Session 4: Solid State Devices

Hetro junction devices

Outline

- A
  - B
  - C
  - D
  - E
- F
  - G
- H
- I
- J

Outline

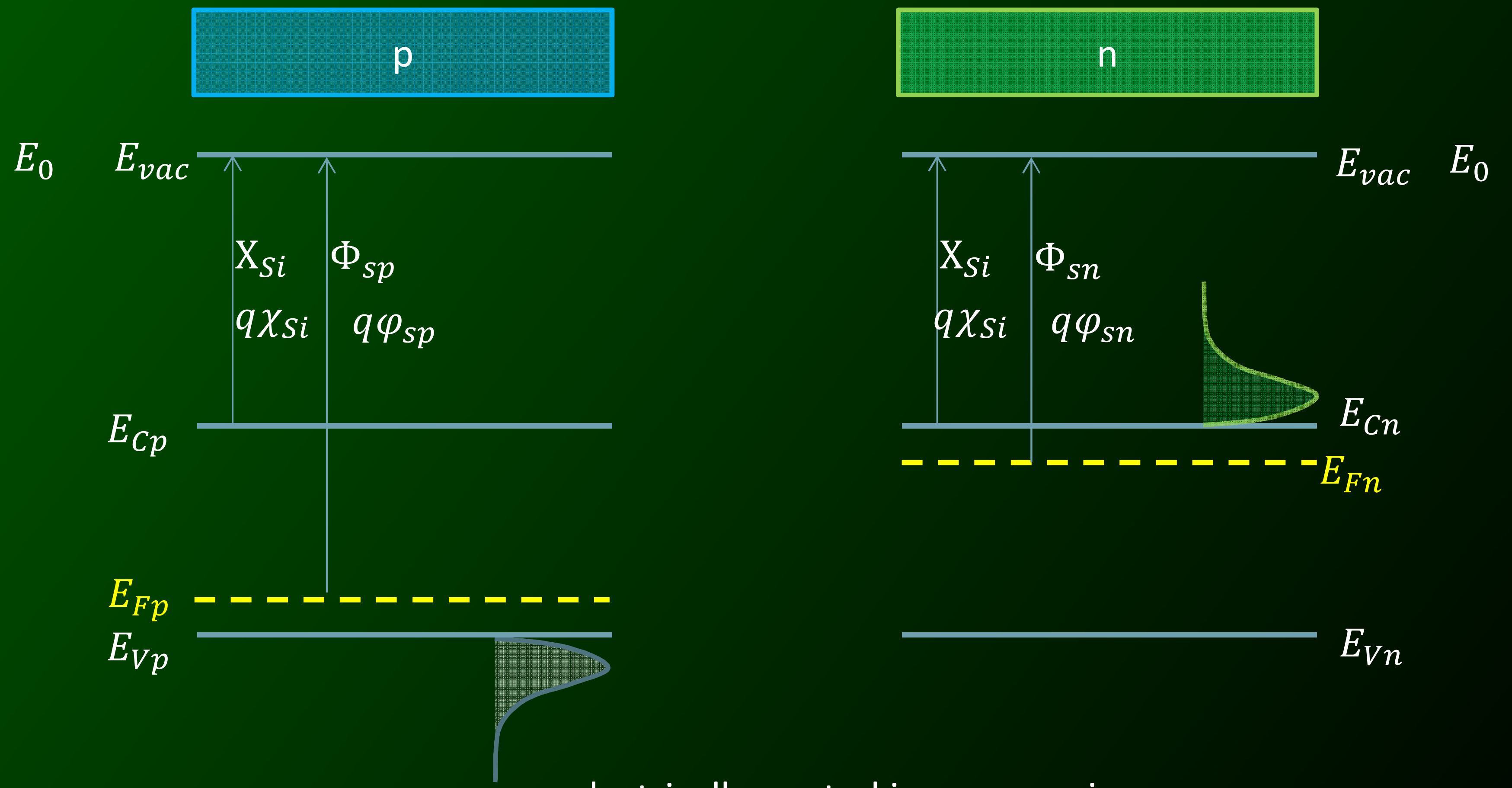
- Ref: Brennan and Brown

Review Homojunction!

