{2\gamma}$$

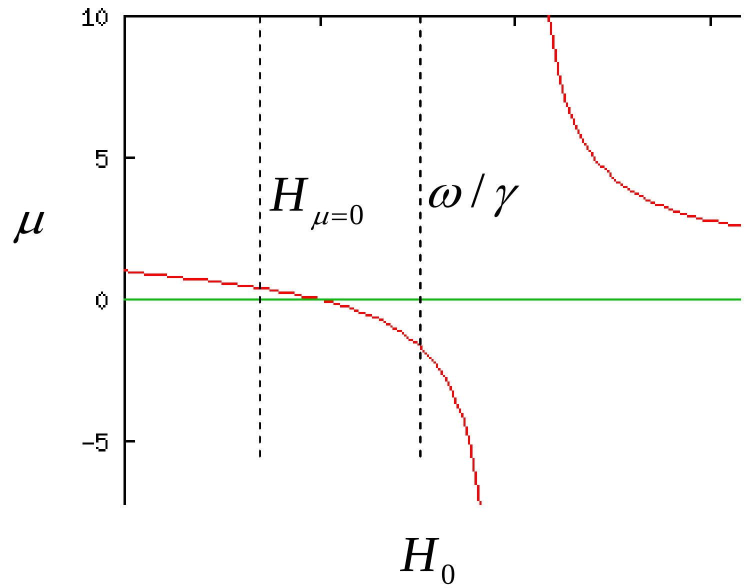


(iv) *Reciprocal Phase shifters*

- For low fields, μ varies between $1+\omega_M/\omega_H$ and 0. One can build a phase shifter by varying H_0 in the range

$$0 < H_0 < H_{\mu=0} = \frac{\sqrt{\omega_M^2 + 4\omega^2} - \omega_M}{2\gamma}$$

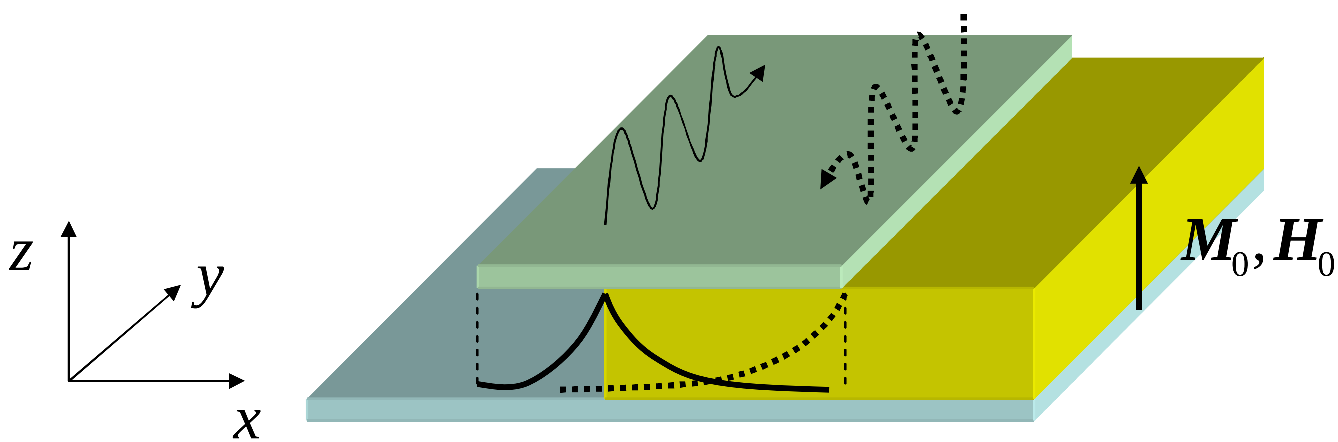
$$0 < |\beta| < k_0 \sqrt{\varepsilon \left(1 + \omega_M / \omega_H\right)}$$



- To use the other branch, we should have $H_0 > \omega/\gamma$. If the operation frequency is too high, the required dc field will be too high as well and difficult to realize.

