Microwave Magnetics
Homework assignment 1

Problem 1:

The saturation magnetization of a lossless ferromagnetic material is $M_s = 10^5$ Amp/meter. An external dc magnetic field of $H_{ext} = 2\times10^5$ Amp/meter is applied to magnetize and saturate the ferromagnet in the z-direction. The gyrotropic constant of the material is $\gamma = g\mu_0 e/2m_e$ with $g = 2$, μ_0 the vacuum permeability, e the electron charge, and m_e the electron mass.

- Plot the diagonal and off-diagonal components of the permeability tensor as function of frequency

Now consider a small ellipsoid built using this magnetic material. The demagnetization tensor of an ellipsoid with its axes along the x,y, and z directions is in general given by

$$\bar{\bar{\boldsymbol{N}}} = \begin{bmatrix} N_x & 0 & 0 \\ 0 & N_y & 0 \\ 0 & 0 & N_z \end{bmatrix}$$

where $N_x + N_y + N_z = 1$. Consider the dc case where the same external field as in problem 1 is uniformly applied to the ellipsoid in the z-direction.

- Find the total dc magnetic field inside the ellipsoid when $N_x = N_y = 0.25, N_z = 0.5$ (ellipsoid will become magnetized along the external field)

Next, in addition to the external dc field, a small, uniform external ac field of $\boldsymbol{h} = h_x\hat{\boldsymbol{x}}$, $h_x = h_0 \exp(j\omega t)$ is applied to the ellipsoid.

- Find the resulting ac magnetization $\boldsymbol{m}$ (phasor representation) as function of the frequency ω.

Problem 2:

The motion of magnetization inside a small, uniformly magnetized magnetic ellipsoid (see figure) is governed by the Landau-Lifshitz equation

$$\frac{d\boldsymbol{M}}{dt} = -\gamma \boldsymbol{M}\times\boldsymbol{H}$$

where $\boldsymbol{M} = \boldsymbol{M}(t)$ denotes the spatially uniform, time-dependent magnetization inside the sample, and γ is the gyrotropic constant (losses are neglected). We assume that no external field is applied so that the magnetic field is given by the uniform demagnetization field $\boldsymbol{H}(t) = -\bar{\bar{\boldsymbol{N}}}\cdot\boldsymbol{M}(t)$ where

$$\bar{\bar{\boldsymbol{N}}} = \begin{bmatrix} N_x & 0 & 0 \\ 0 & N_y & 0 \\ 0 & 0 & N_z \end{bmatrix}$$

is the demagnetization tensor of the ellipsoid with $N_x + N_y + N_z = 1$. We restrict ourselves to ellipsoids which are symmetric with respect to rotations around the z-axis so that $N_x = N_y$.

Use the representation

$$M_x = M_s \sin\theta \cos\varphi,\; M_y = M_s \sin\theta \sin\varphi,\; M_z = M_s \cos\theta$$

in which $M_s = |\boldsymbol{M}|$ is the constant saturation magnetization, and find the general solution of the Landau-Lifshitz equation for the ellipsoid (do not linearize, we are looking for the general solution).