Homojunction: the junction is between two regions of the same material

Heterojunction: the junction is between two different semiconductors

PN junctions – Before Being Joined



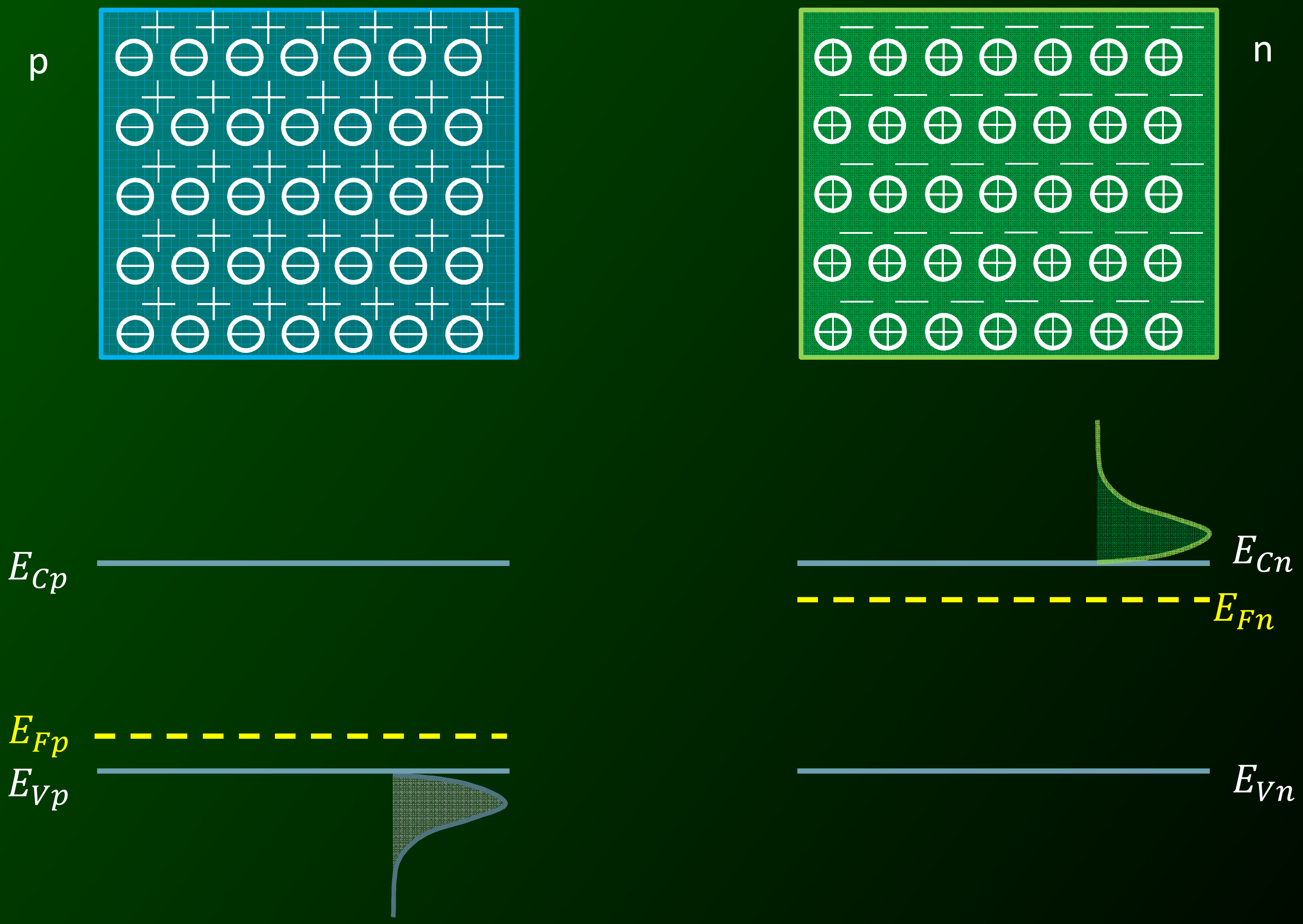
electrically neutral in every region

electron affinity : X_{Si}

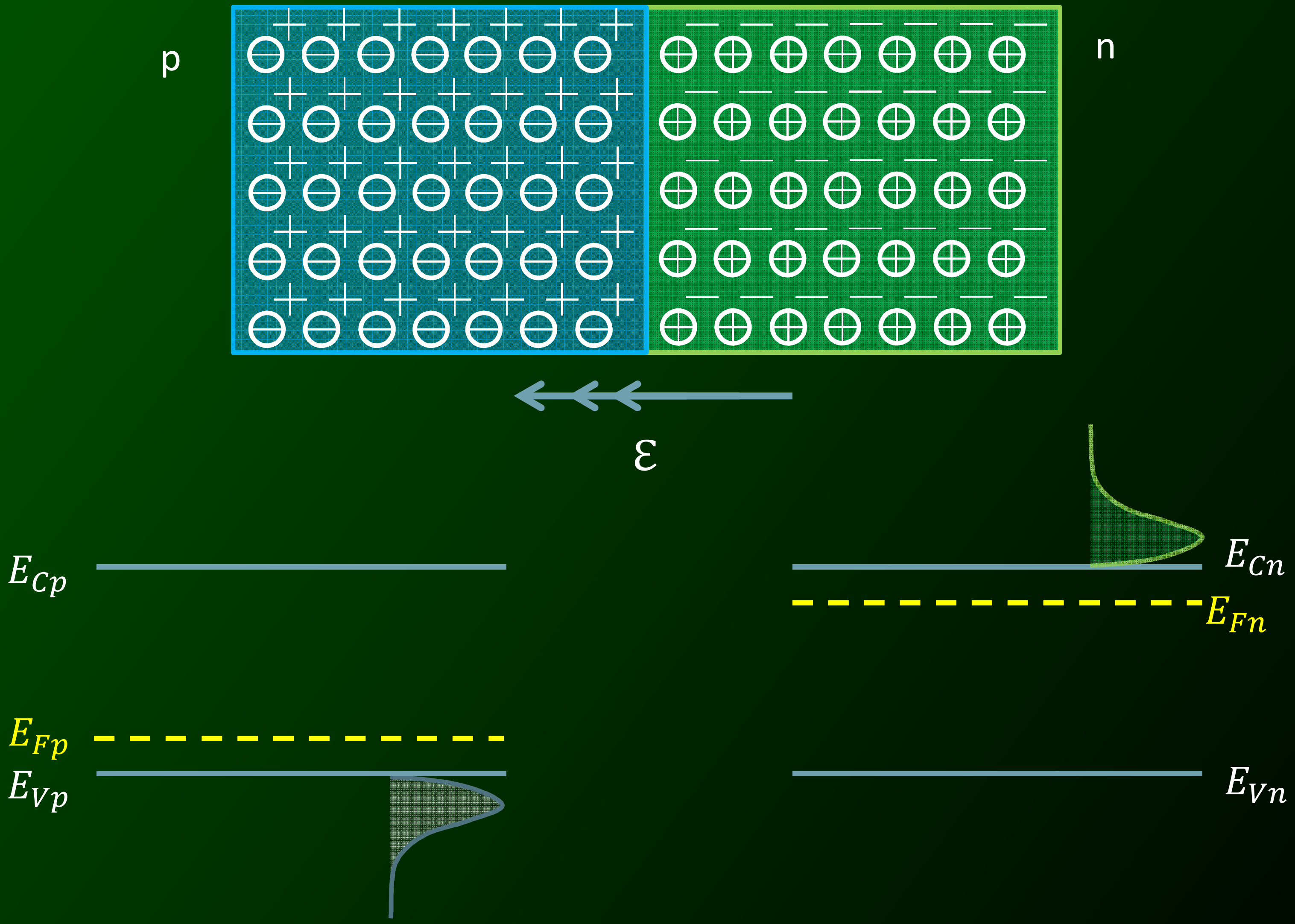
work function Φ: $\Phi = E_{vac} - E_F$

$$\Phi_n \neq \Phi_p$$

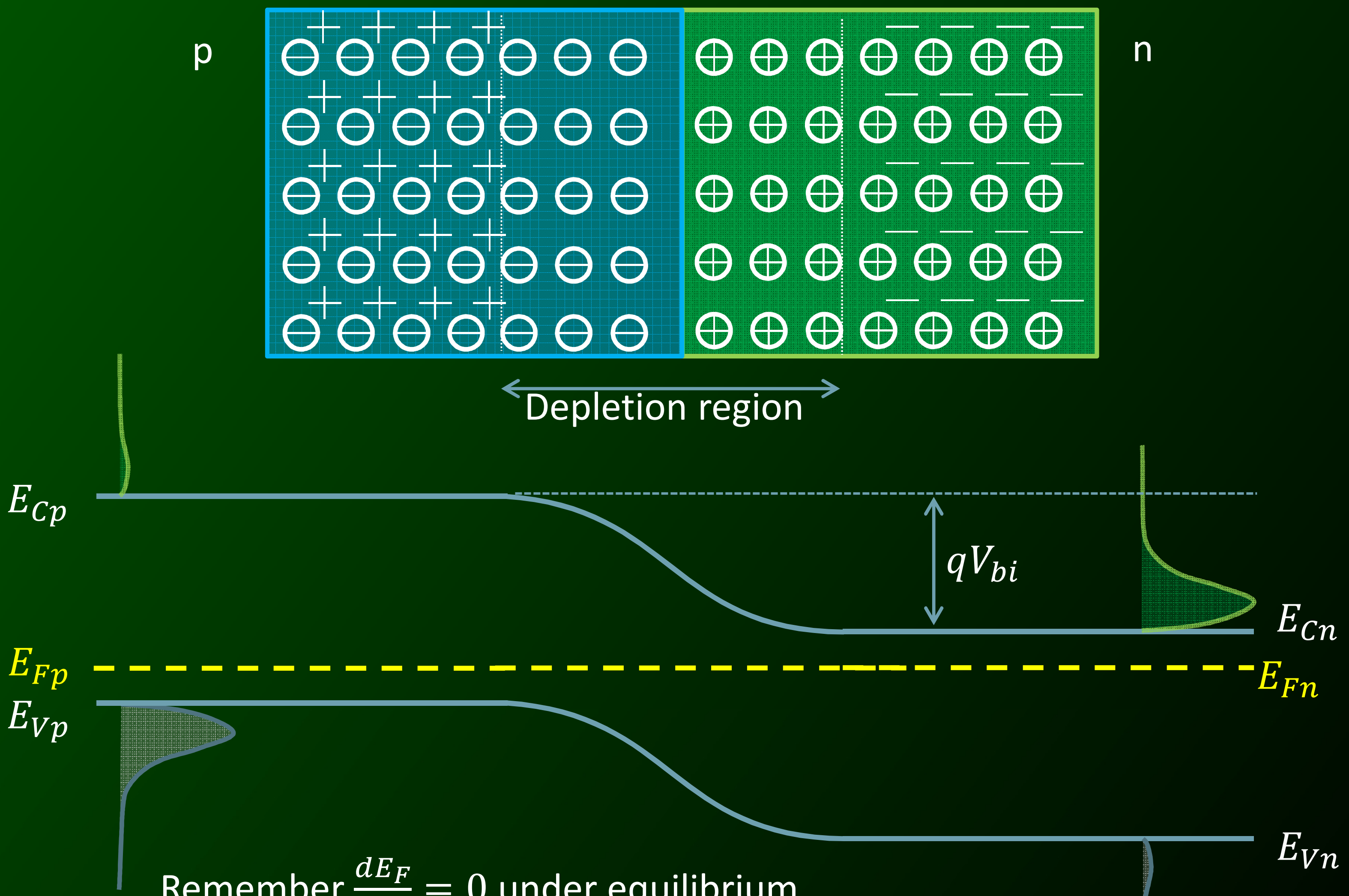
PN junctions (Qualitative)



PN junctions (Qualitative)



PN junctions (Qualitative)



Remember $\frac{dE_F}{dx} = 0$ under equilibrium.

Band bending occurs around the metallurgical junction!

PN junctions (Qualitative)

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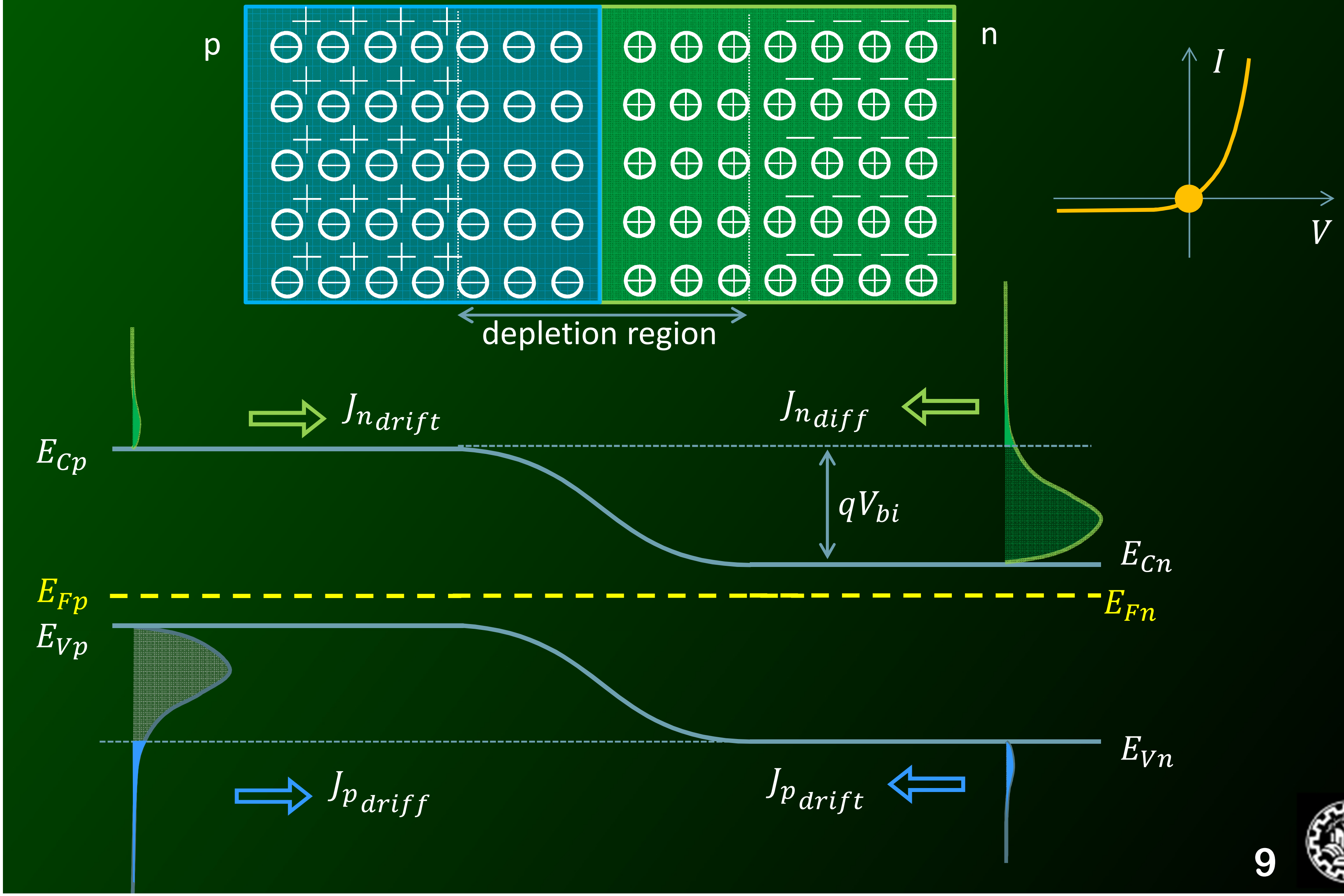

PN junctions (Qualitative)
Reverse Biased

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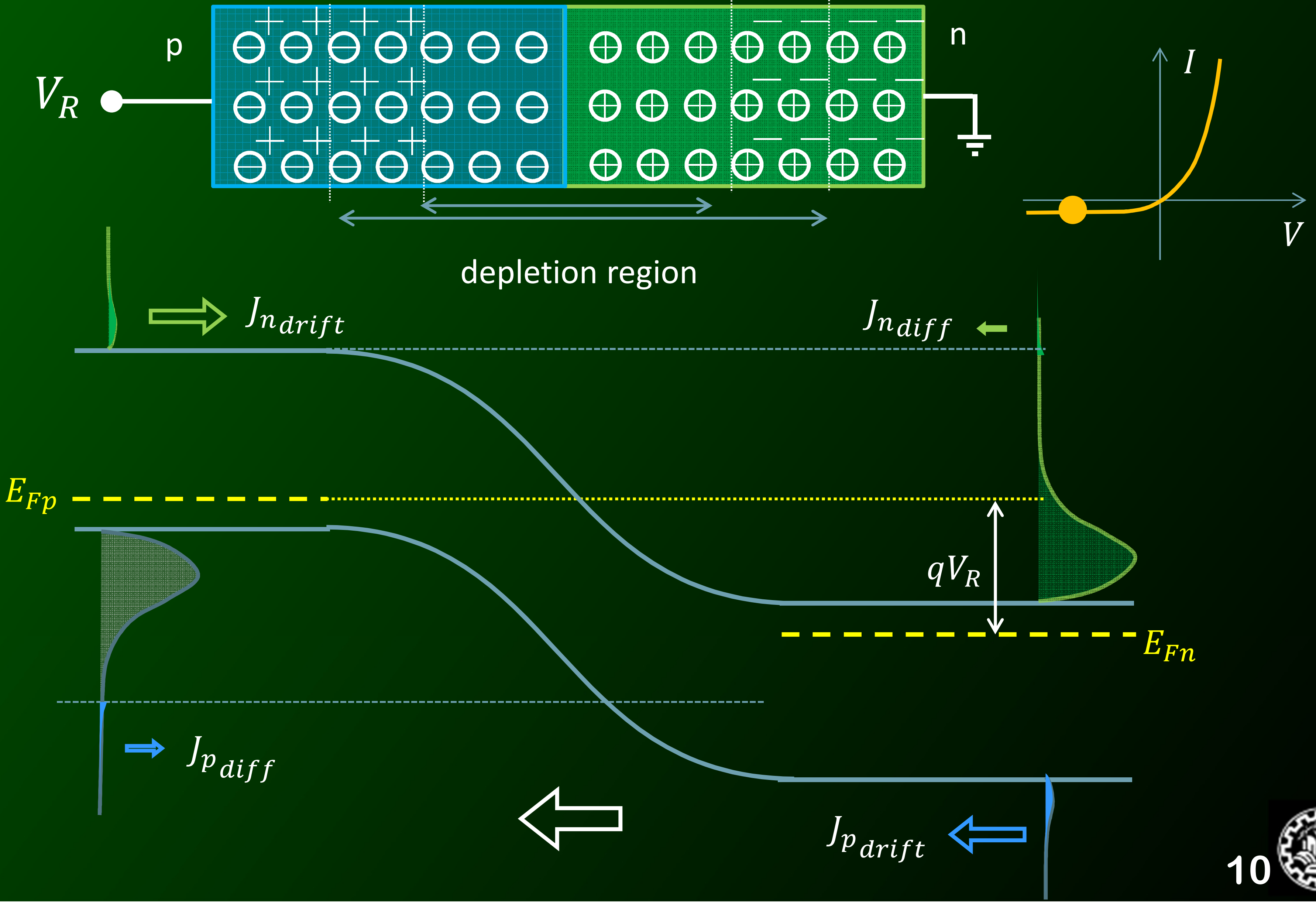