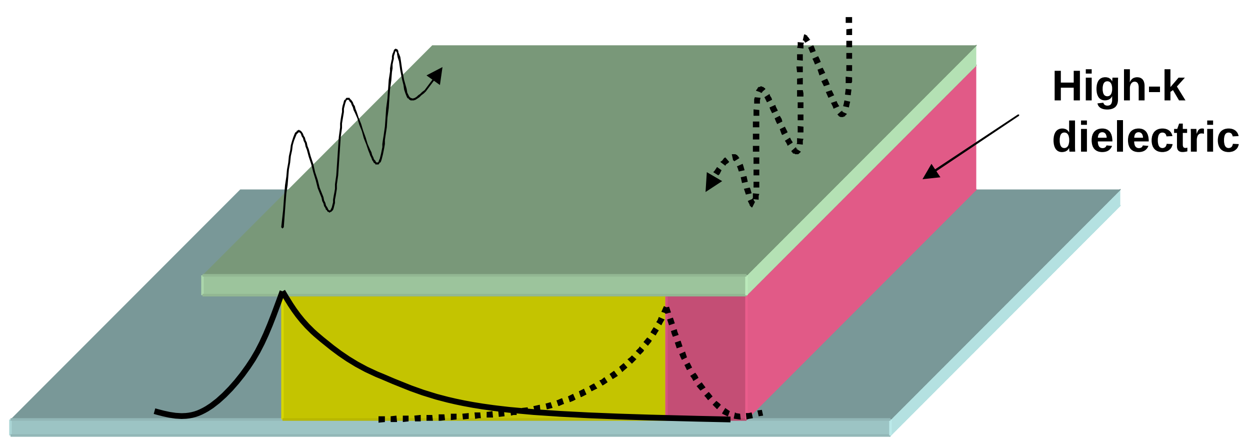
(iv) *Non-reciprocal Phase shifters*

- But, how can one build a non-reciprocal phase shifter?
- We should implement some kind of asymmetry in the configuration
- One example: use a finite ferrite layer and let one edge of the microstrip float in the air



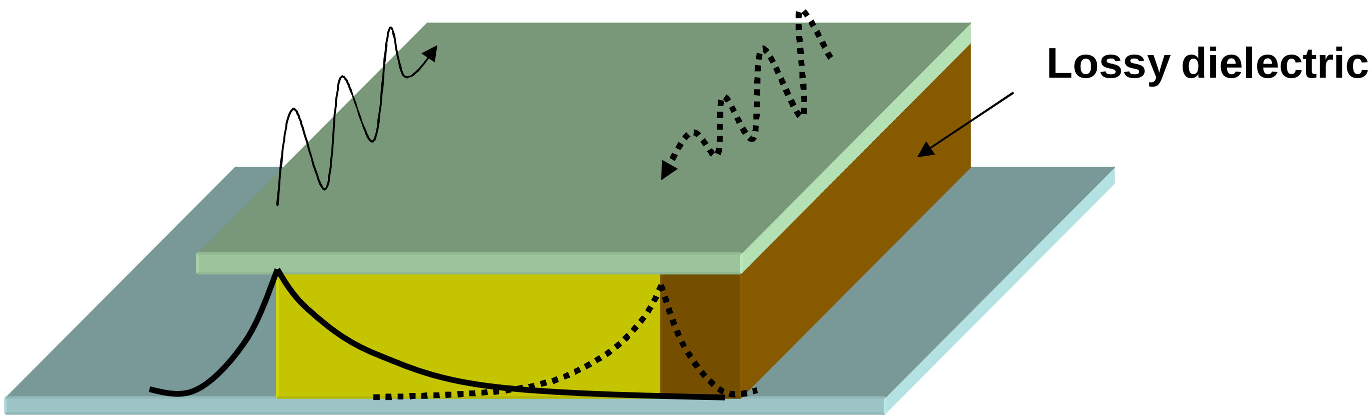
(iv) *Non-reciprocal Phase shifters*

- Another example: use a finite ferrite sample, load it asymmetrically with a dielectric with a high dielectric constant
- Since electric field concentrated near the edges of the ferrite, they will effectively see different dielectric constants when they move in opposite directions



