Microwave Magnetics

Graduate Course
Electrical Engineering (Communications)
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Sharif University of Technology
General information

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Planar magnetic (ferrite) devices

- The devices we have discussed so far utilize metallic waveguides (partially) loaded by magnetic materials.
- But waveguides are difficult to build in integrated microwave circuits where devices are flat, and are implemented on a horizontal substrate.
Planar magnetic (ferrite) devices

- Late 1960’s: attention turned to devices which could be built on a ferrite substrate.
- In particular, waveguides became less popular, and attention was shifted to devices based on planar transmission lines like microstrip and stripline.
- Those were easily realized in microwave integrated circuits.
(i) *Microstrip magnetic (ferrite) devices*

- Here we shall focus on microstrip magnetic devices which are more popular due to the ease of fabrication.

- Analysis will be based on the paper:


![Microstrip line model](image)
(i) *Microstrip magnetic (ferrite) devices*

- Consider a microstrip line on a grounded ferrite substrate, magnetized **perpendicular** to the substrate.
- Other cases (in-plane magnetization along or transverse to the microstrip) have also been investigated, but their full electromagnetic analysis is difficult!
(i) *Microstrip magnetic (ferrite) devices*

- We assume that the width of the microstrip is much larger than the thickness of the (ferrite) substrate.
- Then most of the field is confined to the substrate region between the two metals.
- Fringing field becomes less important and can be replaced by boundary conditions.
(i) Microstrip magnetic (ferrite) devices

- A qualitative way to formulate approximate boundary conditions for microstrips (in general) is as follows:
- At the edges \((x = 0, x = a)\) the transverse component \((J_x)\) of current density is very small. The magnetic field \(h_y\) induced by current should also be small at the edges.
- Also \(h_z\) is small (actually everywhere). 

![Diagram of microstrip magnetic devices](image)
(i) *Microstrip magnetic (ferrite) devices*

- Magnetic wall approximation: assume the magnetic field in the **y** and **z** directions vanishes on the vertical walls between the microstrip edges and the ground plane.

- These walls are magnetic walls: tangential magnetic field vanishes on these walls.

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Planar magnetic devices
(i) Microstrip magnetic (ferrite) devices

- Resulting boundary conditions:

\[ h_y = h_z = 0 \]

when \( x = 0 \) and \( x = a \)

- We only have to solve the field equations in the region enclosed by the microstrip, the ground plane, and the two magnetic walls.

- This problem is similar to that of a rectangular waveguide completely filled with magnetic material, only the lateral boundary conditions are different.
(ii) Field analysis

- As for waveguides, neglect the variation along \( z \) (for simplicity), and look for solutions of the type

\[
e = \tilde{\mathbf{e}}(x) \exp(-j \beta y) \quad \mathbf{h} = \tilde{\mathbf{h}}(x) \exp(-j \beta y)
\]

- Again, only solutions with the electric field in the \( z \)-direction will be acceptable.
(ii) **Field analysis**

- **Field equations**

\[
\frac{\partial^2 \tilde{e}_z}{\partial x^2} + \left( k_0^2 \varepsilon \mu_\perp - \beta^2 \right) \tilde{e}_z = 0
\]

\[
\begin{bmatrix}
\tilde{h}_x \\
\tilde{h}_y
\end{bmatrix} = \frac{1}{j \omega \mu_0 (\mu^2 - \mu_a^2)} \begin{bmatrix}
\mu & -j \mu_a \\
 j \mu_a & \mu
\end{bmatrix} \begin{bmatrix}
j \beta \tilde{e}_z \\
d \tilde{e}_z / dx
\end{bmatrix}
\]

Planar magnetic devices
(ii) Field analysis

- General solution

\[ \tilde{e}_z = A \sin(k_x x) + B \cos(k_x x) \]

\[ k_x^2 = k_0^2 \varepsilon \mu_\perp - \beta^2 \]

\[ \gamma = \frac{\mu_a}{\mu} \beta \]

\[ \tilde{h}_y = \frac{1}{j \omega \mu_0 \mu_\perp} \left( \frac{d\tilde{e}_z}{dx} - \frac{\mu_a}{\mu} \beta \tilde{e}_z \right) = \]

\[ \frac{A}{\omega \mu_0 \mu_\perp} \left[ k_x \cos(k_x x) - \gamma \sin(k_x x) \right] \]

\[ - \frac{B}{\omega \mu_0 \mu_\perp} \left[ k_x \sin(k_x x) + \gamma \cos(k_x x) \right] \]
(ii) Field analysis

