An EKF-Based Joint Estimator for Interference, Multipath, and Code Delay in a DS Spread-Spectrum Receiver

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Abstract—In direct-sequence (DS) spread-spectrum communications, it is often necessary to obtain pseudo-noise (PN) code synchronization in the presence of both narrowband interference and frequency-selective multipath. However, conventional delay-lock code tracking loops are not optimized for such applications, nor are they well-suited to digital implementations. A code tracking algorithm based on the extended Kalman filter (EKF) is described that provides both code synchronization and joint estimates of interferer and channel parameters. It is first assumed that the interference can be modeled by an N-th order autoregressive (AR) process, and the multipath by a finite impulse response filter. A composite channel, equivalent to the convolution of the prewhitening filter and multipath coefficients is then constructed. The received waveform is shown to be a linear function of the composite channel parameters, that can therefore be directly estimated by an extended Kalman filter. The code delay \( \tau \) is viewed as a nonlinear parameter, that can likewise be estimated, after an appropriate linearization, using the EKF. The performance of the algorithm is first evaluated by computing the average bit-error rate (BER) of the joint EKF estimator, yielding a "learning curve" that the average bit-error rate can be computed at each iteration of the EKF estimator, yielding a "learning curve" that provides an overall performance measure of the DS receiver.

The joint EKF estimator is analyzed further by modeling the code delay estimate and error covariance by a vector Markov process. The bivariate p.d.f. of these parameters is evaluated using an adaptive numerical integration technique that solves the Chapman-Kolmogorov equation. From the joint p.d.f. of the nonlinear code delay error variance, in order to investigate the effects of linearization error and estimator bias.

In the following, the signal, channel and interferer model assumptions are stated in Section II. The joint parameterization of the channel and interferer is reviewed in Section...
III, and the EKF equations summarized in Section IV. The 
bit-error rate analysis for the digital RAKE receiver is 
presented in Section V, with results given in Section VI for 
BER. Section VII outlines the nonlinear analysis of the 
EKF, with a detailed description of the numerical solution 
to the Chapman-Kolmogorov equation given in Appendix 
A. An approximation for the measurement noise power in 
the Kalman filter equations is developed in Appendix B.

II. SIGNAL, CHANNEL AND INTERFERER MODELS

In the remainder of this paper, we will assume discrete-
time signal processing is to be used exclusively. Thus, 
al all signals and interferers need to be defined in terms of 
their Nyquist samples. The analog transmitted bandpass 
spread-spectrum signal can be defined as

\[ s_{BP}(t) = \Re \left\{ \sqrt{2E_b} \sum_{n=-\infty}^{\infty} d_n P N(t - nT_b) e^{j2\pi f_c t} \right\} , \]

where \( f_c \) denotes the carrier frequency, the \( d_n \) are binary 
valued (±1) information symbols, and \( P N(t) \) is a wide-
band, pseudo-noise sequence defined by

\[ P N(t) = \sum_{k=0}^{L_{PN}-1} c_k P \delta_r(t - kT_c) . \]

The binary digits \( c_k \) are ±1 valued, with \( T_c \) the chip 
duration satisfying the relation \( T_c = T_s / L_{PN} \). The bit duration 
is \( T_s \) sec, and the processing gain is thus \( L_{PN} \) chips per bit. 
Finally, \( P \delta_r(t) \) is a rectangular pulse with unit height and 
duration \( T_c \) seconds.

In the digital DS receiver considered here (Fig. 1), the 
incoming waveform is downconverted in quadrature and 
sampled at the Nyquist rate. The PN bandwidth is approx-
imately \( 1/T_c \), yielding a Nyquist sampling interval, de-
noted by \( T_s \) of \( T_s = T_c / 2 \) sec. The bandlimited nature of 
the DS waveform facilitates the derivation of an equivalen-
t discrete-time channel as follows. In [6, Chap. 7], it is 
shown that a multipath channel can be represented by a 
tapped-delay line with spacing equal to \( 1/2B \), where \( B \) is 
the signal bandwidth. Thus, the equivalent discrete-time 
channel impulse response for the DS signal is given as fol-
low\[ h(k) = \sum_{i=0}^{N_f-1} f_i \delta_{k,i} , \]

where \( \delta_{k,i} \) denotes the Kronecker delta. The coefficients \( f_i \) 
are complex-valued, and represent the amplitude and phase 
of each multipath component. In general, the \( f_i \) are time-
varying, however, we will assume that the \( f_i \) are constant 
parameters in order to make the simulations and nonlinear 
EKF analysis feasible. This assumption corresponds to a 
channel with a Doppler spread that is negligible compared 
to the information bandwidth of \( 1/T_b \) Hz.

In the presence of multipath and interference, the re-
ceived, complex-valued Nyquist samples are described by

\[ r(k) = \sum_{i=0}^{N_f-1} f_i s_{ip}((k - i)T_s + \tau) + j(k) + n(k) , \]

with the code delay denoted by \( \tau \). The variables \( j(k) \) and 
n\( n(k) \) represent the narrowband interferer and thermal noise 
respectively. These processes are assumed to be zero-mean 
and circular Gaussian with covariance functions denoted by

\[ E\{j(k)^* j(i)\} = R_{jj} (l - k) , \]

and

\[ E\{n(k)^* n(i)\} = \sigma_n^2 \delta_{k,l} , \]

where \( \delta_{k,l} \) is the Kronecker delta function, and

\[ \sigma_n^2 = 2N_0 / T_c . \]

Again, the DS waveform \( P N(t) \) is bandlimited to \( 1/T_c \) Hz 
prior to Nyquist sampling, and the resulting low-pass fil-
tered signal, \( P N_{lp}(t) \) is given by [4]

\[ P N_{lp}(t) = \sum_{k=0}^{L_{PN}-1} c_k 1/p \left[ S_i \left( 2\pi \left( t - kT_c \right) \right) - S_i \left( 2\pi \left( t - (k + 1)T_c \right) \right) \right] , \]

where

\[ S_i(x) = \int_{0}^{x} \sin \frac{y}{y} dy . \]