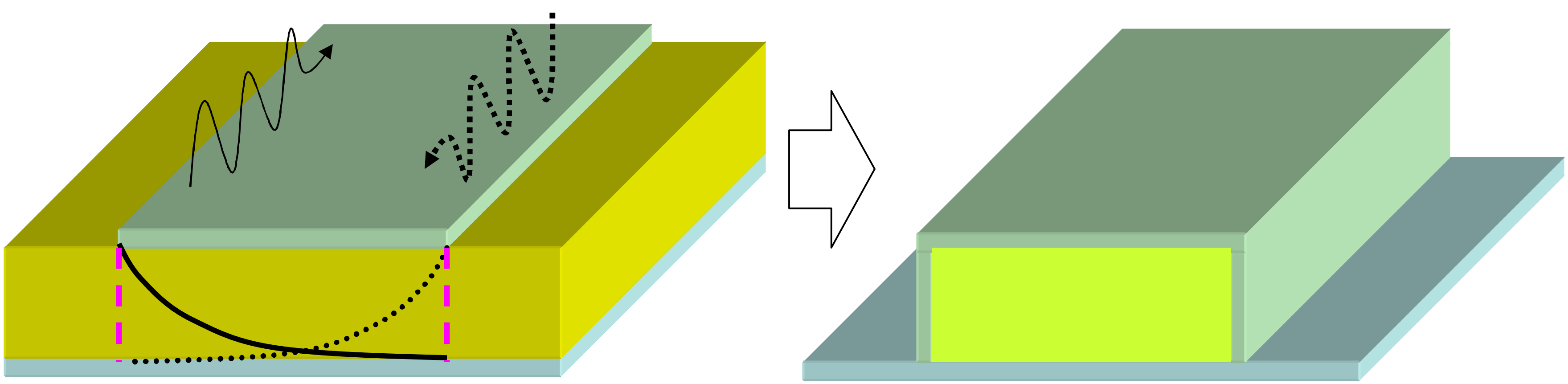
(v) *Field displacement isolators*

- Easy to realize: load one edge of the ferrite with a lossy dielectric material
- The two waves will have different attenuations over a broad range of frequencies



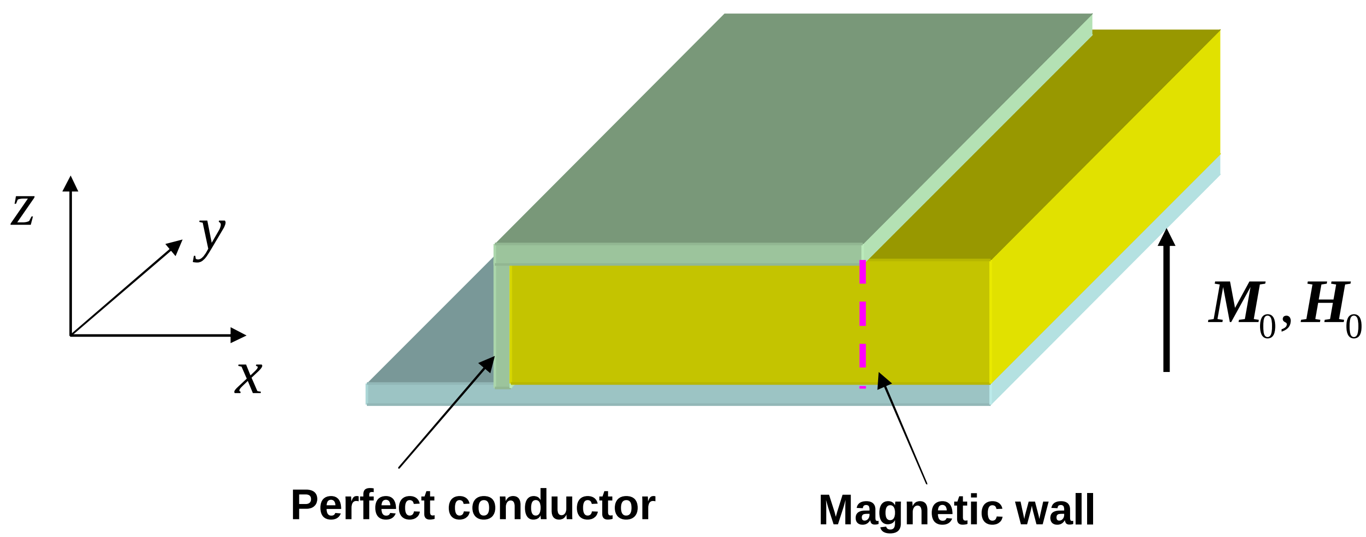
(vi) *Mixed boundary conditions*

- We saw that in a microstrip on a magnetic, vertically magnetized substrate, zero-cutoff edge modes exist
- But if the magnetic wall boundary conditions were replaced by perfect conductors, like a in a completely-filled waveguide, all modes have a cutoff and do not propagate when $\mu_{\perp} < 0$



(vi) *Mixed boundary conditions*

- Now, what happens when we metallize just one edge of the magnetic layer?
- Do we have modes with a zero cutoff?