PN junctions (Qualitative)
Forward Biased

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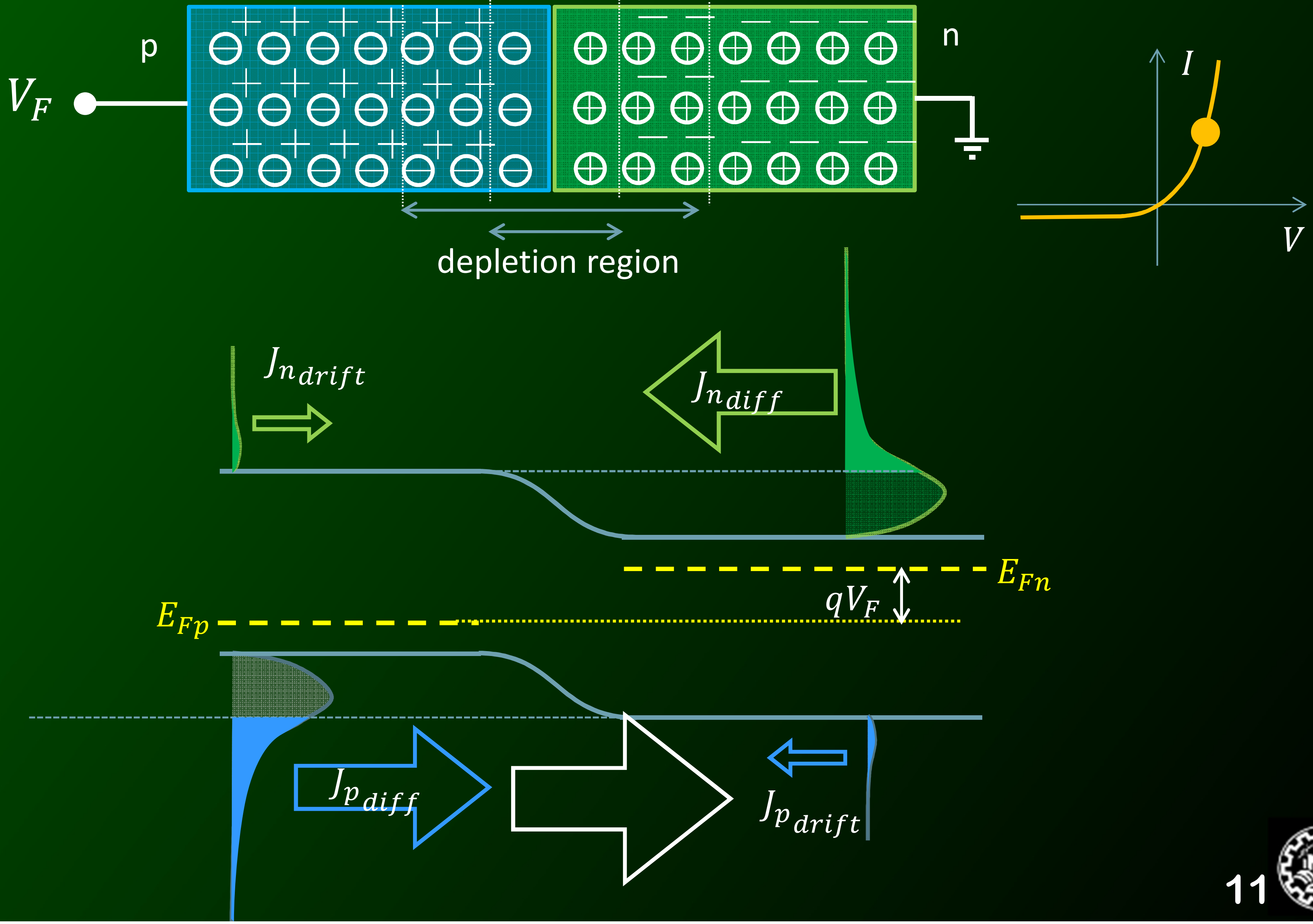

PN junctions (Qualitative)

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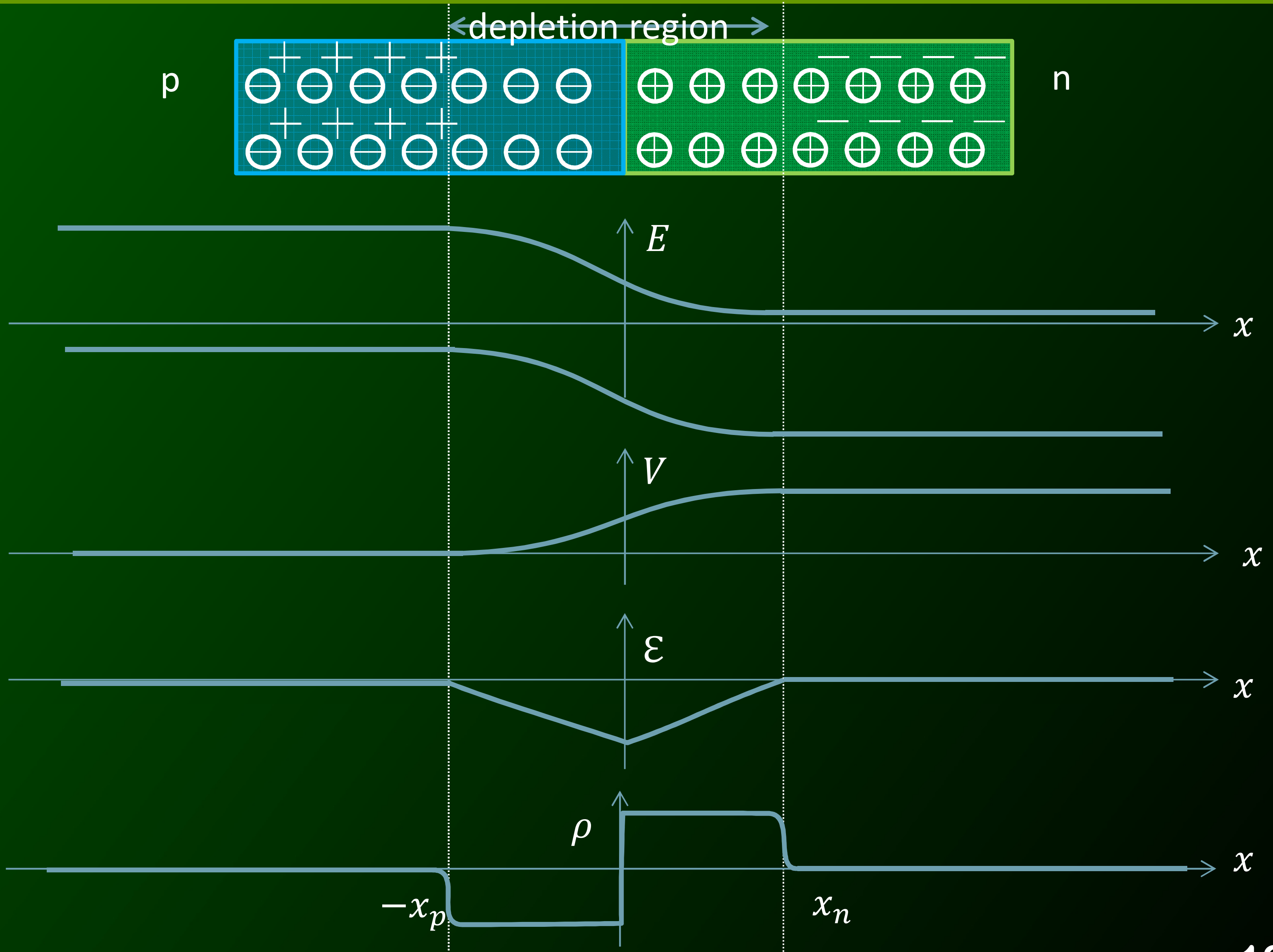
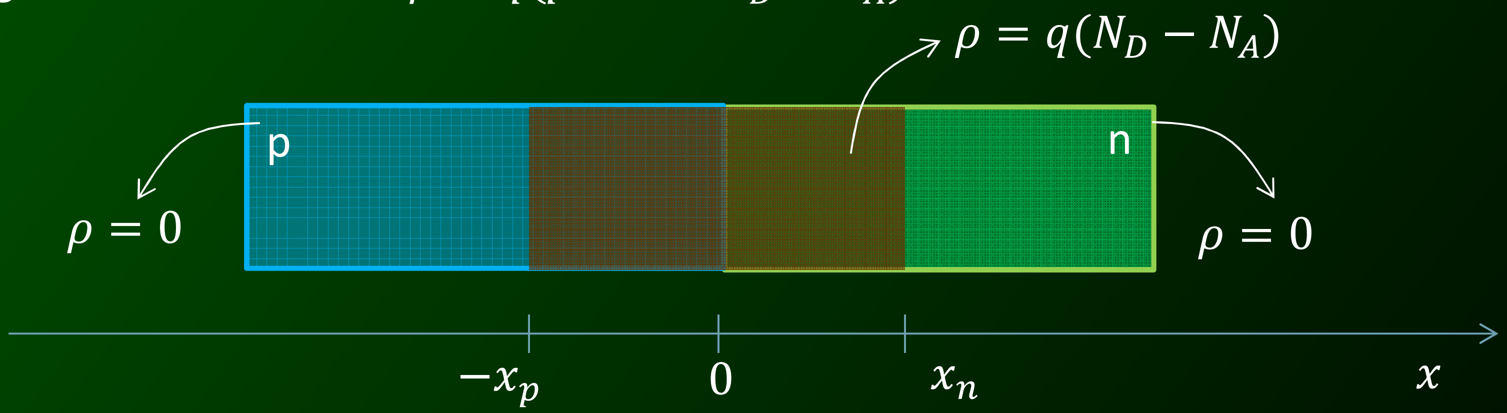


PN junctions - Assumptions

The Depletion Approximation : Obtaining closed-form solutions for the electrostatic variables

Charge Distribution : $\rho = q(p - n + N_D - N_A)$

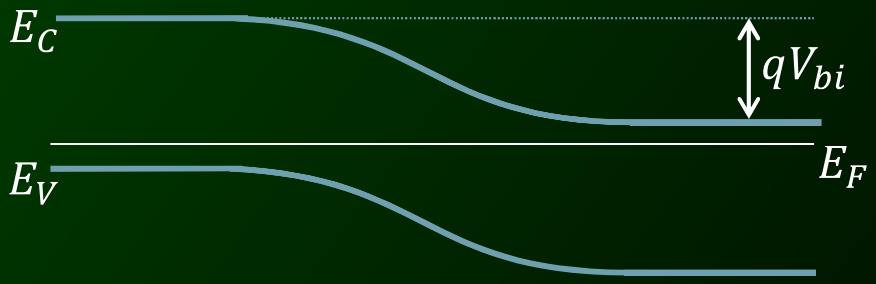


Note that

(1) $-x_p \leq x \leq x_n$: p & n are negligible ($\because \mathcal{E}$ exist).