At this point, the channel is parameterized by the linear 
f\( f \) coefficients, and the unknown PN code delay is rep-
and thermal noise is approximated by an N-th order autoregressive (AR) process, as in [1]. Thus

$$j(k) + n(k) \approx \sum_{n=1}^{N_a} \alpha_n (j(k-n) + n(k-n)) + \epsilon(k), \quad (5)$$

where $\epsilon(k)$ is a discrete-time, complex-valued white noise process, with variance $\sigma^2$. Equation (2) is now rewritten in the following form, using the AR model for the interferer plus thermal noise.

$$r(k) = \sum_{l=0}^{N_f-1} f_l \delta_p ((k-l)T_s + \tau) + \sum_{n=1}^{N_a} \alpha_n \left\{ r(k-n) - \sum_{l=0}^{N_f-1} f_l \delta_p ((k-l-n)T_s + \tau) \right\} + \epsilon(k). \quad (6)$$

III. FORMULATION OF THE JOINT PARAMETER ESTIMATION PROBLEM

In order to compute joint parameter estimates, $r(k)$ must be expressed in the following manner, with $g(\bullet)$ denoting a general nonlinear function.

$$r(k) = g(r, f_1, \alpha_n) + \epsilon(k).$$

Equation (6) is in the above form, however, the dependence between $f_1$ and $\alpha_n$ is nonlinear. In order to decouple the channel and AR parameter estimates, we define a new composite channel, with coefficients $\beta_m$ as follows (see also [1].)

$$\beta_m = f_m - \sum_{n=1}^{N_a} \alpha_n f_{m-n} \quad (7)$$

for $m = 0, 1, \ldots, N_a + N_f - 1$, and with $f_1$ implicitly taken to be zero for $l < 0$ and $l > N_f - 1$. That is, the coefficients $\beta_m$ represent a channel that is the convolution of a prewhitening filter, corresponding to the AR model, and the original multipath channel. Now $r(k)$ can be rewritten as

$$r(k) = \sum_{m=0}^{N_p-1} \beta_m \delta_p ((k-m)T_s + \tau) + \sum_{n=1}^{N_a} \alpha_n r(k-n) + \epsilon(k), \quad (8)$$

where $N_p \equiv N_a + N_f$. The measurement $r(k)$ is taken to be a linear function of the composite channel coefficients $\beta_m$ and AR coefficients $\alpha_n$. Since the previous measurements $\{r(k-1), \ldots, r(0)\}$ are completely determined at sample $k$, an ordinary Kalman filter could be employed to jointly estimate the $\beta_m$ and $\alpha_n$ terms, provided that $\tau$ was known. In order to estimate the delay, however, the measurement equation (8) must be linearized about a predicted value of $\tau$ in order to implement the EKF, as discussed in the next section. Note that the data sequence, $d_n$ is unknown a-priori. As will be discussed in Section V, the overall receiver operates in a decision-directed mode, and the unknown $d_n$ are replaced by decisions $d_n$.

The composite channel, $\beta_m$, is clearly only an approximation to the true convolved channel depending on $f_1$ and $\alpha_n$. It should be emphasized that this approximation is made in the interests of obtaining a tractable recursive estimation algorithm. The joint estimation of $f_1$ and $\alpha_n$ individually is a very difficult nonlinear filtering problem.

IV. EKF JOINT ESTIMATOR

The extended Kalman filter equations for joint estimation of the parameters $\tau$, $\alpha_n$, and $\beta_m$ are now presented. The derivation closely follows that in [4, 8, 9], and will only be summarized here. In the EKF, the following first-order autoregressive models for the parameters are implicitly assumed.

$$\tau(k+1) = F_\tau \tau(k) + w_\tau(k),$$

$$\beta_m(k+1) = F_\beta \beta_m(k) + w_\beta(k),$$

$$\alpha_n(k+1) = F_\alpha \alpha_n(k) + w_\alpha(k),$$

where $w(k)$ represents white Gaussian process noises. Associated with the time-varying models for the parameters is the one-step transition matrix

$$F = \text{diag}\{F_\tau, F_\beta, \ldots, F_\beta, F_\alpha, \ldots, F_\alpha\}, \quad (10)$$

and process noise covariance matrix,

$$Q = \text{diag}\{\sigma^2_\tau, \sigma^2_\beta, \ldots, \sigma^2_\beta, \sigma^2_\alpha, \ldots, \sigma^2_\alpha\}. \quad (11)$$

Note that both $F$ and $Q$ have dimension $N_\alpha + N_\beta + 1$. The elements of $Q$ are defined by

$$\sigma^2_\tau = E\{[w_\tau(k)]^2\},$$

$$\sigma^2_\beta = E\{[w_\beta(k)]^2\},$$

and

$$\sigma^2_\alpha = E\{[w_\alpha(k)]^2\}.$$
with respect to the delay variable \( r \), the following measurement update equation is obtained.

\[
\begin{bmatrix}
\hat{r}(k|k) \\
\hat{\beta}_0(k|k) \\
\vdots \\
\hat{\beta}_{N_p-1}(k|k) \\
\hat{\alpha}_1(k|k) \\
\vdots \\
\hat{\alpha}_{N_a}(k|k)
\end{bmatrix} =
\begin{bmatrix}
\hat{r}(k|k-1) \\
\hat{\beta}_0(k|k-1) \\
\vdots \\
\hat{\beta}_{N_p-1}(k|k-1) \\
\hat{\alpha}_1(k|k-1) \\
\vdots \\
\hat{\alpha}_{N_a}(k|k-1)
\end{bmatrix} +
\begin{bmatrix}
\frac{\partial \hat{\beta}_j(k)}{\partial \hat{r}(k)} \\
s_{ip}(kT_s + \hat{r}) \\
\vdots \\
s_{ip}((k - N_p + 1)T_s + \hat{r}) \\
r(k - 1) \\
\vdots \\
r(k - N_a)
\end{bmatrix}
\]  

\[
\frac{1}{\sigma(k|k-1)} P(k|k-1)
\]

\[
\left[
\begin{array}{c}
\hat{r}(k) - \sum_{n=1}^{N_a} \hat{\alpha}_n(k|k-1)r(k - n) - \hat{\beta}_j(k) - \eta(k)
\end{array}
\right].
\]  

The one-step prediction of the prewhitened, received waveform is defined by

\[
\hat{\beta}_j(k) = \sum_{m=0}^{N_p-1} \hat{\beta}_m(k|k-1)s_{ip}((k - m)T_s + \hat{r}(k)).
\]  

Note that \( \hat{r}(k) \) corresponds to the real part of the one-step predictions, that is,

\[
\hat{r}(k) = \text{Re}\{\hat{r}(k|k-1)\}.
\]  

The correction term, \( \eta(k) \) arises since we are estimating a real-valued parameter, \( r \), using complex measurements \[9\], \[8\].