(vi) *Mixed boundary conditions*

- General solution inside magnetic layer:

$$\tilde{e}_z = A\sin(k_x x) + B\cos(k_x x)$$

$$\tilde{h}_y = \frac{A}{\omega\mu_0\mu_\perp}\left[k_x\cos(k_x x) - \gamma\sin(k_x x)\right] - \frac{B}{\omega\mu_0\mu_\perp}\left[k_x\sin(k_x x) + \gamma\cos(k_x x)\right]$$

$$k_x^2 = k_0^2\varepsilon\mu_\perp - \beta^2$$

$$\gamma = \frac{\mu_a}{\mu}\beta$$

(vi) *Mixed boundary conditions*

- Imposing boundary conditions:

$x = 0:$

$$\tilde{e}_z = A\sin(k_x x)$$

$$\tilde{h}_y = \frac{A}{\omega\mu_0\mu_\perp}\left[k_x\cos(k_x x) - \gamma\sin(k_x x)\right]$$

$x = a:$

$$k_x\cos(k_x a) - \gamma\sin(k_x a) = 0$$

$$\Rightarrow \quad k_x\cot(k_x a) = \gamma$$

(vi) *Mixed boundary conditions*

- Equation for propagation constant:

$$\sqrt{k_0^2 \varepsilon \mu_\perp - \beta^2} \cot\left(a\sqrt{k_0^2 \varepsilon \mu_\perp - \beta^2}\right) = \frac{\mu_a}{\mu}\beta$$

- Consider the particular case where $\omega_\perp < \omega < \omega_H + \omega_M$

$$\mu_\perp < 0$$

$$\Rightarrow \quad \sqrt{\beta^2 + k_0^2 \varepsilon |\mu_\perp|} \coth\left(a\sqrt{\beta^2 + k_0^2 \varepsilon |\mu_\perp|}\right) = \frac{\mu_a}{\mu}\beta$$

- Bear in mind that $\dfrac{\mu_a}{\mu} = \dfrac{\omega_M \omega}{\omega_\perp^2 - \omega^2} < 0$