(2) $x \leq -x_p$ or $x \geq x_n$: $\rho = 0$

How to Find $\rho(x), \mathcal{E}(x), V(x)$



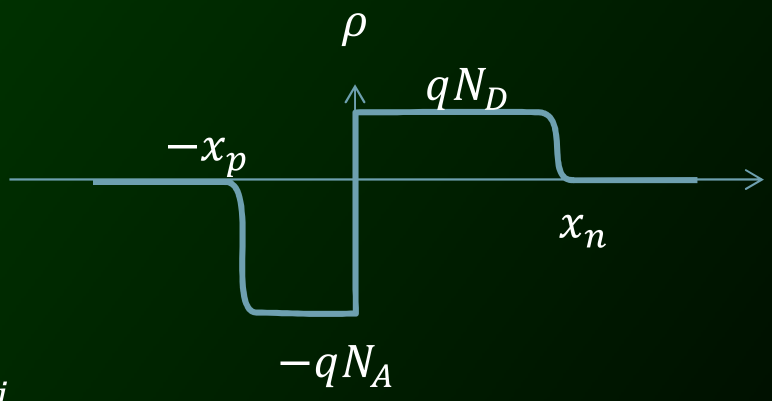
1. Find the built-in potential V_{bi}

2. Use the depletion approximation → $\rho(x)$
(depletion-layer widths x_p, x_n unknown)

3. Integrate $\rho(x)$ to find $\mathcal{E}(x)$
boundary conditions $\mathcal{E}(-x_p) = 0, \mathcal{E}(x_n) = 0$

4. Integrate $\mathcal{E}(x)$ to obtain $V(x)$
boundary conditions $V(-x_p) = 0, V(x_n) = V_{bi}$

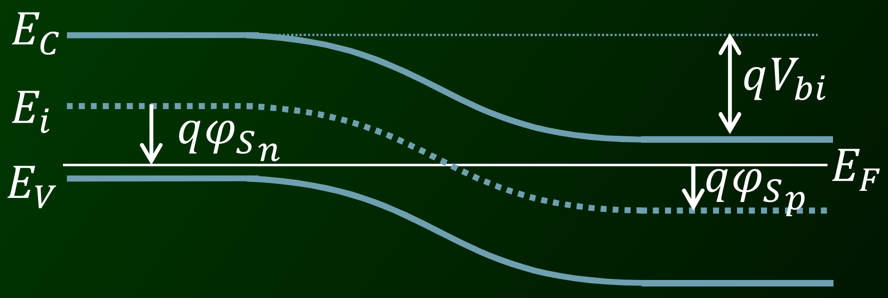
5. For $\mathcal{E}(x)$ to be continuous at $x = 0$,
$N_A x_p = N_D x_n$ → solve for x_p, x_n



Built-In Potential V_{bi}

$$qV_{bi} = q\varphi_{S_p} + q\varphi_{S_n}$$

$$= (E_i - E_F)_p + (E_F - E_i)_n$$



For non-degenerately doped material:

$$\left.\begin{aligned}(E_i - E_F)_p &= kT\ln\left(\frac{p}{n_i}\right) = kT\ln\left(\frac{N_A}{n_i}\right)\\ (E_F - E_i)_n &= kT\ln\left(\frac{n}{n_i}\right) = kT\ln\left(\frac{N_D}{n_i}\right)\end{aligned}\right\} \rightarrow V_{bi} = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

What shall we do for $p^+ - n$ (or $n^+ - p$) junction?!?!?

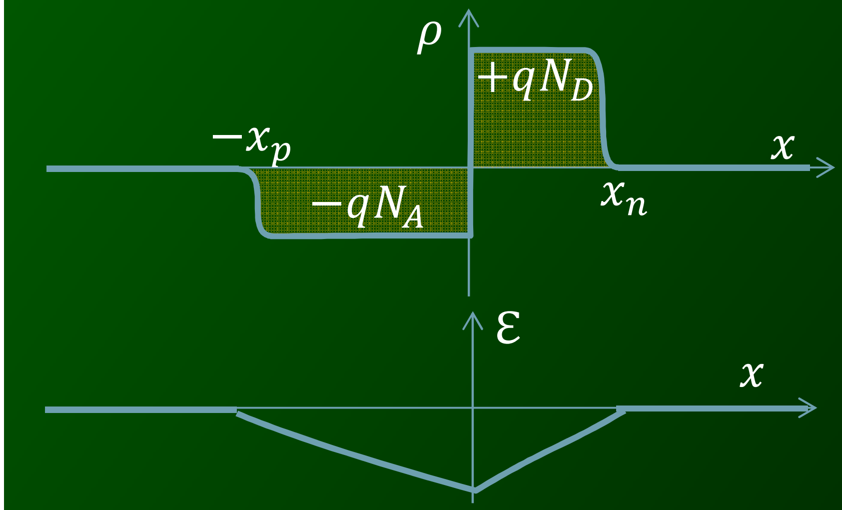
p^+:

$$(E_i - E_F)_p = \frac{E_G}{2}$$

n^+:

$$(E_F - E_i)_n = \frac{E_G}{2}$$

The Depletion Approximation



$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\epsilon}$$

$$\rho = -qN_A \rightarrow$$

$$\mathcal{E}(x) = \frac{-qN_A}{\epsilon} + C = \frac{-qN_A}{\epsilon}(x + x_p)$$

$$\rho = \ qN_D \rightarrow$$

$$\mathcal{E}(x) = \frac{qN_D}{\epsilon} + C' = \frac{qN_D}{\epsilon}(x - x_n)$$

The electric field is continuous at $x\ = \ 0$

$$x_p N_A = x_n N_D$$

Charge neutrality condition as well!

Electrostatic Potential in the Depletion Layer

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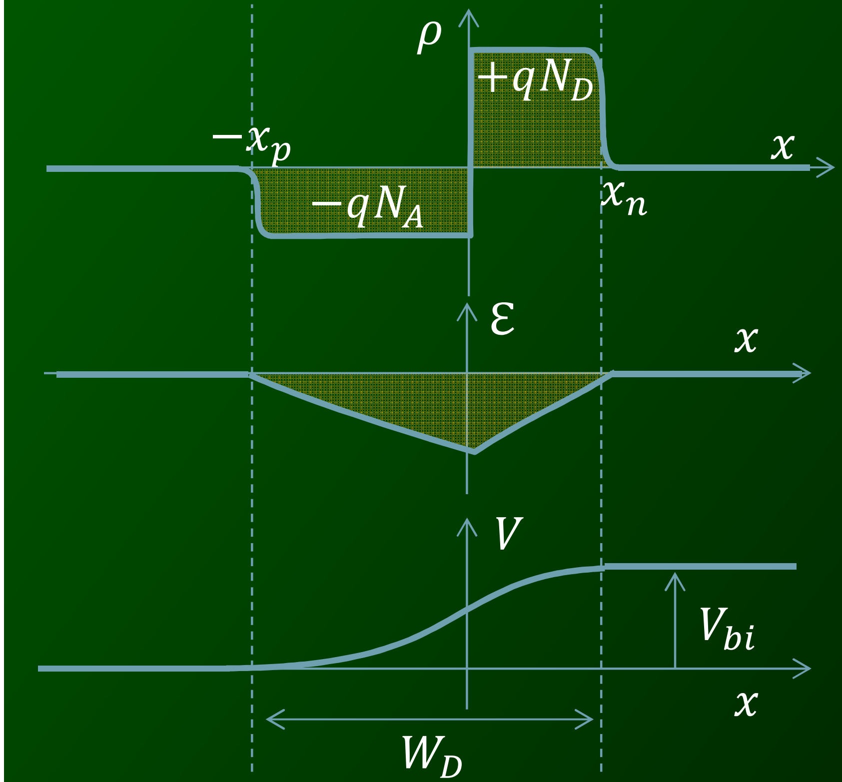


$$\frac{dV}{dx} = -\mathcal{E}$$

$-x_p < x < 0$:

$$\mathcal{E}(x) = \frac{-qN_A}{\epsilon}(x + x_p)$$

$$V(x) = \frac{qN_A}{2\epsilon}\left(x + x_p\right)^2 + C = \frac{qN_A}{2\epsilon}\left(x + x_p\right)^2$$

$0 < x < x_n$:

$$\mathcal{E}(x) = -\frac{qN_D}{\epsilon}(x_n - x)$$

$$V(x) = -\frac{qN_D}{2\epsilon}(x_n - x)^2 + C' = V_{bi} - \frac{qN_D}{2\epsilon}(x_n - x)^2$$

Depletion Layer Width

$$-x_p < x < 0: \quad V(x) = \frac{qN_A}{2\epsilon}\left(x + x_p\right)^2$$

$$0 < x < x_n: \quad V(x) = V_{bi} - \frac{qN_D}{2\epsilon}(x_n - x)^2$$

$$\left.\begin{array}{l} V(0) = \dfrac{qN_A}{2\epsilon}x_p^2 = V_{bi} - \dfrac{qN_D}{2\epsilon}x_n^2 \\ x_pN_A = x_nN_D \end{array}\right\} \rightarrow \left\{\begin{array}{l} x_n = \sqrt{\dfrac{2\epsilon_s V_{bi}}{q}\left(\dfrac{N_A}{N_D(N_A + N_D)}\right)} \\ x_p = \sqrt{\dfrac{2\epsilon_s V_{bi}}{q}\left(\dfrac{N_D}{N_A(N_A + N_D)}\right)} \end{array}\right.$$

Summing, we have:

$$\boxed{W = x_p + x_n = \sqrt{\frac{2\epsilon_s V_{bi}}{q}\left(\frac{1}{N_D} + \frac{1}{N_A}\right)}}$$

Depletion Layer Width

If $N_A \gg N_D$ as in a $p^+ - n$ junction:

$$W = \sqrt{\frac{2\epsilon_s V_{bi}}{q}\left(\frac{1}{N_D} + \frac{1}{N_A}\right)} \quad \rightarrow W = \sqrt{\frac{2\epsilon_s V_{bi}}{qN_D}} \approx x_n$$

$$x_p N_A = x_n N_D \quad \rightarrow \quad x_p \ll x_n \quad \rightarrow x_p \approx 0$$

Note:

$$V_{bi} = \frac{E_G}{2q} + \frac{kT}{q} \ln \frac{N_D}{n_i}$$

Example

A $p^{+} - n$ junction has $N_A = 10^{20}\ cm^{-3}$ and $N_D = 10^{17} cm^{-3}$. What is

a) its built in potential,

$$V_{bi} = \frac{E_G}{2q} + \frac{kT}{q} \ln \frac{N_D}{n_i} \approx 1V$$

b) W ,

$$W \approx \sqrt{\frac{2\epsilon_s V_{bi}}{qN_D}} = \sqrt{\frac{2 \times 12 \times 8.85 \times 10^{-14} \times 1}{1.6 \times 10^{19} \times 10^{17}}} = 0.12\mu m$$