This term is given by

\[
\eta(k) = \frac{\partial \hat{\beta}_j(k)}{\partial \hat{r}(k)} Im\{\hat{r}(k|k-1)\}.
\]  

The EKF implicitly rejects the narrowband interference, by computing the following term in the innovations.

\[
r(k) - \sum_{n=1}^{N_a} \hat{\alpha}_n(k|k-1)r(k - n).
\]  

The above difference represents the output of a prewhitening filter operating on the received samples \( r(k) \), and thus subtracts out the interference. By removing the interference in the innovations, its effect on the measurement updates is minimized.

The error covariance matrices and innovations variance propagate as follows.

\[
\begin{align*}
P(k|k-1) &= FP(k-1|k-1)F^T + Q, \\
P(k|k) &= \\
&\left[I - \frac{1}{\sigma(k|k-1)} P(k|k-1)h(k)h(k)^H\right] P(k|k-1), \\
&\sigma(k|k-1) = h(k)^H P(k|k-1)h(k) + \sigma_n^2.
\end{align*}
\]  

The vector \( h(k) \) represents the gradient of the estimated signal \( \delta_f(k) \) with respect to the parameters, and is given by

\[
h(k) = \begin{bmatrix}
\frac{\partial \delta_f(k)}{\partial \hat{r}(k)} \\
s_{ip}(kT_s + \hat{r}) \\
\vdots \\
s_{ip}((k - N_p + 1)T_s + \hat{r}) \\
r(k - 1) \\
\vdots \\
r(k - N_a)
\end{bmatrix}^H.
\]  

V. Receiver Design and Bit-Error Rate Analysis

A block diagram of the DS receiver is shown in Fig. 1. This receiver follows the same digital RAKE design presented in \[9\], \[8\]. Essentially, the RAKE correlates the incoming prewhitened samples \( r(k) \) with a replica of the transmitted waveform, prefiltered by the estimated composite channel with coefficients \( \hat{\beta}_m \). However, in \[9\] and \[8\], it was assumed that the data sequence \( d_n \) was known a-priori in updating the EKF. Here, the receiver operates in a decision-directed mode, with the unknown symbols \( d_n \) replaced by decisions \( \hat{d}_n \) during EKF updates.

The overall decision-directed algorithm and demodulator is described as follows. The RAKE correlator uses EKF estimates during bits \( d(n) \) and \( d(n-1) \) that were computed at the end of bit \( d(n-2) \). The RAKE outputs, \( X(n) \) and \( X(n-1) \) corresponding to bits \( d(n), d(n-1) \) are given by the following.

\[
X(n) = \\
\sum_{k=(n+1)2L_PN-1}^{(n+1)2L_PN-1} (r(k) - \\
\sum_{l=1}^{N_a} \hat{\alpha}_l((n-1)2L_PN|(n-1)2L_PN-1)r(k-l)) \times \\
\sum_{l=1}^{n2L_PN-1} s_c(k - n2L_PN),
\]  

\[
X(n-1) = \\
\sum_{k=(n-1)2L_PN}^{(n-1)2L_PN-1} (r(k) - \\
\sum_{l=1}^{N_a} \hat{\alpha}_l((n-1)2L_PN|(n-1)2L_PN-1)r(k-l)) \times \\
\sum_{l=1}^{n2L_PN-1} s_c(k - (n-1)2L_PN).
\]  

The correlating waveform, \( s_c(k) \), is given by

\[
s_c(k) = \\
\sum_{m=0}^{N_p-1} \hat{\beta}_m((n-1)2L_PN|(n-1)2L_PN-1) \times \\
P(2L_PN)(k-m)T_s + \hat{\tau}((n-1)2L_PN),
\]  

The quantity \( \sigma_n^2 \) represents the variance of the measurement noise. In the assumed AR model (equation (5)), the measurement noise process is actually \( e(k) \), with unknown variance, \( \sigma_e^2 \). In Appendix B, it is shown that \( \sigma_e^2 \) tends to \( \sigma_n^2 \), in the limit as the interferer bandwidth tends to zero. For narrowband interferers, it is thus reasonable to approximate \( \sigma_e^2 \) by \( \sigma_n^2 \), in the interest of obtaining a practical estimation algorithm.
for \( k = 0, 1, \ldots, 2LPN - 1 \). Thus, \( s_c(k) \) represents one period, and one bit duration, of the PN sequence, that has been prefiltered by an estimate of the composite channel. A preliminary BPSK decision is then made on bit \( d(n-1) \), given by

\[
d(n) = \text{sgn}(Re\{X(n)\}).
\]  

(20)

The EKF updates are next propagated over bit \( d(n-1) \), corresponding to samples \( k = (n-1)2LPN \ldots n2LPN - 1 \). Finally, a DPSK decision is made using correlator outputs \( X(n) \) and \( X(n-1) \) as follows

\[
U(n) = Re\{X(n)X(n-1)^*\},
\]  

(21)

with \( U(n) \) compared to a zero threshold.

Note that although the BPSK decision could also be used to demodulate the data sequence, the DPSK decisions are more reliable when phase errors are present in the composite channel estimates, and when the channel itself is subject to rapid phase variations. However, it was found in the course of simulation, that the decision-directed EKF estimator performed somewhat better using BPSK decisions, since a DPSK error propagates over two successive bits. Thus, BPSK demodulation is used to obtain decisions for the EKF estimator, while DPSK decisions are employed to obtain the final sequence estimate. Once the DPSK decision has been made, the process is repeated for bits \( d(n+1), d(n) \), as summarized below.