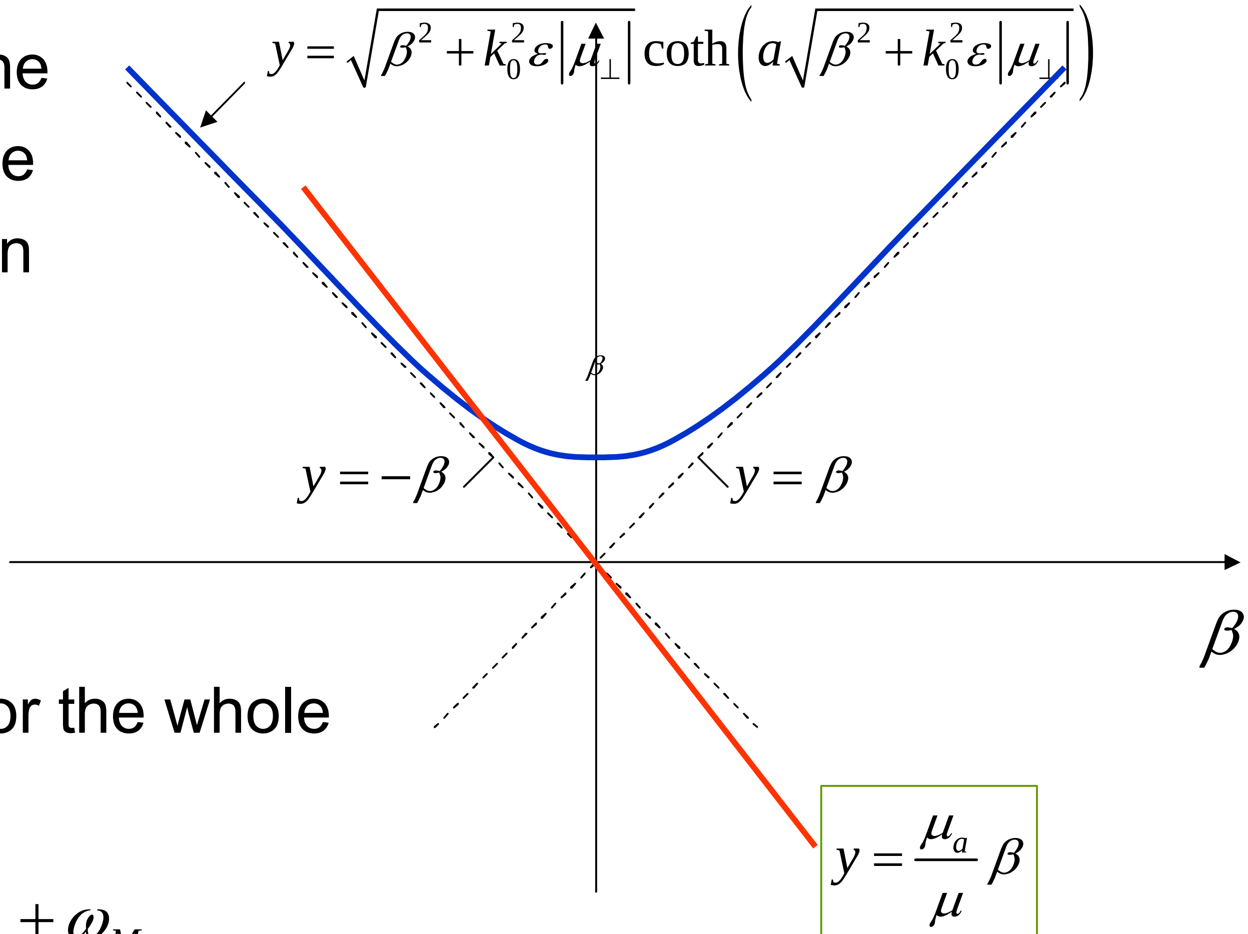
(vi) *Mixed boundary conditions*

- We have a solution for any width of the microstrip (a) if the following condition holds:

$$\left|\frac{\mu_a}{\mu}\right| > 1$$

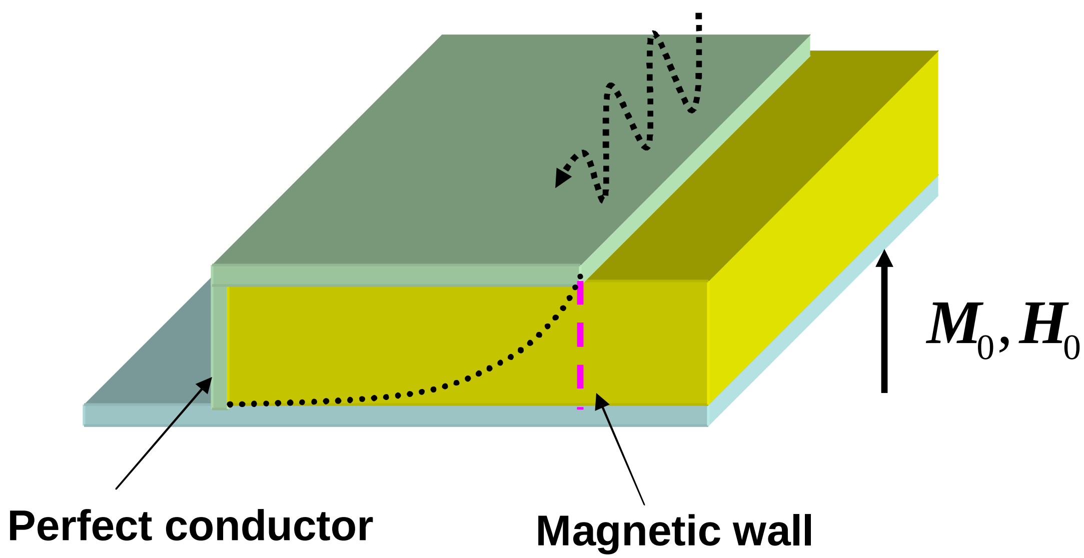
- This is satisfied for the whole range

$$\omega_\perp < \omega < \omega_H + \omega_M$$



(vi) *Mixed boundary conditions*

- But the propagation is ***uni-directional*** : only in the –y direction
- Putting a metal wall at the edge kills the corresponding edge mode whose electric field is concentrated near that edge→ propagation in one-direction, no cutoff