- Magnetic wall boundary conditions

\[ k_x A - \gamma B = 0 \]

\[
\left[ k_x \cos(k_x a) - \gamma \sin(k_x a) \right] A \\
- \left[ k_x \sin(k_x a) + \gamma \cos(k_x a) \right] B = 0
\]

\[ (\gamma A + k_x B) \sin(k_x a) = 0 \]
(ii) Field analysis

- Conventional solutions:

\[
\sin(k_x a) = 0 \rightarrow k_x = \frac{n \pi}{a} \rightarrow \beta = \pm \sqrt{k_0^2 \varepsilon \mu_\perp - \left(\frac{n \pi}{a}\right)^2}
\]

\[n = 1, 2, 3, \ldots\]

- Note that \(n = 0\) is not a solution (why?)

- These modes are identical (in propagation constant) to those of a transversely magnetized, completely filled rectangular waveguide

- They have a cutoff frequency, and they cannot propagate when \(\mu_\perp < 0\)
(iii) *Edge-guided modes*

- The ‘other’ solution:

\[
\begin{align*}
\gamma A + k_x B &= 0 \\
k_x A - \gamma B &= 0
\end{align*}
\]

\[
k_x^2 + \gamma^2 = 0 \quad \Rightarrow \quad \beta = \pm k_0 \sqrt{\varepsilon \mu}
\]

- This mode has no cutoff
- It’s propagation constant is determined by \( \mu \) instead of \( \mu_\perp \)
- It propagates if \( \mu > 0 \):

\[
\omega < \omega_H \quad \text{or} \quad \omega > \omega_\perp = \sqrt{\omega_H (\omega_H + \omega_M)}
\]
(iii) *Edge-guided modes*

- It is the dominant mode at frequencies where the conventional modes cannot propagate due to negative $\mu_\perp$ or cutoff (e.g. due to small microstrip width)

- In particular, if the internal dc field is very small, so that both $\omega_H$ and $\omega_\perp$ are small compared to operation frequency then
  \[ \mu = \frac{\omega_\perp^2 - \omega^2}{\omega_H^2 - \omega^2} \approx 1 \]

- We will have dispersionless propagation of this mode
(iii) *Edge-guided modes*

- Electric field profile:

\[
\frac{B}{A} = -\frac{\gamma}{k_x}, \quad k_x = j\gamma \rightarrow
\]

\[
\hat{e}_x(x) = A\left[\sin(k_x x) + j \cos(k_x x)\right]
= jA \exp(\gamma x) = jA \exp\left(\frac{\mu_a}{\mu} \beta x\right)
\]

\[
\frac{\mu_a}{\mu} = \frac{\omega M \omega}{\omega^2 - \omega^2}
\]

- Electric field shows exponential behavior along \(x\)
(iii) *Edge-guided modes*

- In the often used range of operation where $\omega > \omega_{\perp}$ we will have the following situation:
- For propagation in $+y$ direction, electric field is concentrated near the left edge, whereas for waves moving in $-y$ direction the field is near the right edge.

\[
\begin{align*}
\beta > 0: & \quad x = 0 & x = a \\
\beta < 0: & \quad x = 0 & x = a
\end{align*}
\]
(iii) Edge-guided modes

- Magnetic field profile:

\[ \tilde{h}_y = 0 \]

\[ \tilde{h}_x = \frac{A}{\omega \mu_0 \mu_\perp} \left[ \beta \sin(k_x x) - \frac{\mu_a}{\mu} k_x \cos(k_x x) \right] \]

\[ + \frac{B}{\omega \mu_0 \mu_\perp} \left[ \beta \cos(k_x x) + \frac{\mu_a}{\mu} k_x \sin(k_x x) \right] \]

\[ = \frac{\beta}{\omega \mu_0 \mu} jA \exp(\gamma x) = \frac{\beta}{\omega \mu_0 \mu} jA \exp \left( \frac{\mu_a}{\mu} \beta x \right) \]
(iii) **Edge-guided modes**