For each bit \( d(n) \)

- For samples \( k = (n-1)2LPN \), \( (n-1)2LPN + 1 \ldots (n+1)2LPN - 1 \)
  - Form reference signal \( s_c(k-n2LPN) \) and
  - \( s_c(k-(n-1)2LPN) \) using previous EKF estimates
  - \( \beta_m((n-1)2LPN|n-1-2LPN) \)
  - \( \alpha_0((n-1)2LPN|n-1-2LPN) \)
  - Next sample
  - Compute RAKE outputs \( X(n) \) and \( X(n-1) \)
  - Compute BPSK decision \( d(n-1) = Re\{X(n-1)\} \)
  - For samples \( k = (n-1)2LPN \), \ldots, \( n2LPN - 1 \)
  - Update EKF estimates using \( d(n-1) \)
  - and obtain \( \beta_m(n2LPN|n2LPN-1) \) and
  - \( \alpha_0(n2LPN|n2LPN-1) \)
  - Next sample
  - Compute DPSK decision \( \text{sgn}\{\{U(n)\}\} \)
  - Next bit

A complete Monte-Carlo simulation of the decision-directed algorithm is impractical, due to the large number of bits, and hence DS waveform chips and Nyquist samples that must be generated. However, note that if the EKF estimates were deterministic, then \( X(n) \) and \( X(n-1) \) would be Gaussian random variables, since \( s_c(k) \) would then be deterministic, and \( r(k) \) would be circular Gaussian. We can obtain the analytic bit-error rate, for a specific simulated trajectory of the EKF estimator, as follows. The correlator outputs, \( X(n) \) and \( X(n-1) \) are jointly Gaussian random variables, conditioned on particular values of the prior EKF estimates \( \alpha, \beta, \) and \( \tau \). Furthermore, these estimates are deterministic functions of the cumulative measurements \( r(n-1)2LPN-1 \), where

\[
r^k \equiv \{r(0), r(1) \ldots r(k)\}.
\]

Thus, we can use the bit error rate analysis in [6, Chap. 4] and [9], to obtain a conditional BER for a specific measurement history. That is, for bit \( d(n) \), the conditional DPSK error probability, denoted

\[
P(E(n)|r((n-1)2LPN-1),
\]

is solely a function of the means, variances, and cross-covariance of the terms \( X(n) \) and \( X(n-1) \), with the prior EKF estimates effectively constant. Thus, \( E(n) \) denotes the event that a DPSK error is made on bit \( d(n) \), using the prior EKF estimates. It is readily shown that the conditional expected value of \( X(n) \) is

\[
X(n)(|r((n-1)2LPN-1)) = \hat{X}(n) = \sum_{k=n2LPN}^{(n+1)2LPN-1} R_{eq}(k-k') \times
\]

(22)

Next sample

\[
\sum_{k=n2LPN}^{(n+1)2LPN-1} \sum_{k'=n2LPN}^{(n+1)2LPN-1} R_{eq}(k-k') \times
\]

(23)

Next sample

\[
\sum_{k=n2LPN}^{(n+1)2LPN-1} \sum_{k'=n2LPN}^{(n+1)2LPN-1} R_{eq}(k-k') \times
\]

(24)

Next sample

\[
\sum_{k=n2LPN}^{(n+1)2LPN-1} \sum_{k'=n2LPN}^{(n+1)2LPN-1} R_{eq}(k-k') \times
\]

(25)

Next sample

\[
\sum_{k=n2LPN}^{(n+1)2LPN-1} \sum_{k'=n2LPN}^{(n+1)2LPN-1} R_{eq}(k-k') \times
\]

(26)

Next sample

\[
\sum_{k=n2LPN}^{(n+1)2LPN-1} \sum_{k'=n2LPN}^{(n+1)2LPN-1} R_{eq}(k-k') \times
\]

(27)

Next sample

\[
\sum_{k=n2LPN}^{(n+1)2LPN-1} \sum_{k'=n2LPN}^{(n+1)2LPN-1} R_{eq}(k-k') \times
\]

(28)

Next sample

\[
\sum_{k=n2LPN}^{(n+1)2LPN-1} \sum_{k'=n2LPN}^{(n+1)2LPN-1} R_{eq}(k-k') \times
\]

(29)

Next sample

\[
\sum_{k=n2LPN}^{(n+1)2LPN-1} \sum_{k'=n2LPN}^{(n+1)2LPN-1} R_{eq}(k-k') \times
\]

(30)
The covariance function of the interferer, \( R_{jj}(k) \), depends on the technique used to generate \( j(k) \). In the following simulations, \( j(k) \) is obtained by digital filtering of a discrete-time white noise process. The filter transfer function corresponds to a fourth-order elliptic filter, designed to approximate a rectangular, bandpass power spectral density, and is thus of the form

\[
H(z) = \frac{\sum_{m=0}^{4} b_m z^{-m}}{1 - \sum_{n=1}^{4} a_n z^{-n}}.
\]

The covariance function is then readily determined by a partial fraction expansion of the power-spectral density, given by

\[
\Phi_{jj}(z) = H(z) H(z^{-1}) \sigma_r^2,
\]

\[
R_{jj}(k) = \frac{1}{2\pi} \int \Phi_{jj}(z) e^{-i k z} dz.
\]

where \( \sigma_r^2 \) is the variance of the discrete-time white noise process driving the interference generation filter \( H(z) \).

The final bit-error rate is obtained from the following expression [6].

\[
P(E(n)|\mu(n-1)2LPN)^{-1}) = Q(a, b) - \frac{1}{2} f_0(ab) e^{-\frac{a^2 + b^2}{2}},
\]

where the terms \( a \) and \( b \) are completely determined from the conditional means \( \tilde{X}(n) \), \( \tilde{X}(n-1) \), covariances \( \mu_{xx}(n) \), and cross-covariance \( \mu_{xx}(n, n-1) \), and with \( Q(a, b) \) the Marcum-Q function. Given a specific trajectory of prewhitening filter, channel, and delay estimates, the analytic BER can then be evaluated using equation (28). This procedure yields a learning curve displaying bit-error rate versus iterations of the EKF joint estimator. The unconditional BER can be estimated, at each iteration, by averaging the conditional error probability over \( N_r \) independent runs. That is,

\[
P(E(n)) \approx \frac{1}{N_r} \sum_{i=1}^{N_r} P(E(n)|\mu(n-1)2LPN)\).
\]

The overall method thus is a compromise between a complete Monte Carlo simulation, which would be impractical, and a complete analysis of the unconditional BER, which would likewise be impractical due to the random nature of the EKF estimates.

