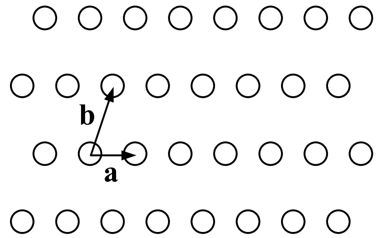
In the Name of God

Photonic Crystals – Final Exam | Time 3:00 Hours – Open Book

1. Prove the orthogonality and inversion properties of Wannier functions.
2. For the 2D crystal shown below



   a. Obtain the reciprocal lattice vectors in the general case when $\mathbf{a} \neq \mathbf{b}$.
   b. Plot the reciprocal lattice, identify the angles and dimensions.
   c. Show the first and irreducible Brillouin Zones.
   d. Examine the crystal symmetry groups for all high-symmetry points.
3. The alternate definition of Density of States (DOS) function $D(\omega)$ reads

$$D(\omega) = \frac{1}{V_{WSC}} \int_{V_{WSC}} \varepsilon(\mathbf{r}) \rho(\mathbf{r};\omega) d^3 r \,,$$

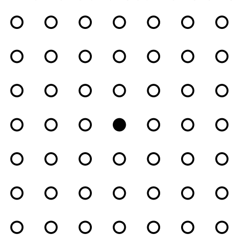
   in which

$$\rho(\mathbf{r};\omega) = -\frac{2\omega}{\pi c^2} \operatorname{Im}\left\{ \operatorname{tr}\left[\overleftrightarrow{G}(\mathbf{r},\mathbf{r};\omega) \right] \right\}$$

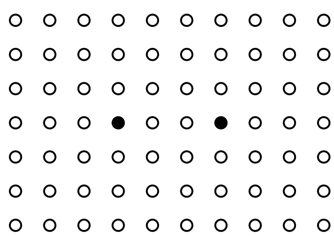
   represents the local DOS (LDOS) function in terms of the Green's function $\overleftrightarrow{G}(\mathbf{r},\mathbf{r}';\omega)$. Obtain an expression for $D(\omega)$ and simplify the result as much as possible.
4. Suppose that the nonlinear polarization due to $\chi^{(3)}$ effect of a field at frequency $\omega_n(\boldsymbol{\kappa})$ is given by

$$P_{NL}(\mathbf{r},t) = \frac{1}{6} A^3 \chi^{(3)}(\mathbf{r}) \left| E_{n\boldsymbol{\kappa}}(\mathbf{r}) \right|^2 E_{n\boldsymbol{\kappa}}(\mathbf{r}) \exp\left[j\omega_n(\boldsymbol{\kappa}) t \right],$$

   where the 3rd harmonic is neglected. Obtain an expression for position and time-dependent of the total propagating field. For the sake of simplicity take $\chi^{(3)}(\mathbf{r}) \propto \varepsilon(\mathbf{r})$, and assume in-plane propagation in 2D system.
5. Suppose that we have a lossless single-mode monopole cavity in an infinite 2D square lattice, with the resonant frequency ω_0, as shown in the structure below

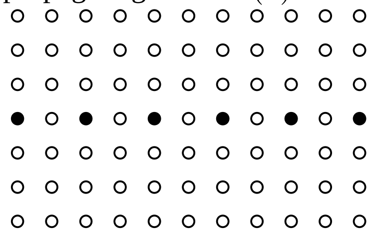


   a. If we have two of such cavities where the coupling rate is α_m, where m is the number of layers separating the two, then what would be the frequency and symmetries of the resulting super modes?



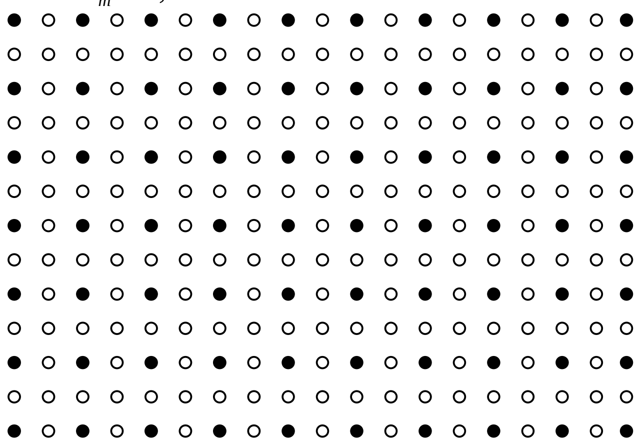
In the above figure, $m = 2$; suppose furthermore that $\alpha_m = \alpha_0 \exp(-m\zeta)$, with α_0 and ζ being constants.

b. Obtain the dispersion of the propagating mode $\omega(\kappa)$ in the CROW shown below



Discuss the number of branches and their symmetry. How does the result change, if $m > 1$?

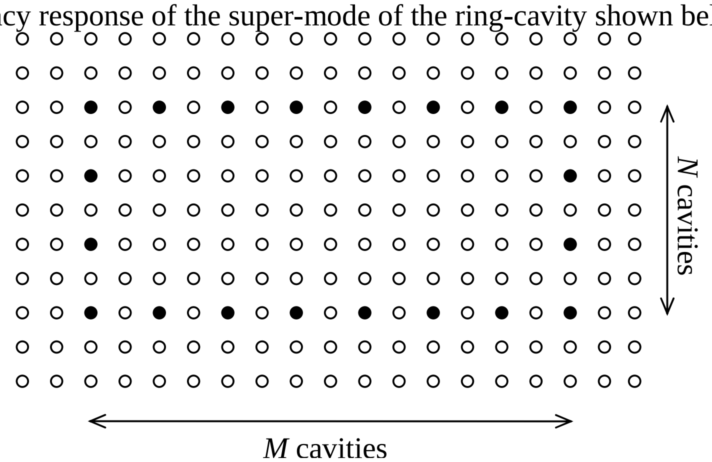
c. * Extend the approach to analyze the Coupled Planar Cavity Resonator Array (CPCRA) and find the frequency $\omega(\boldsymbol{\kappa})$ of the mode propagating having the Bloch wavevector $\boldsymbol{\kappa}$. Assume $\alpha_m = 0$, $m > 1$. Discuss the number of branches.



d. * If the single cavity in part (a) has a Lorentzian response as

$$R(\omega) = \frac{\gamma^2}{(\omega - \omega_0)^2 + \gamma^2},$$

due to a loss rate γ, what would be the Quality Factor Q for this cavity? Calculate the frequency response of the super-mode of the ring-cavity shown below



e. Discuss the changes one should consider in the above answers if the single cavity was originally

i. Non-degenerate Quadrupole
ii. Double-degenerate Dipole

Email answers to 5.c and 5.d till Friday July 7, 2006 noon to pbg@ee.sharif.edu.

Spring 2006 Semester Good Luck