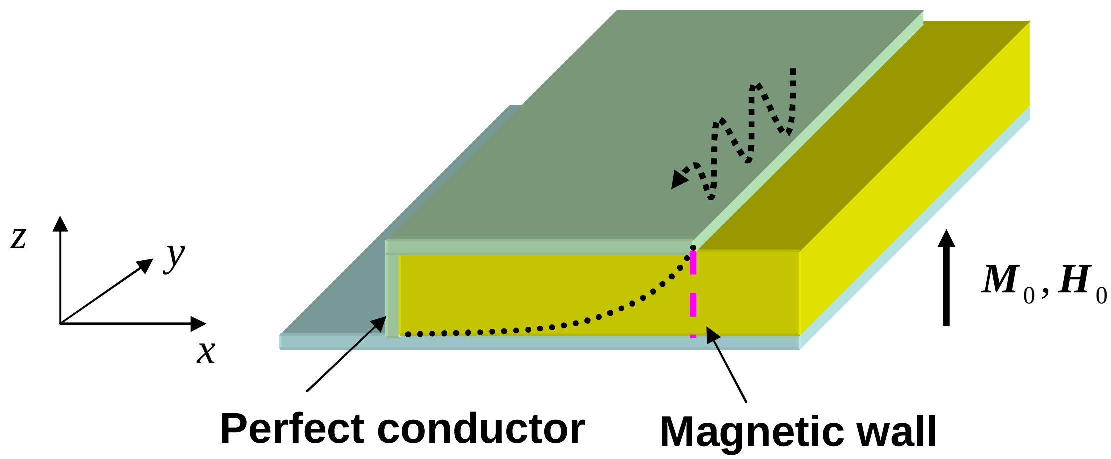
(vi) *Mixed boundary conditions*

- What about the fields?

$$\tilde{e}_z = A\sin\left(k_x x\right) = jA\sinh\left(jx\sqrt{\beta^2 + k_0^2\varepsilon\left|\mu_\perp\right|}\right)$$

$$\tilde{h}_y = \frac{A\gamma}{\omega\mu_0\mu_\perp}\frac{\sin\left[k_x(a-x)\right]}{\cos\left(k_x a\right)} = \frac{jA\gamma}{\omega\mu_0\mu_\perp}\frac{\sinh\left[(a-x)\sqrt{\beta^2 + k_0^2\varepsilon\left|\mu_\perp\right|}\right]}{\cosh\left(a\sqrt{\beta^2 + k_0^2\varepsilon\left|\mu_\perp\right|}\right)}$$

- Clearly this cannot be a TEM wave (how could you see this without calculation?)



(vi) *Mixed boundary conditions*

- In case where $\omega < \omega_\perp$ or $\omega > \omega_H + \omega_M \rightarrow \mu_\perp > 0$

$$\sqrt{k_0^2 \varepsilon \mu_\perp - \beta^2} \cot\left(a\sqrt{k_0^2 \varepsilon \mu_\perp - \beta^2} \right) = \frac{\mu_a}{\mu} \beta$$

- The right hand side of the above equation has a number of singularities as function of β at points where

$$a\sqrt{k_0^2 \varepsilon \mu_\perp - \beta^2} = n\pi \ , n = 1, 2, 3, \cdots$$

- Consider now the particular case where the width a of the microstrip is so small that $k_0 a \sqrt{\varepsilon \mu_\perp} < \pi$

- Is there a solution?

(vii) *Remarks*

- Microstrip on a vertically magnetized substrate is not the only possible configuration utilizing transmission lines
- One can think of microstrips with transversely magnetized substrates
- In addition, one can consider other transmission lines on ferrite substrates (coplanar waveguides)
- But these structures are difficult to analyze.