VI. Bit-Error Rate Results

The EKF joint estimator was simulated over 3000 iterations, corresponding to 1500 chips, or 100 bits with the processing gain, \( LPN \), set to 15 chips/bit. In all cases, the signal-to-noise ratio, \( E_b/N_0 \), was set to 10 dB. The interferer was generated using the fourth-order elliptic filter with a 3 dB cutoff of .1 Hz, and the jammer-to-signal power ratio, \( J/S \), was also equal to 10 dB. The DS signal bandwidth, \( 1/T_e \), was normalized to 1 Hz, and thus the interferer jammed approximately ten percent of the band.

Two channels were simulated, the first corresponding to no fading, with \( f_0 = 1 \), and the remaining \( f_t \) coefficients set to zero. The second channel consisted of four rays (\( N_f = 4 \)), with the resulting transfer function given by

\[
F(z) = f_0 + f_1 z^{-1} + f_2 z^{-2} + f_3 z^{-3}.
\]

The frequency response of this channel is plotted in Fig. 2. Throughout the EKF simulations, a 4 coefficient prewhitening filter was used (\( N_\alpha = 3 \)), and thus \( N_j \) was set to 4 for the direct-path only channel, and to 7 for the 4-ray channel case. The decision-directed algorithm was used in all cases.

For the case of no fading, a tracking error trajectory is shown in Figure 3, with the error defined by \( \tau - \hat{\tau}(k) \). Note that without prewhitening, the tracking error eventually diverges to about one chip, representing a loss of lock condition. However, when the \( \alpha_n \) coefficients are estimated using the decision-directed EKF equations, and the input samples thus prewhitened, the tracking error remains within one-quarter of a chip. A similar plot is shown for the 4-ray channel in Figure 4. Note that the tracking error diverges even more rapidly in the presence of multipath, when prewhitening is not used, but again remains within a quarter of a chip, when both the interferer and channel parameters are estimated.

The BER versus iterations of the joint estimator is shown in Figure 5 for the direct-path only case, with narrowband interference present. The lower dashed line represents the optimal BER resulting when the interferer estimates, \( \hat{\alpha}_n \), are fixed at the values obtained from the Wiener-Hopf equation. That is, the \( \hat{\alpha}_n \) are given by the solution of the following set of equations, that assumes exact knowledge of the interferer correlation function \( R_{jj}(k) \).

\[
R_{jj}(l) = \sum_{n=1}^{N_\alpha} \alpha_n \{ R_{jj}(n - l) + R_{nn}(n - l) \}
\]

for \( l = 1, 2, ... N_\alpha \). Note that the BER quickly falls from one-half to about \( .5 \times 10^{-3} \) within 300 iterations, correspond-
The receiver performance without prewhitening (the EKF estimates only \( \tau \) and the \( \beta \) coefficients,) is also illustrated in Fig. 5. Due to the lack of interference rejection, and divergence of the delay estimate, the BER remains at 1/2 for this system. An estimate of the unconditional BER, \( P(E(n)) \), was also obtained by averaging over \( N_r = 50 \) independent runs. Note that the unconditional BER closely tracks the optimal BER, after convergence, and that convergence is again quite rapid.

For the 4-ray channel example, the BER is plotted in Figure 6. The optimum bit-error rate is computed using the Wiener solution for the \( \alpha \) coefficients, and with \( \beta \), given by the convolution of the optimum prewhitening filter response and the true channel coefficients \( f_t \). Again, the BER quickly falls to the vicinity of the optimal BER within 10 bits (300 iterations.) The BER is actually lower than in the previous example (roughly 0.5 \times 10^{-4} instead of 0.5 \times 10^{-3}, since the digital RAKE correlator effectively “coherently combines” the multipath, thus obtaining a diversity gain. The unconditional BER is also obtained by averaging \( P(E(n))p^{(n-1)LPN-1} \) over 50 runs.

Finally, the frequency response of the prewhitening filter, using the coefficient estimates \( \tilde{\alpha}_0 \), computed by the EKF, is shown in Fig. 7, along with the interferer power spectral density, computed from equation (27).

VII. NONLINEAR ANALYSIS OF THE EKF

The performance of digital phase-locked and delay-locked loops is typically evaluated by numerical solution of the Chapman-Kolmogorov equation that characterizes the phase or timing error process [10]. A similar analysis of the extended Kalman filter estimator is useful in order to determine the exact nonlinear timing error variance. However, the nonlinear analysis of the EKF is complicated by the fact that the time-delay estimate, \( \tilde{\tau}(k|k) \) is not by itself a Markov process, due to the stochastic nature of the scalar Riccati equation defining the error covariance update, and the presence of the unknown channel and interferer pa-
In the approximate analysis of [4], the above stochastic Riccati equation was replaced by the following deterministic covariance update.

\[
p(k) = F_r^2 \left[ \frac{p(k-1)}{p(k-1)} - \frac{p(k-1)^2 |s_d(kT_s + \tau)|^2}{p(k-1)[s_d(kT_s + \tau)]^2 + \sigma_n^2} \right] + \sigma_e^2.
\]

Note that this is the error covariance that would be computed by the EKF if the timing estimate was exact, and hence the linearization error was zero. We can thus view this deterministic update for \(p(k)\) as a lower bound on the nonlinear timing error variance.