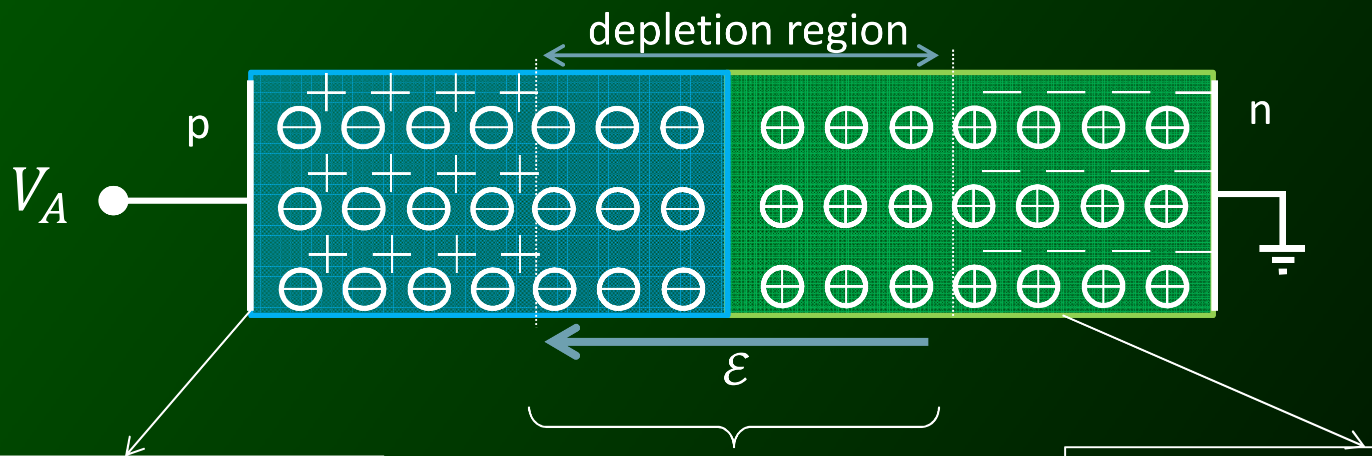
c) x_n , and

$$x_n \approx W = 0.12\mu m$$

d) x_p

$$x_p = x_n \frac{N_D}{N_A} = 1.2 \times 10^{-4}\ \mu m = 1.2\ \text{Å} \sim 0$$

Biases pn Junction (assumptions)



Negligible voltage drop
(Ohmic contact)

V_A dropped here

will apply continuity
equation in this region

1) Low level injection
2) Zero voltage drop ($\mathcal{E} = 0$)

Since ($\mathcal{E} = 0$) may apply
minority carrier diffusion
equations

Note: V_A should be significantly smaller than V_{bi}(Otherwise, we cannot assume low-level injection)

Effect of Bias on Electrostatics

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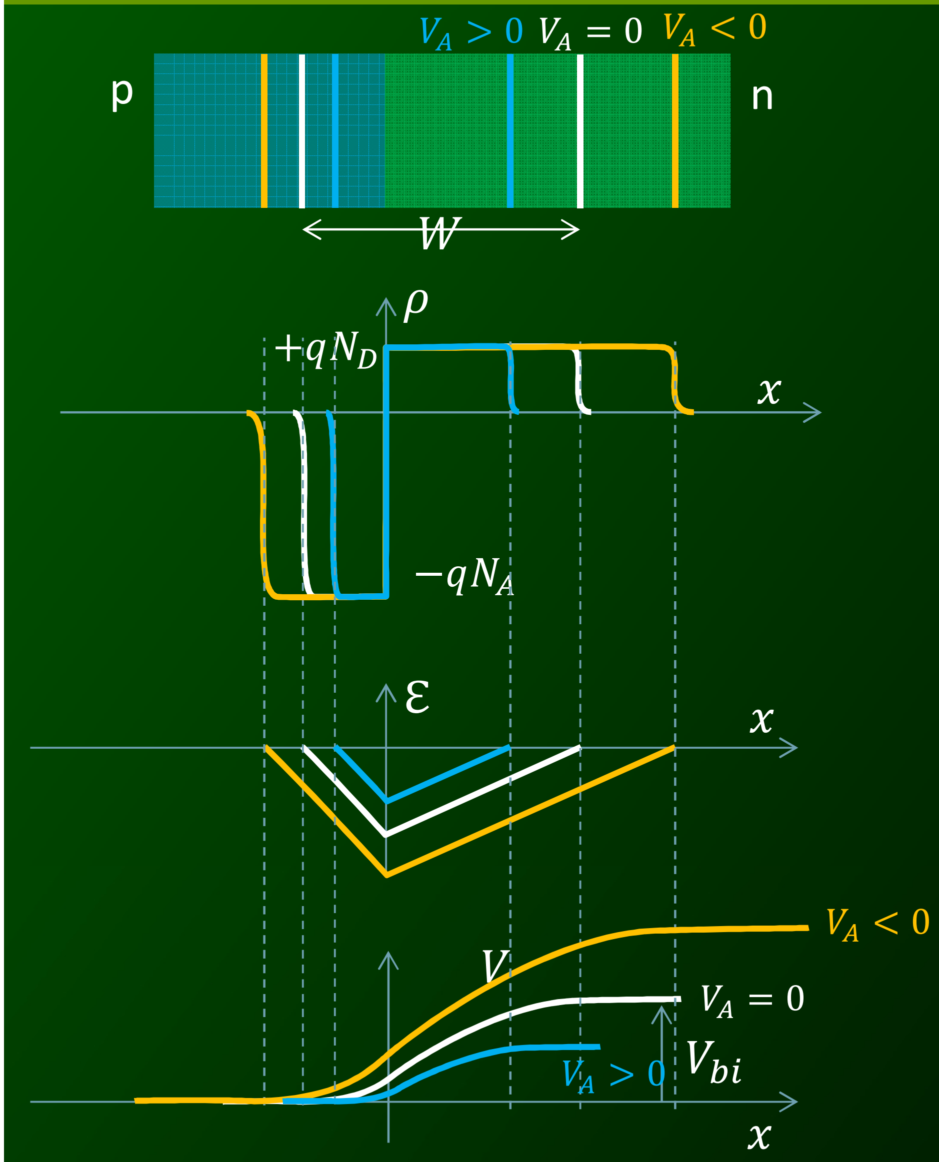
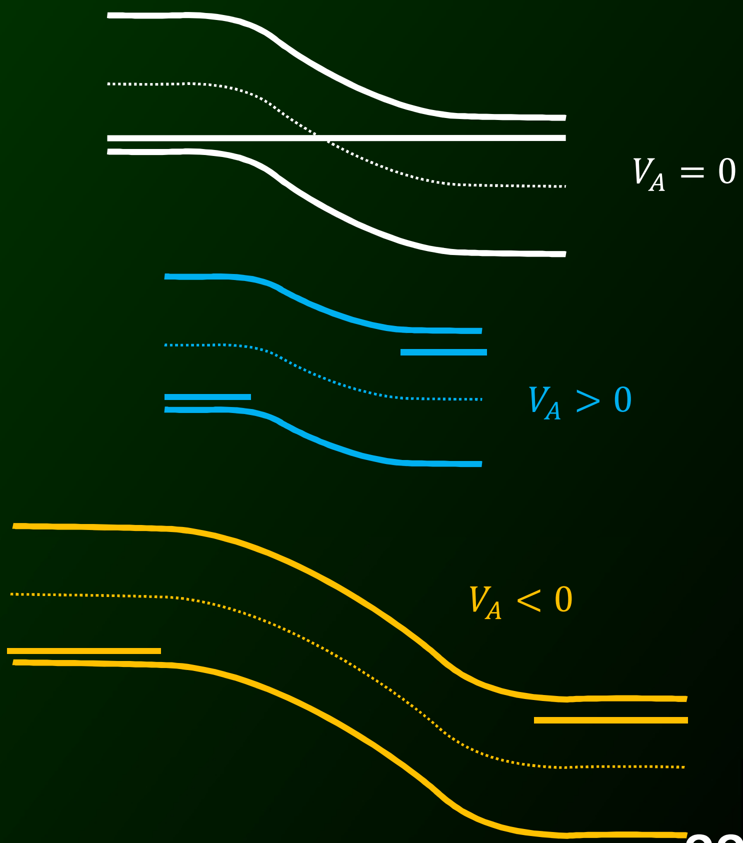


Energy Band Diagram

1) The Fermi level is omitted from the depletion region because the device is no longer in equilibrium: We need the quasi Fermi energy level.

2) $E_{fp} - E_{fn} = -qV_A$



Va Applied Voltage

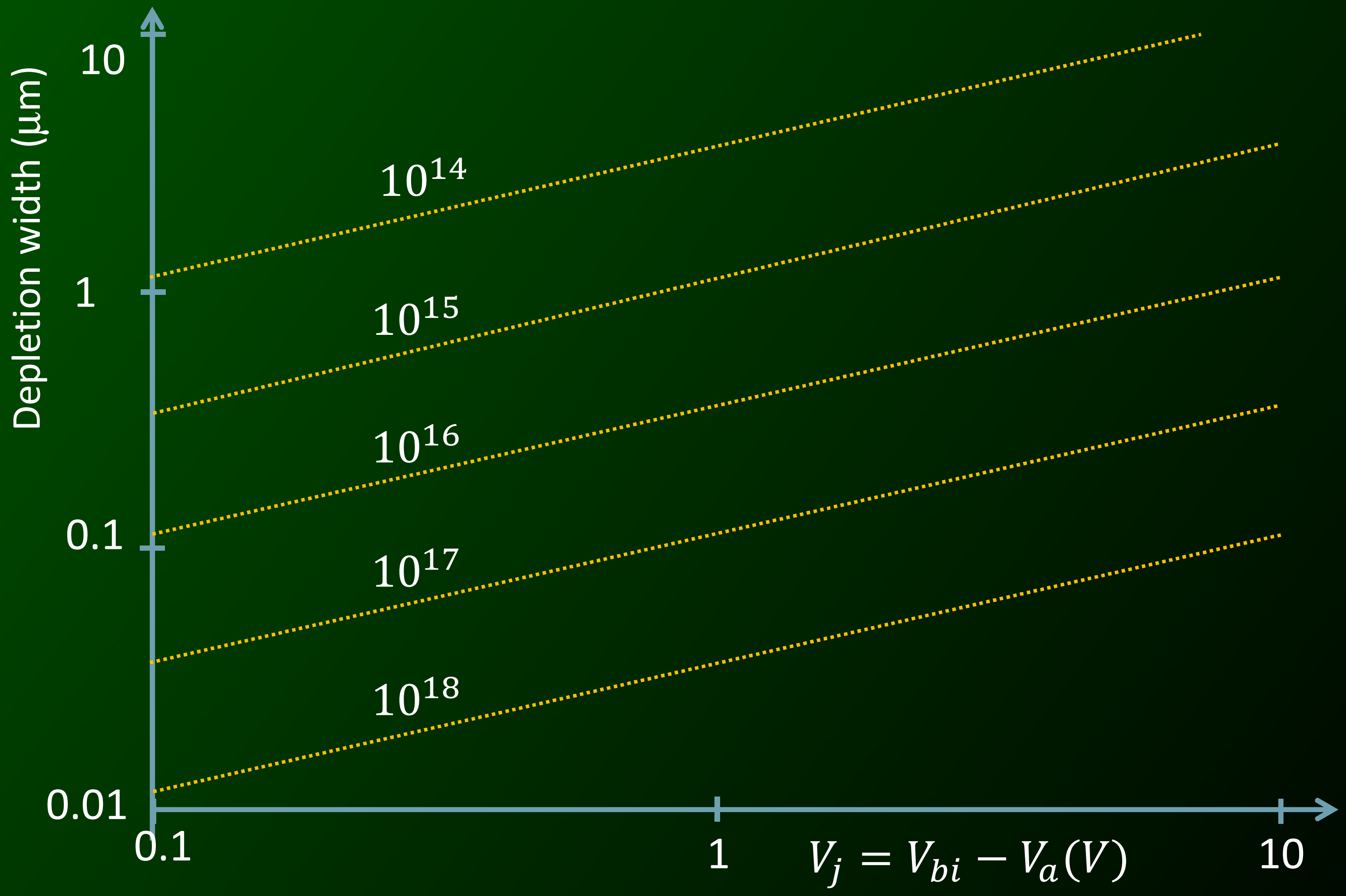
Now as we assumed all voltage drop is in the depletion region (Note that VA ≤ Vbi)