- This is a TEM wave (both electric and magnetic fields perpendicular to the direction of propagation)

- The relation between electric and magnetic field for each wave is

\[
\tilde{h}_x = \pm \tilde{e}_z \sqrt{\frac{\varepsilon_0 \varepsilon}{\mu_0 \mu}}
\]
(iii) *Edge-guided modes*

- Note that although we neglected the fringe field by using the magnetic wall boundary conditions, it is not true that there are no fields beyond the edges of the microstrip.
- But they rapidly drop away from the edges.
- Fringe fields are difficult to calculate: better approximations or numerical methods are required.
(iii) *Edge-guided modes*

- These modes are called edge-guided modes.
- The exponential drop of fields away from the edges is similar to surface waves in a partially filled waveguide.
- But there, surface waves were formed at the magnetic slab-air interface. Here there is no such interface.
- The advantage of these modes is their zero cutoff.
- Besides, the concentration of fields near the edges and the field displacement effect allows the realization of a number of devices.
(iv) Reciprocal Phase shifters

- The edge-guided mode can be used to design a reciprocal phase shifter simply by the application of an external dc field.

\[ \beta = \pm k_0 \sqrt{\varepsilon \mu} \]

\[ \mu = \frac{\omega_\perp^2 - \omega^2}{\omega_H^2 - \omega^2} \]

\[ H_{\mu=0} = \frac{\sqrt{\omega_M^2 + 4\omega^2} - \omega_M}{2\gamma} \]
(iv) Reciprocal Phase shifters

- For low fields, $\mu$ varies between $1 + \frac{\omega_M}{\omega_H}$ and 0. One can build a phase shifter by varying $H_0$ in the range

$$0 < H_0 < H_{\mu=0} = \frac{\sqrt{\omega_M^2 + 4\omega^2} - \omega_M}{2\gamma}$$

$$0 < |\beta| < k_0 \sqrt{\varepsilon \left(1 + \frac{\omega_M}{\omega_H}\right)}$$

- To use the other branch, we should have $H_0 > \frac{\omega}{\gamma}$. If the operation frequency is too high, the required dc field will be too high as well and difficult to realize.
(iv) **Non-reciprocal Phase shifters**

- But, how can one build a non-reciprocal phase shifter?
- We should implement some kind of asymmetry in the configuration.
- One example: use a finite ferrite layer and let one edge of the microstrip float in the air.
(iv) **Non-reciprocal Phase shifters**

- Another example: use a finite ferrite sample, load it asymmetrically with a dielectric with a high dielectric constant.

- Since electric field concentrated near the edges of the ferrite, they will effectively see different dielectric constants when they move in opposite directions.
(v) Field displacement isolators

- Easy to realize: load one edge of the ferrite with a lossy dielectric material.
- The two waves will have different attenuations over a broad range of frequencies.

![Diagram of field displacement isolators with a lossy dielectric material.]
(vi) **Mixed boundary conditions**

- We saw that in a microstrip on a magnetic, vertically magnetized substrate, zero-cutoff edge modes exist.
- But if the magnetic wall boundary conditions were replaced by perfect conductors, like a in a completely-filled waveguide, all modes have a cutoff and do not propagate when $\mu_\perp < 0$.  

![Diagram showing a microstrip on a magnetic substrate with an arrow indicating the transition to a completely-filled waveguide]
(vi) \textit{Mixed boundary conditions}

- Now, what happens when we metallize just one edge of the magnetic layer?
- Do we have modes with a zero cutoff?
(vi) Mixed boundary conditions