The signal \(s_p(t)\) represents the low-pass equivalent, transmitted waveform, prefiltered by the channel with coefficients \(f_1\), and optimum prewhitening filter with coefficients \(a_\alpha\). Equivalently, \(s_p(t)\) can then written as the original low-pass equivalent signal, \(s_{lp}(t)\) in equation (3), filtered by the composite channel \(\beta_m\). Thus

\[
s_p(t) = \sum_{m=0}^{N_\alpha-1} \beta_m s_{lp}(t - mT_s).
\]

The signal derivative, \(s_d(t)\) is defined by

\[
s_d(t) = \frac{\partial s_p(t)}{\partial t}.
\]

Examination of (31) reveals that \(\tau(k)\) is not Markov, since it depends on the entire past timing estimation trajectory \(\tau(k)\) through \(p(k)\). However, define the two-dimensional vector process \(x(k)\) as follows.

\[
x(k) \equiv \begin{bmatrix} \tau(k) \\ p(k) \end{bmatrix}.
\]

The vector \(x(k)\) depends only on \(x(k-1)\) and the circular white Gaussian process \(e(k)\). Thus, \(x(k)\) is a two-dimensional Markov process, and obeys the following Chapman-Kolmogorov equation, where \(p(x(k)|x(k-n))\) denotes the general n-step transition density.

\[
p(x(k)|x(0)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(x(k)|x(k-1))p(x(k-1)|x(0))dx(k-1).
\]

The joint one-step transition density is determined as follows. First, observe that \(\tau(k)\) is Gaussian, when conditioned on \(\tau(k-1)\) and \(p(k-1)\), since the residual error \(e(k)\) is itself Gaussian. Furthermore, \(p(k)\) is just a constant when conditioned on the previous values of the error covariance and time-delay estimate, \(\tau(k-1)\) and \(p(k-1)\). In terms of \(\tau(k)\) and \(p(k)\), therefore, the bivariate transition density can be written as

\[
p(\tau(k), p(k)|\tau(k-1), p(k-1)) = N(\tau(k); m_\tau, \sigma_\tau^2) \delta(p(k) - m_p(k)),
\]

\[1\]

Note that we are implicitly assuming that the interferer is exactly an \(N_\alpha\)-order Gaussian AR process, so that the prewhitening filter output, \(e(k)\) is circular white Gaussian.
where \( N(x; m, \sigma^2) \) is the univariate Gaussian density with mean \( m \) and variance \( \sigma^2 \), and \( \delta(x) \) is the dirac-delta function. The mean function \( m_r(k) \) is given by the deterministic portion of the measurement-update equation (31).

\[
m_r(k) = F_r \tau(k - 1) + \text{Re} \left\{ \frac{p(k-1) s^T(kT_s + F_r \tau(k-1))}{p(k-1) s^2(kT_s + F_r \tau(k-1))^2 + \sigma_n^2} \times [s_{\theta}(kT_s + \tau) + s_\theta(kT_s + F_r \tau(k-1))] \right\}.
\]

The variance can be written in terms of the Kalman gain and prediction error residual power as

\[
\sigma^2(k) = \frac{1}{2} |K(k)|^2 \sigma_n^2
\]

\[
K(k) = \frac{p(k-1) s^T(kT_s + F_r \tau(k-1))}{p(k-1) s^2(kT_s + F_r \tau(k-1))^2 + \sigma_n^2}.
\]

Finally, the function \( m_p(k) \) defining \( p(k) \) is just the r.h.s. of (32).

In principle, the evolution in time of the joint p.d.f. can be evaluated by numerical integration of (36). However, the dirac-delta function that appears in the one-step transition density (37) introduces complications, as does the computational burden of two-dimensional numerical integration. In Appendix A, an adaptive, two-dimensional integration algorithm is presented that propagates probability masses instead of the density function itself, and thus eliminates the dirac-delta function in the kernel of the C-K equation.

VIII. NONLINEAR EKF ANALYSIS - RESULTS

The two-dimensional Chapman-Kolmogorov equation was evaluated by numerical integration, to obtain the joint and marginal p.d.f.s of the time delay estimate and error covariance, as well as the nonlinear timing error variance. The interferer of Section VI, with an approximately rectangular spectral density was used to compute the optimum prewhitening coefficients \( \alpha \), using the Wiener-Hopf equations (30). The processing gain, signal-to-noise ratio, and signal to jammer ratio were kept at the same values used in the BER analysis. Specifically, \( L_{PN} \) was set to 15 chips/bit, with \( E_b/N_0 \) and \( J/S \) both equaling 10 dB. However, to lessen computing time, a three-ray, instead of 4-ray channel was used, with transfer function of the form

\[
F(z) = f_0 + f_1 z^{-1} + f_2 z^{-2}.
\]

The joint density of the time-delay estimate \( \tau(k) \), and the error covariance \( p(k) \) is shown in Fig. 8, after 100 iterations of the EKF, with both the 3-ray channel and interferer present. In this case, the density in the delay dimension is sharply peaked at \( \tau \), which agrees with the true value of the time delay. The density of the error covariance is confined to the narrow region from \( 5 \times 10^{-3} \) to \( 5.4 \times 10^{-3} \). It is interesting to note that the corresponding solution of the deterministic Riccati equation (33) is \( 5.11 \times 10^{-3} \). This reflects the fact that the bulk of the probability mass in the \( \hat{\tau} \) dimension is centered about the true value of \( \tau \), and thus

\[
\text{Fig. 8 Joint PDF from numerical integration of the two-dimensional C-K equation. Interferer with 4-th order elliptic filter spectral density, 3-ray channel, } E_b/N_0 = 10 \text{ dB, } J/S = 10 \text{dB.}
\]

linearization error is minimal. Thus, there is close agreement between the deterministic covariance, which assumes no linearization error, and the actual covariance computed using the stochastic Riccati equation. The marginal p.d.f. of the delay estimate \( \tau(k) \) can be computed by numerical integration of the joint p.d.f. in Fig. 8. The evolution in time up to 100 iterations is shown in Fig. 9. The nonlinear timing error variance can similarly be computed, and is shown in Fig. 10 for the interferer plus 3-ray channel case. The error variance is also shown for the case of no interferer and no fading. It is seen that this error variance is actually somewhat larger than that for the case with multipath and interference, since the 3-ray channel has greater than unity gain, and the EKF is effectively coherently combining the multipath to obtain better performance. The covariance \( p(k) \) computed by the deterministic Riccati equation (33) is also plotted, and it is seen that close agreement between the stochastic and deterministic results is obtained after 100 iterations. Thus, we can conclude that the linearization error in the EKF has become negligible, and that the deterministic Riccati equation accurately predict the timing error variance by 100 iterations.

