Microwave Magnetics
Homework assignment 3

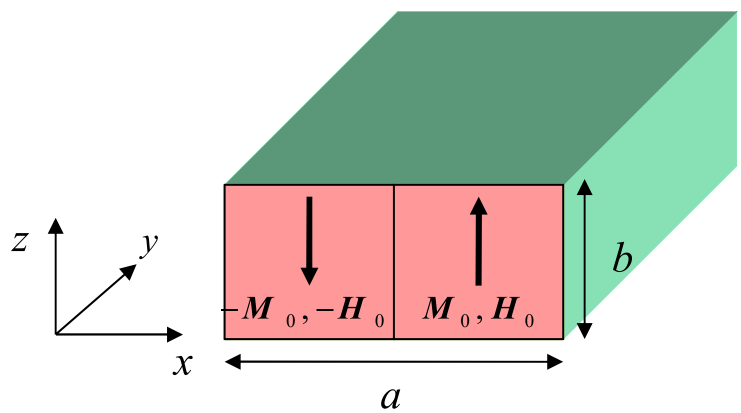
Problem 1:

Each half of a (perfectly conducting) rectangular waveguide with the width a and height b is filled with a magnetic slab with the thickness $d = a/2$ as shown in the figure below. The saturation magnetization of the slabs is M_s and they have a relative dielectric constant of 1. The left and right slabs are saturated in the –z and +z directions by applying a dc magnetic field of H_0 in respective directions in each region. We are interested in solutions of the type

$$\mathbf{e} = \tilde{\mathbf{e}}(x)\exp(-j\beta y) \quad \mathbf{h} = \tilde{\mathbf{h}}(x)\exp(-j\beta y)$$

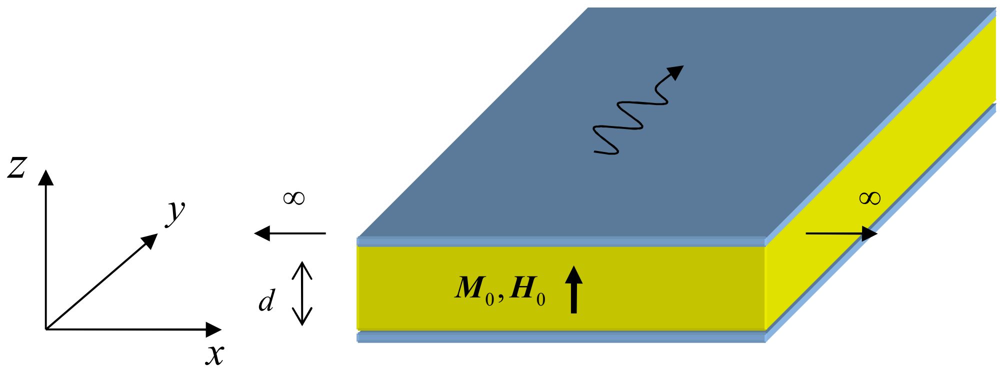
in which the electromagnetic field is uniform in the z-direction and propagates along the waveguide (y-direction) with a propagation constant β. We confine ourselves to solutions where the electric field is parallel to the z-axis.

- Find an equation for β
- Specify the range of frequencies for which solutions exist
- Can one distinguish between conventional and "surface wave" solutions?



Problem 2:

Consider the parallel-plate waveguide shown below consisting of a lossless magnetic slab with the thickness d in between two perfectly conducting, infinite plates. The slab has a relative dielectric constant of ε, a saturation magnetization of M_s and a dc magnetic field of H_0 both normal to the slab. We would like to find the waveguide modes which are propagating along the *y*-direction and are independent of x. But we take into account the variation in the vertical direction (along *z*). Find the propagation constant β of these modes.



Problem 3:

Consider a perfectly conducting, air-filled cylindrical waveguide of radius a. The fields in the TM modes of the waveguide are given by

$$\mathbf{e} = \tilde{\mathbf{e}}(r,\varphi)\exp\left(-j\beta z\right), \mathbf{h} = \tilde{\mathbf{h}}(r,\varphi)\exp\left(-j\beta z\right)$$

$$\tilde{\mathrm{e}}_z = \exp(jm\varphi)J_m\left(k_c r\right),\ \tilde{\mathrm{e}}_r = \frac{-j\beta}{k_c^2}\frac{\partial \tilde{\mathrm{e}}_z}{\partial r},\ \tilde{\mathrm{e}}_\varphi = \frac{-j\beta}{k_c^2}\frac{\partial \tilde{\mathrm{e}}_z}{r\partial \varphi}$$

$$\tilde{\mathrm{h}}_z = 0,\ \tilde{\mathrm{h}}_r = \frac{j\omega\varepsilon_0}{k_c^2}\frac{\partial \tilde{\mathrm{e}}_z}{r\partial \varphi},\ \tilde{\mathrm{h}}_\varphi = \frac{-j\omega\varepsilon_0}{k_c^2}\frac{\partial \tilde{\mathrm{e}}_z}{\partial r}$$

$$k_c = \frac{\xi_{mp}}{a}, \beta = \sqrt{k_0^2 - k_c^2}$$

where ξ_{mp} is the *p*-th root of the Bessel function J_m. We put a thin, infinitely long, cylindrical magnetic rod of radius b inside the waveguide such that its axis coincides with that of the guide (z-axis). The rod is magnetized along its axis and has a saturation magnetization of M_s and an internal dc magnetic field of H_0. The rod is lossless and has a relative dielectric constant of one.

(i) Use perturbation theory to calculate the change in propagation constant after inserting the rod. (You do not have to evaluate the resulting integrals)

(ii) Does this change depend on the direction of propagation? Does it depend on the sign of m?