$$x_n + x_p = W = \sqrt{\frac{2\epsilon_s(V_{bi} - V_A)}{q}\left(\frac{1}{N_D} + \frac{1}{N_A}\right)}$$

$$x_p N_A = x_n N_D$$

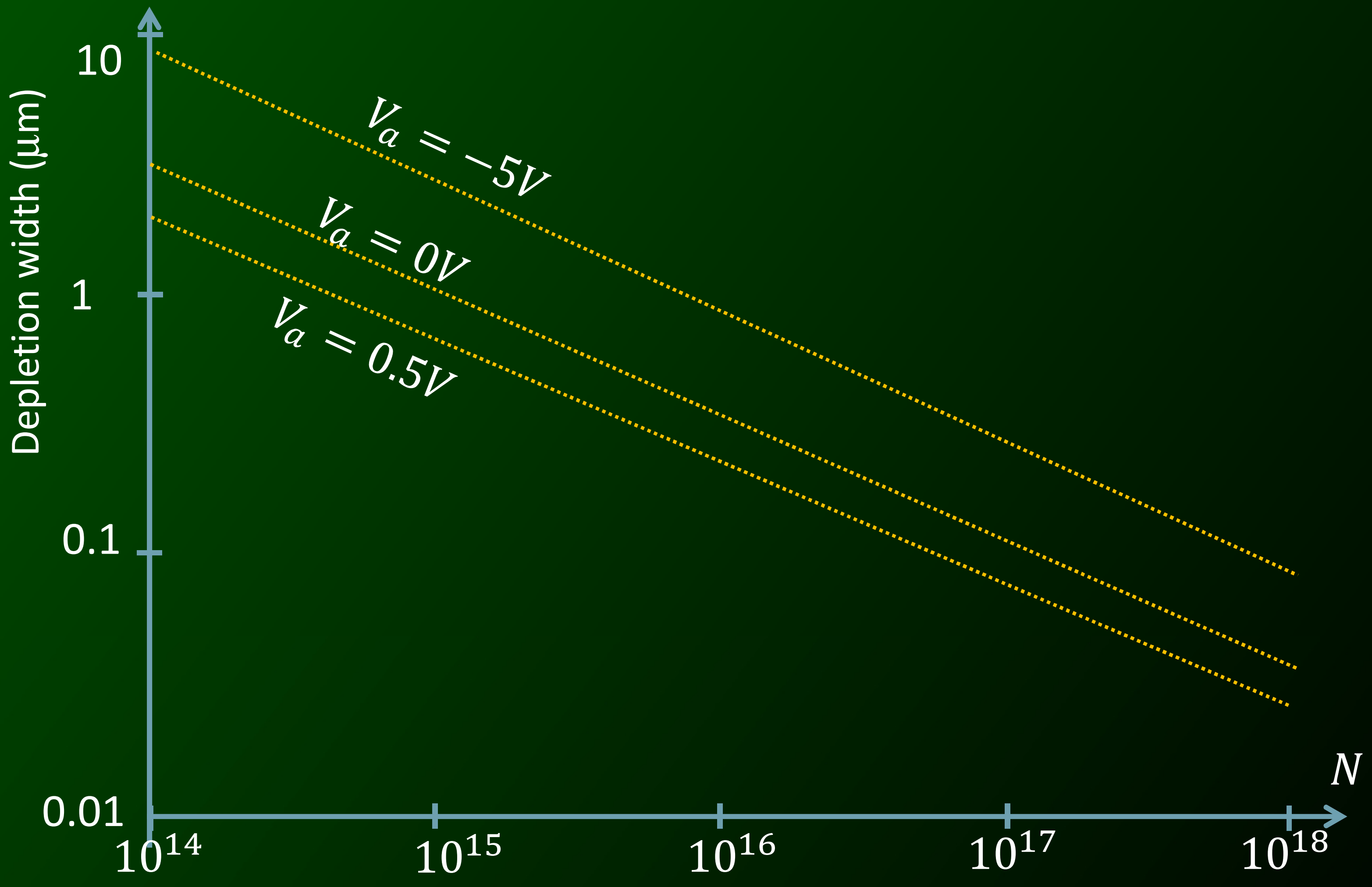
W vs. Va

The junction width for one-sided step junctions in silicon as a function of junction voltage with the doping on the lightly doped side as a parameter.



W vs. Na

Junction width for a one-sided junction is plotted as a function of doping on the lightly doped side for three different operating voltages.



pn Junction:
I-V Characteristic (assumptions)

Assumption :

1) low-level injection: $n_p \ll p_p \sim N_A$(or $\Delta n \ll p_0$, $p \sim p_0$ in p-type)

$p_n \ll n_n \sim N_D$(or $\Delta p \ll n_0$, $n \sim n_0$ in n-type)

2) In the bulk, $n_n \sim n_{n0} = N_D$, $p_p \sim p_{p0} = N_A$

3) For minority carriers $J_{drift} \ll J_{diff}$ in quasi-neutral region

4) Nondegenerately doped step junction

5) Long-base diode in 1-D (both sides of quasi-neutral regions are much longer than their minority carrier diffusion lengths, L_n or L_p)

6) No Generation/Recombination in depletion region

7) Steady state $d/dt = 0$

8) $G_{opt} = 0$

pn Junction: I-V Characteristic

Game plan:

i) continuity equations for minority carriers

$$\frac{\partial \Delta n_p}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{\Delta n_p}{\tau_n} + \cancel{G_{ext}}$$

$$\frac{\partial \Delta p_n}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\Delta p_n}{\tau_p} + \cancel{G_{ext}}$$

ii) minority carrier current densities in the quasi-neutral region

$$J_p = J_{p_{drift}} + J_{p_{diff}} = qp\mu_p \cancel{\mathcal{E}} - qD_p\frac{dp}{dx} \sim -\ qD_p\frac{dp}{dx}$$

$$J_n = J_{n_{drift}} + J_{n_{diff}} = qn\mu_n \cancel{\mathcal{E}} + qD_n\frac{dn}{dx} \sim qD_n\frac{dn}{dx}$$

pn Junction: I-V Characteristic

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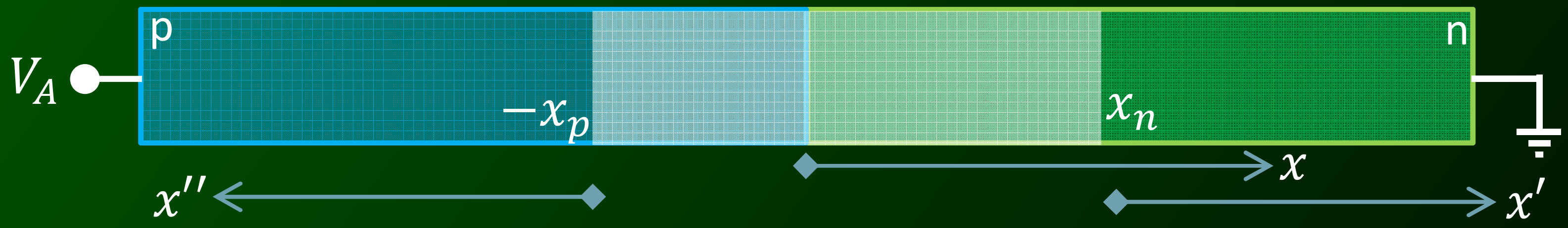
Steady-State solution is: $\dfrac{d^2\Delta n_p}{dx^2} = \dfrac{\Delta n_p}{D_n\tau_n} \rightarrow \Delta n_p = Ae^{x/L_n} + Be^{-x/L_n} \quad \left(L_n = \sqrt{D_n\tau_n}\right)$

diode is long enough!