IX. CONCLUSIONS

An EKF-based algorithm was presented that computes joint estimates of composite channel coefficients, interferer AR parameters, and PN code delay in a DS spread-spectrum receiver. The overall bit-error rate performance of the receiver was evaluated versus iterations of the joint estimator. Results showed that favorable performance could be obtained in the presence of strong interference (J/S equal to 10 dB). Simulation results indicated that the code delay estimate quickly diverges when interference is present, but only the delay and channel are estimated by
ILTIS: ESTIMATOR FOR INTERFERENCE, MULTIPATH, AND CODE DELAY IN A DS/SS RECEIVER

Fig. 9 Evolution in time of the marginal delay estimate p.d.f. Interferer with 4-th order elliptic filter spectral density, 3-ray channel, $E_b/N_0 = 10$ dB, $J/S = 10$dB.

Fig. 10 Nonlinear timing error variance for interferer plus multipath case, and no interferer, no multipath case.

The nonlinear tracking variance closely agreed with that computed using a deterministic Riccati equation for the EKF error covariance after 100 iterations, and thus we can conclude that the linearization error became negligible at that point. Examination of the two-dimensional p.d.f. of the delay estimate and error covariance confirmed that in the $p(k)$ dimension, the density was sharply peaked about the value predicted by the deterministic Riccati equation.

The extended Kalman filter joint estimator and RAKE receiver presented here provide a comprehensive design for an all-digital direct-sequence receiver. Despite the apparent complexity of the EKF algorithm, it should be emphasized that matrix inversions are not required, since the measurements are scalars. However, for large signal bandwidths and sampling rates, the matrix multiplication operations in the covariance and measurement updates are computationally intensive. For HF channel applications, where typical DS bandwidths would not exceed one megahertz, it would be feasible to implement the digital receiver in programmable digital signal processing devices. For VHF and UHF applications, implementation of the EKF and digital RAKE is potentially feasible in custom VLSI circuitry. In general, although the computational burden of the algorithm can become quite large for wideband spread-spectrum applications, the EKF has a significant advantage over conventional analog synchronization circuitry, in that it can cope with a wide range of fading and co-channel interference conditions. As the capabilities of VLSI signal processing devices expand, it appears that the type of all-digital receiver design considered here should warrant increased consideration.

A. NUMERICAL INTEGRATION METHOD FOR A TWO-DIMENSIONAL CHAPMAN-KOLMOGOROV EQUATION

The Chapman-Kolmogorov equation defining the delay estimate process and associated EKF error covariance is given by

\begin{equation}
p(\hat{\tau}(k), p(k) | \hat{\tau}(0), p(0)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} N(\hat{\tau}(k); m_\tau(k), \sigma^2(k)) \delta(p(k) - m_p(k)) \times p(\hat{\tau}(k - 1), p(k - 1) | \hat{\tau}(0), p(0)) d\tau(k - 1) dp(k - 1).
\end{equation}

The numerical solution of this equation is complicated by the presence of the dirac-delta function in the kernel, and the fact that the joint density function has infinite support in the $\tau$ dimension. Thus, we seek to develop an adaptive numerical integration procedure that eliminates these difficulties.

To obtain a summation approximation to the above integral, first approximate the joint density function by its probability mass in a set of rectangular regions. Let $y_1 = \hat{\tau}(k)$, $y_2 = p(k)$, $x_1 = \hat{\tau}(k - 1)$, and $x_2 = p(k - 1)$. Then define a probability mass of dimension $\Delta y_1 \times \Delta y_2$, centered at the point $(y_1, y_2)$ as follows.

\begin{equation}
g(y_1, y_2 | 0) \equiv \ldots
\end{equation}
The function $m_r(k)$, $\sigma^2(k)$ and $m_p(k)$ are the EKF measurement and covariance updates, conditioned on $\tau(k - 1) = x_1$ and $p(k - 1) = x_2$. Specifically,

$$
m_r(k) = F_r x_1 + \underbrace{Re\left\{\frac{x_2 s_d(k T_s + F_r x_1)}{x_2 s_d(k T_s + F_r x_1)^2 + \sigma^2} \times [s_\beta(k T_s) - s_\beta(k T_s + F_r x_1)]\right\}}_{(A.8)}
$$

and

$$
\sigma^2(k) = \frac{1}{2} \left\{ \frac{x_2 s_d(k T_s + F_r x_1)^2}{x_2 s_d(k T_s + F_r x_1)^2 + \sigma^2} \right\}^2
$$

At each iteration, equation (A.7) yields $q(y_1, y_2|0)$ on a grid of points $y_1, y_2$ given the joint probability mass function $q(x_1, x_2|0)$. However, the limits $y_{1,\text{min}}, y_{1,\text{max}}$ and $y_{2,\text{min}}, y_{2,\text{max}}$ must still be determined. From equation (A.10), it can be seen that in the $p(k)$ dimension, the joint mass function has finite support, since the signal derivative $s_\beta(t)$ is a bounded function. However, since the transition p.d.f. has a Gaussian form in the $y_1$ dimension, clearly $q(y_1, y_2|0)$ has infinite support in this dimension. Under the assumption that the shape of the marginal delay estimate p.d.f. is unimodal, the following algorithm is used to set limits on the double summation.