- General solution inside magnetic layer:

\[ \tilde{\mathbf{e}}_z = A \sin(k_x x) + B \cos(k_x x) \]

\[ \tilde{h}_y = \frac{A}{\omega \mu_0 \mu_\perp} \left( k_x \cos(k_x x) - \gamma \sin(k_x x) \right) \]

\[ \quad - \frac{B}{\omega \mu_0 \mu_\perp} \left( k_x \sin(k_x x) + \gamma \cos(k_x x) \right) \]

\[ k_x^2 = k_0^2 \varepsilon \mu_\perp - \beta^2 \]

\[ \gamma = \frac{\mu_a}{\mu} \beta \]
(vi) **Mixed boundary conditions**

- Imposing boundary conditions:

\[ \tilde{e}_z = A \sin(k_x x) \]

\[ x = 0 : \quad \tilde{h}_y = \frac{A}{\omega \mu_0 \mu_\perp} [k_x \cos(k_x x) - \gamma \sin(k_x x)] \]

\[ x = a : \quad k_x \cos(k_x a) - \gamma \sin(k_x a) = 0 \]

\[ k_x \cot(k_x a) = \gamma \]
(vi) **Mixed boundary conditions**

- Equation for propagation constant:

\[
\sqrt{k_0^2 \varepsilon \mu_\perp - \beta^2} \cot \left( a \sqrt{k_0^2 \varepsilon \mu_\perp - \beta^2} \right) = \frac{\mu_a}{\mu} \beta
\]

- Consider the particular case where \( \omega_\perp < \omega < \omega_H + \omega_M \) and \( \mu_\perp < 0 \):

\[
\sqrt{\beta^2 + k_0^2 \varepsilon |\mu_\perp|} \coth \left( a \sqrt{\beta^2 + k_0^2 \varepsilon |\mu_\perp|} \right) = \frac{\mu_a}{\mu} \beta
\]

- Bear in mind that \( \frac{\mu_a}{\mu} = \frac{\omega_M \omega}{\omega_\perp - \omega^2} < 0 \)
(vi) **Mixed boundary conditions**

- We have a solution for any width of the microstrip $(a)$ if the following condition holds:

\[
\left| \frac{\mu_a}{\mu} \right| > 1
\]

- This is satisfied for the whole range

\[
\omega_\perp < \omega < \omega_H + \omega_M
\]
(vi) **Mixed boundary conditions**

- But the propagation is *uni-directional*: only in the $-y$ direction.
- Putting a metal wall at the edge kills the corresponding edge mode whose electric field is concentrated near that edge $\rightarrow$ propagation in one-direction, no cutoff.
(vi) Mixed boundary conditions

- What about the fields?

\[ \tilde{e}_z = A \sin(k_x x) = jA \sinh \left( jx \sqrt{\beta^2 + k_0^2 \varepsilon} |\mu_\perp| \right) \]

\[ \tilde{h}_y = \frac{A \gamma}{\omega \mu_0 \mu_\perp} \frac{\sin[k_x (a-x)]}{\cos(k_x a)} = \frac{jA \gamma}{\omega \mu_0 \mu_\perp} \frac{\sinh[(a-x) \sqrt{\beta^2 + k_0^2 \varepsilon} |\mu_\perp|]}{\cosh(a \sqrt{\beta^2 + k_0^2 \varepsilon} |\mu_\perp|)} \]

- Clearly this cannot be a TEM wave (how could you see this without calculation?)
(vi) **Mixed boundary conditions**

- In case where \( \omega < \omega_\perp \) or \( \omega > \omega_H + \omega_M \) \( \Rightarrow \) \( \mu_\perp > 0 \)

\[
\sqrt{k_0^2 \varepsilon \mu_\perp - \beta^2} \cot \left( a\sqrt{k_0^2 \varepsilon \mu_\perp - \beta^2} \right) = \frac{\mu_a}{\mu} \beta
\]

- The right hand side of the above equation has a number of singularities as function of \( \beta \) at points where

\[
a\sqrt{k_0^2 \varepsilon \mu_\perp - \beta^2} = n\pi , \ n = 1, 2, 3, \ldots
\]

- Consider now the particular case where the width \( a \) of the microstrip is so small that \( k_0 a \sqrt{\varepsilon \mu_\perp} < \pi \)

- Is there a solution?
(vii) Remarks

- Microstrip on a vertically magnetized substrate is not the only possible configuration utilizing transmission lines.
- One can think of microstrips with transversely magnetized substrates.
- In addition, one can consider other transmission lines on ferrite substrates (coplanar waveguides).
- But these structures are difficult to analyze.