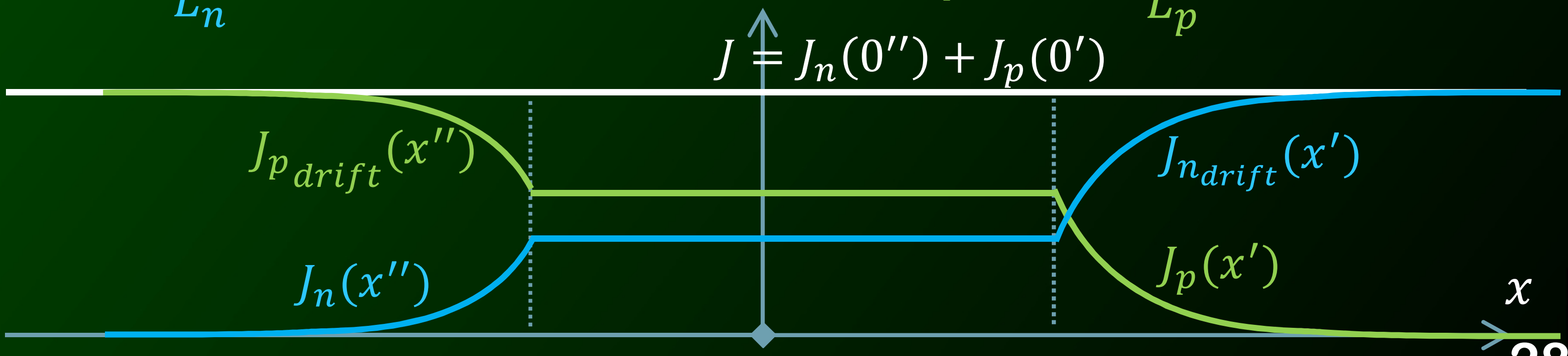


$$\Delta n_p(x'') = A''e^{-x''/L_n} \qquad \Delta p_n(x') = A'e^{-x'/L_p}$$

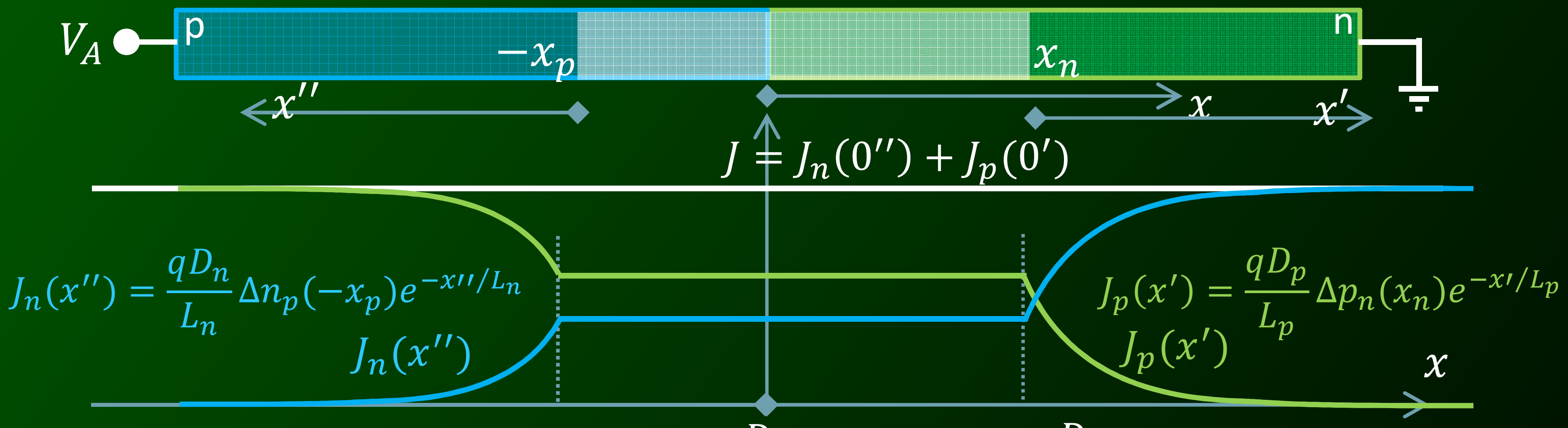
$$\Delta n_p(x'') = \Delta n_p(-x_p)e^{-x''/L_n} \qquad \Delta p_n(x') = \Delta p_n(x_n)e^{-x'/L_p}$$

$$J_n = qD_n\frac{dn}{dx} \qquad J_p = -\,qD_p\frac{dp}{dx}$$

$$J_n(x'') = \frac{qD_n}{L_n}\Delta n_p(-x_p)e^{-x''/L_n} \qquad J_p(x') = \frac{qD_p}{L_p}\Delta p_n(x_n)e^{-x'/L_p}$$



pn Junction: I-V Characteristic



$$J = J_n(0'') + J_p(0')$$

$$J_n(x'') = \frac{qD_n}{L_n}\Delta n_p(-x_p)e^{-x''/L_n}$$

$$J_p(x') = \frac{qD_p}{L_p}\Delta p_n(x_n)e^{-x'/L_p}$$

$$J = J_n(0'') + J_p(0') = \frac{qD_n}{L_n}\Delta n_p(-x_p) + \frac{qD_p}{L_p}\Delta p_n(x_n)$$

Now! we need to find $\Delta n_p(-x_p)$ and $\Delta p_n(x_n)$ vs V

$$V_2 - V_1 = \frac{kT}{q}\ln\frac{n_2}{n_1} = \frac{kT}{q}\ln\frac{p_1}{p_2}$$

$$\rightarrow V_0 - V = \frac{kT}{q}\ln\frac{\overset{N_D}{n(x_n)}}{n(-x_p)} = \frac{kT}{q}\ln\frac{\overset{N_A}{p(-x_p)}}{p(x_n)}$$

$$V_0 = \frac{kT}{q}\ln\frac{N_A N_D}{n_i^2} \quad \rightarrow$$

$$\boxed{\begin{aligned} n(-x_p) &= n_{p0}\, e^{qV/kT} \\ p(x_n) &= p_{n0}\, e^{qV/kT} \end{aligned}}$$

pn Junction: I-V Characteristic

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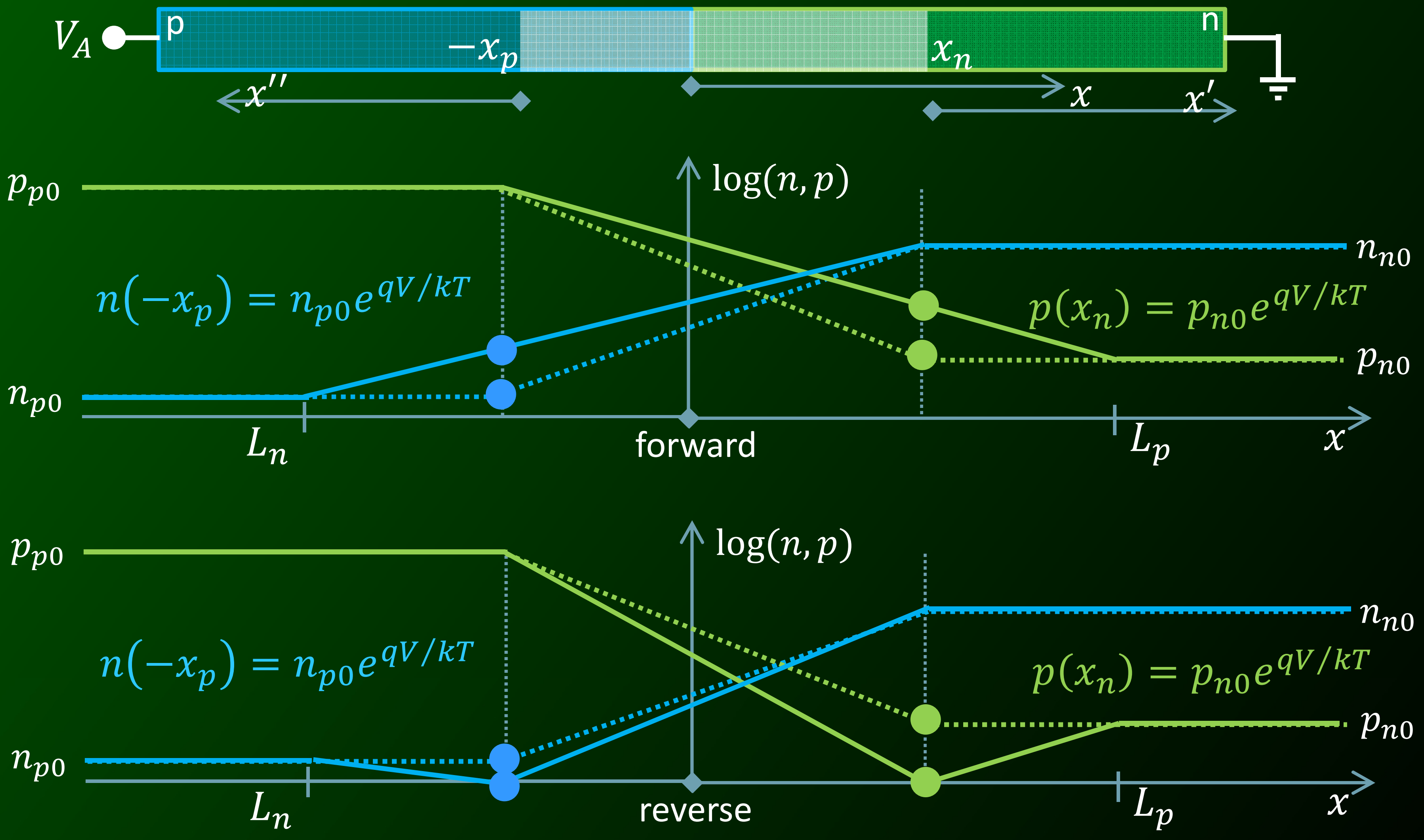

pn Junction: I-V Characteristic

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$$J = J_n(0'') + J_p(0') = \frac{qD_n}{L_n}\Delta n_p(-x_p) + \frac{qD_p}{L_p}\Delta p_n(x_n)$$

$$n(-x_p) = n_{p0}\, e^{qV/kT} \quad ; \quad \Delta n_p(-x_p) = n - n_{p0} = n_{p0}(e^{qV/kT} - 1) \quad ; \quad n_{p0} = n_i^2/N_A$$

$$p(x_n) = p_{n0}\, e^{qV/kT} \quad ; \quad \Delta p_n(x_n) = p - p_{n0} = p_{n0}(e^{qV/kT} - 1) \quad ; \quad p_{n0} = n_i^2/N_D$$

$$J = q\left(\frac{D_n}{L_n}n_{p0} + \frac{D_p}{L_p}p_{n0}\right)\left(e^{qV/kT} - 1\right) \qquad I = AJ$$

$$I = qA\left(\frac{D_n}{L_n}\frac{n_i^2}{N_A} + \frac{D_p}{L_p}\frac{n_i^2}{N_D}\right)\left(e^{qV/kT} - 1\right) = I_0\left(e^{qV/kT} - 1\right)$$

$$I_0 = qAn_i^2\left(\sqrt{\frac{D_n}{\tau_n}}\frac{1}{N_A} + \sqrt{\frac{D_p}{\tau_p}}\frac{1}{N_D}\right)$$

pn Junction: I-V Characteristic

$$I = qA\left(\frac{D_n}{L_n}\frac{n_i^2}{N_A} + \frac{D_p}{L_p}\frac{n_i^2}{N_D}\right)\left(e^{qV/kT} - 1\right) = I_0\left(e^{qV/kT} - 1\right)$$

asymmetrically doped junction

If $p^+ - n$ diode ($N_A \gg N_D$), then $$I_0 \approx qA\frac{D_p}{L_p}\frac{n_i^2}{N_D}$$

If $n^+ - p$ diode ($N_D \gg N_A$), then $$I_0 \approx qA\frac{D_n}{L_n}\frac{n_i^2}{N_A}$$

That is, one has to consider only the lightly doped side of such junction in working out the diode I-V characteristics.