Assume that the prior mass function $q(x_1, x_2|0)$ is given, as well as the previous limits $x_{1,\text{min}}, x_{1,\text{max}}, x_{2,\text{min}}, x_{2,\text{max}}$. The approximate mean and variance in the $x_1 = \tau(k - 1)$ dimension are first computed as follows.

$$
m_1 = \sum_{x_{1,\text{max}}}^{x_{1,\text{max}}} \sum_{x_{2,\text{min}}}^{x_{2,\text{min}}} \sum_{x_{1,\text{min}}}^{x_{1,\text{max}}} \sum_{x_{2,\text{min}}}^{x_{2,\text{min}}} \sum_{x_{1,\text{min}}}^{x_{1,\text{max}}} \sum_{x_{2,\text{min}}}^{x_{2,\text{min}}} (x_1 - m_1)^2 q(x_1, x_2|0)
$$

Then the limits for evaluation of the updated mass function are given by

$$
y_{1,\text{min}} = m_1 - 3 \times \sqrt{\text{var}_1}
$$

$$
y_{1,\text{max}} = m_1 + 3 \times \sqrt{\text{var}_1}
$$

That is, the limits in the $y_1$ dimension are set to three times the standard deviation of the previously computed marginal p.d.f. In the $y_2$ dimension, the density has finite support. Thus

$$
y_{2,\text{min}} = \min_{x_{1,\text{min}}} m_p(k)
$$

$$
y_{2,\text{max}} = \max_{x_{1,\text{max}}} m_p(k)
$$

The summation equation (A.7) is now updated, yielding the new mass function $q(y_1, y_2|0)$. The limits $y_{1,\text{min}}, y_{1,\text{max}}, y_{2,\text{min}}, y_{2,\text{max}}$ replace the previous $x$ quantities, and the algorithm is repeated.
B. LIMITING VALUE OF THE AR PROCESS
RESIDUAL NOISE POWER

In this Appendix, it is shown that the variance $\sigma^2_n$ of the AR process residual in equation (5), can be approximated by the white noise power, $\sigma^2_n$, as the interferer bandwidth tends to zero. Recall that the process $j(k)+n(k)$ is assumed to be autoregressive, of order $N_a$. Thus, repeating (5),

$$j(k)+n(k) \approx \sum_{n=1}^{N_a} \alpha_n (j(k-n)+n(k-n)) + \epsilon(k). \quad (B.1)$$

Since $\epsilon(k)$ is circular white Gaussian, under the AR model assumption, the filter $A(z) = \sum_{n=1}^{N_a} \alpha_n z^{-n}$ is the optimum predictor for $j(k)+n(k)$. The power spectral density of the thermal noise, $n(k)$, is denoted by $\Phi_{nn}(z) = \sigma^2_n$. It is assumed that the interferer, $j(k)$, has a rectangular spectral density, $\Phi_{jj}(z)$ given on the unit circle by

$$\Phi_{jj}(e^{i\omega}) = \left\{ \begin{array}{ll} \eta \sigma^2_n & \text{for } \omega \leq \omega_u \\ 0 & \text{otherwise} \end{array} \right. , \quad (B.2)$$

where $-\pi \leq \omega \leq \omega_u \leq \pi$. The reason for this definition of $\Phi_{jj}(z)$ in terms of $\sigma^2_n$ will become clear shortly, however, note that by varying $\eta$ and $\omega_u$, it is possible to represent an interferer with arbitrary power and bandwidth.

Next, consider the minimum mean-square error, which is just $\sigma^2_n$, at the output of the prediction error filter, $[1-A(z)]$. Let $\Phi_{xx}(z) = \Phi_{jj}(z) + \Phi_{nn}(z)$ represent the power spectral density of the process $j(k)+n(k)$. Then

$$\Phi_{xx}(e^{i\omega}) = \left\{ \begin{array}{ll} \sigma^2_n & \text{for } -\pi \leq \omega \leq \omega_u, \omega_u \leq \omega \leq \pi \\ (1+\eta)\sigma^2_n & \text{for } \omega_u \leq \omega \leq \omega_u \end{array} \right. . \quad (B.3)$$

The mean-square error for the case of a perfect predictor, is given by the Kolmogorov-Szego formula [11, Chap. 14].

$$\sigma^2_e = \exp \left[ \frac{1}{2} \int_{-\pi}^{\pi} \ln \Phi_{xx}(e^{i\omega}) \omega \right] . \quad (B.4)$$

Now, replace $\Phi_{xx}(z)$ by $\Phi_{jj}(z) + \Phi_{nn}(z)$, using the definition of $\Phi_{jj}(z)$ given in (B.2).

$$\sigma^2_e = \exp \left[ \frac{1}{2} \int_{-\pi}^{\pi} \ln \sigma^2_n \omega \omega_u \ln(1+\eta) \omega \right] , \quad (B.5)$$

which simplifies to

$$\sigma^2_e = \sigma^2_n (1+\eta)/(\omega_u - \omega_u)^{1/2}. \quad (B.6)$$

Note that $(\omega_u - \omega_u)$ is just the bandwidth, $B$, of the interferer. Thus

$$\sigma^2_e = \sigma^2_n (1+\eta)^B . \quad (B.7)$$

If the interferer power is maintained at a constant value, for varying $B$ and $\eta$, then $\sigma^2_e$ can be re-expressed as

$$\sigma^2_e = \sigma^2_n (1+\eta)^{c/\eta} \quad (B.8)$$

where $c$ is a constant. Clearly, as $\eta \rightarrow \infty$, $(1+\eta)^{c/\eta} \rightarrow 1$. Thus, as the bandwidth tends to zero, $\sigma^2_e$ tends to $\sigma^2_n$, for a fixed interferer power. Intuitively, this corresponds to the case of a perfectly predictable interferer, $j(k)$, which must be a complex exponential (tone) with zero bandwidth. That is, if the interferer has zero bandwidth, the prediction filter forms an infinitely narrow notch in the frequency domain. The frequency response of $[1-A(z)]$ is then unity except at the interferer location, $\omega_u$, where it is zero. Thus the residual spectral density is $\sigma^2_n$ except at the point $\omega_u$, and the minimum mean-square error approaches $\sigma^2_n$.

The approximation $\sigma^2_e \approx \sigma^2_n$ is quite good for highly narrowband interferers. For example, assume that $B = 0.01$, and $\eta = 1 \times 10^5$ (corresponding to an interferer with power given by $100 \times \sigma^2_n$, or 20 dB above the noise power.) Then $\sigma^2_e = 1.011 \times \sigma^2_n$. For $B = 0.01$, and $\eta = 1 \times 10^4$, we obtain $\sigma^2_e = 1.096\sigma^2_n$, and thus the approximation is valid.

REFERENCES


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