pn Junction: I-V Characteristic

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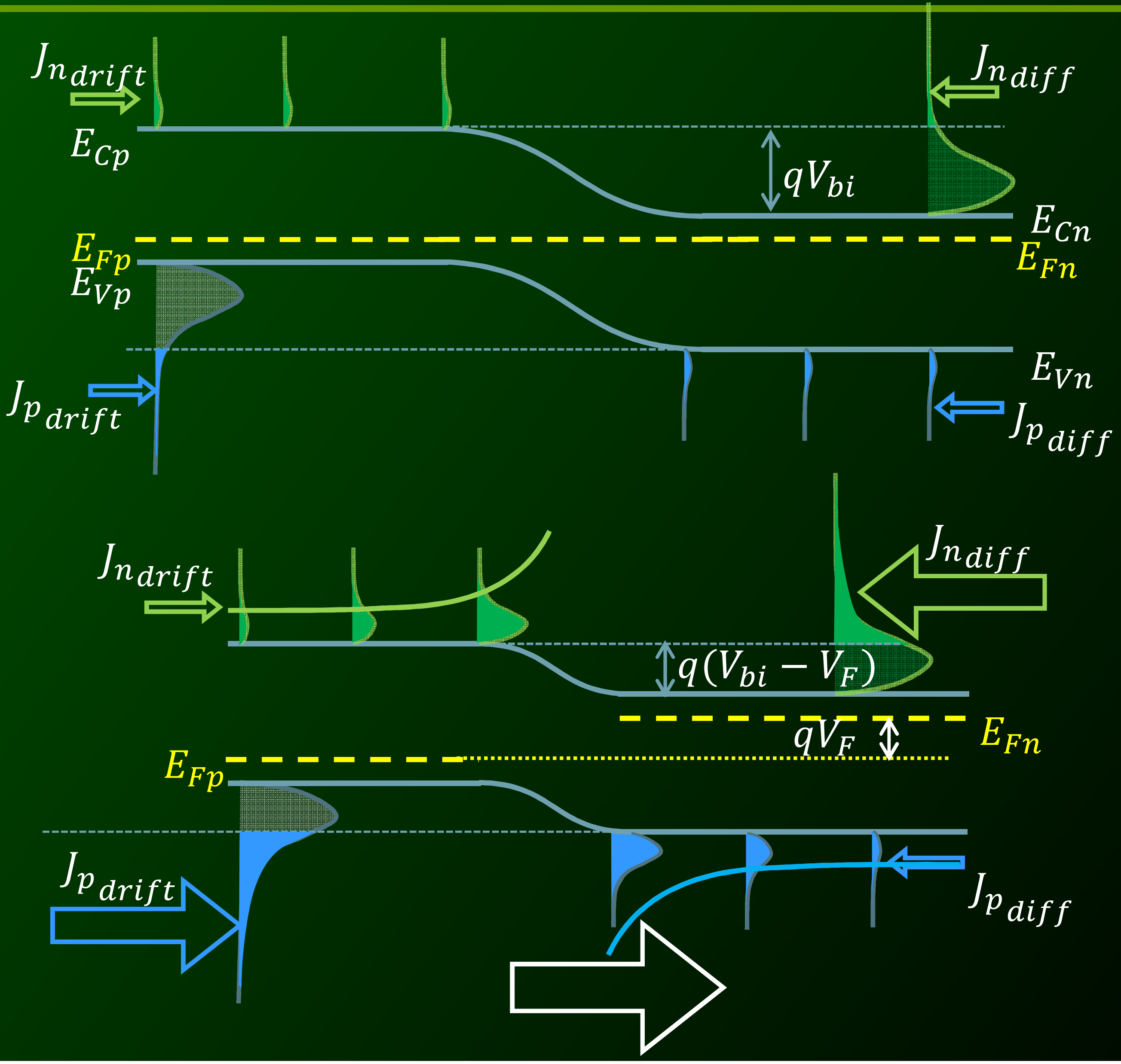
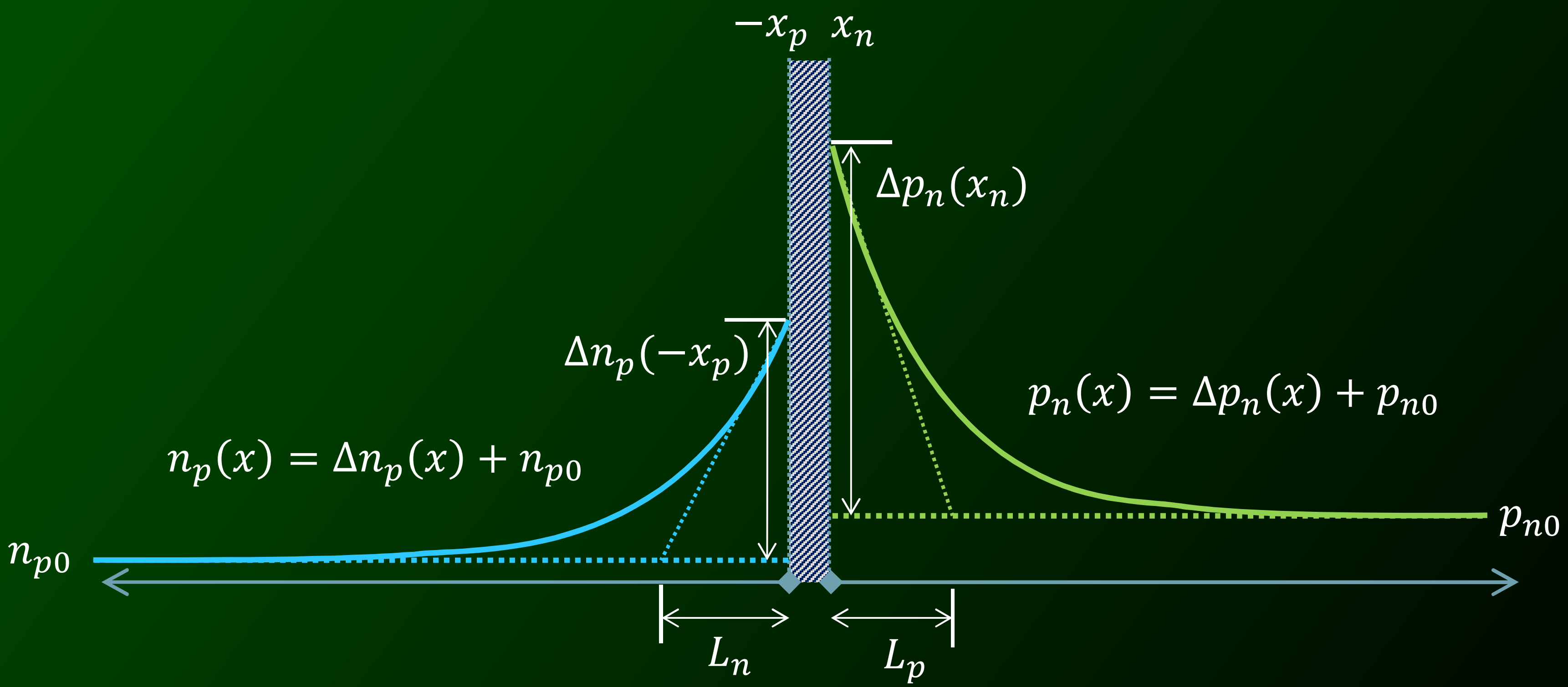
V=0

V>0



pn Junction: I-V Characteristic

The minority carrier concentrations on either side of the junction under forward bias



Minority-Carrier Charge Storage

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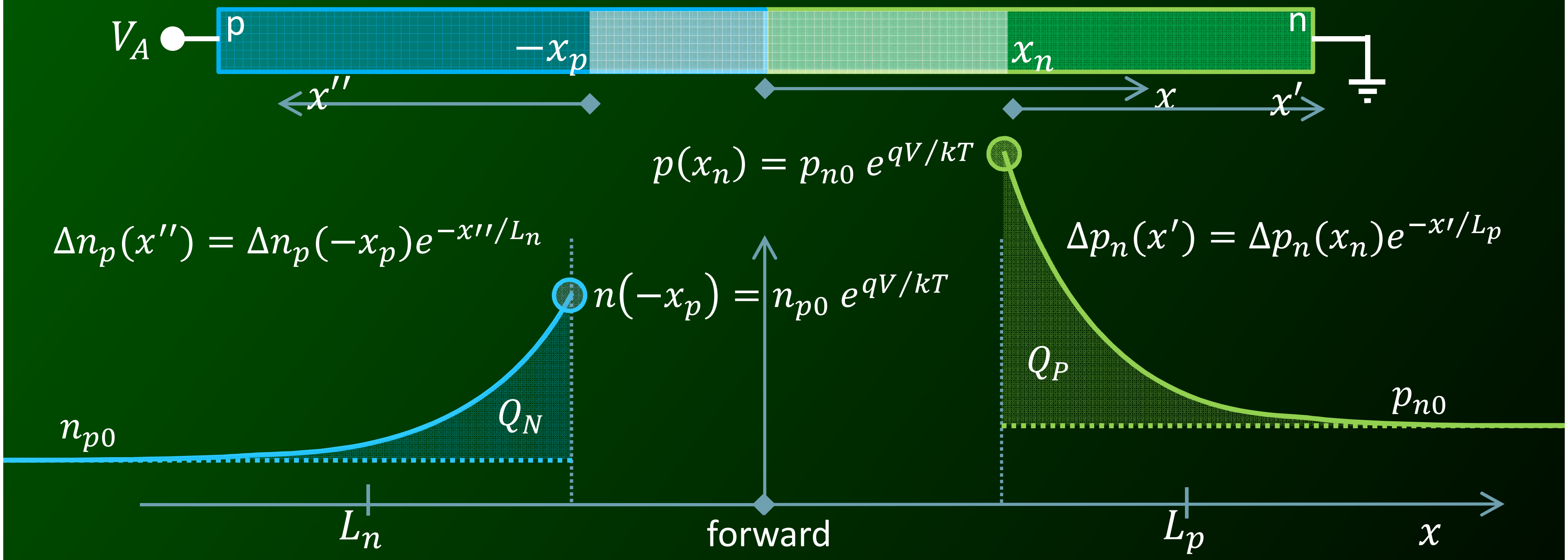

$Q_N = -qA\Delta n_p(-x_p)L_n$

$Q_P = -qA\Delta p_n(x_n)L_P$

$$\frac{\partial \Delta n_p}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{\Delta n_p}{\tau_n} + \cancel{Const}$$

$$\frac{\partial \Delta p_n}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\Delta p_n}{\tau_p} + \cancel{Const}$$

Charge Control Model

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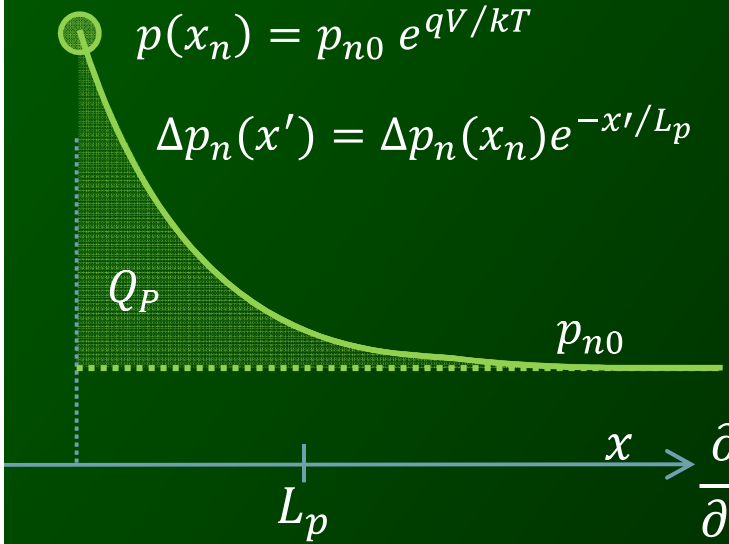
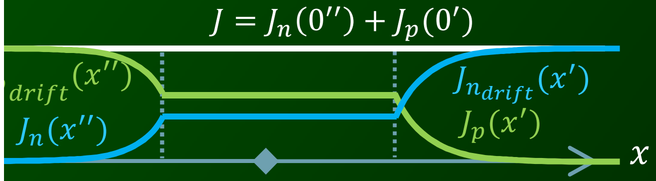


$$p(x_n) = p_{n0}\, e^{qV/kT}$$

$$\Delta p_n(x') = \Delta p_n(x_n) e^{-x\prime/L_p}$$



In general: $\Delta p_n(x,t)$

$$\frac{\partial \Delta p_n}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\Delta p_n}{\tau_p}$$

$$\frac{\partial (qA\Delta p_n)}{\partial t} = -A\frac{\partial J_p}{\partial x} - \frac{qA\Delta p_n}{\tau_p}$$

$$\frac{\partial}{\partial t}\underbrace{\left[qA\int_{x_n}^{\infty}\Delta p_n dx\right]}_{Q_P} = -A\int_{J(x_n)}^{J(\infty)} dJ_p - \frac{1}{\tau_p}\left[qA\int_{x_n}^{\infty}\Delta p_n dx\right]$$

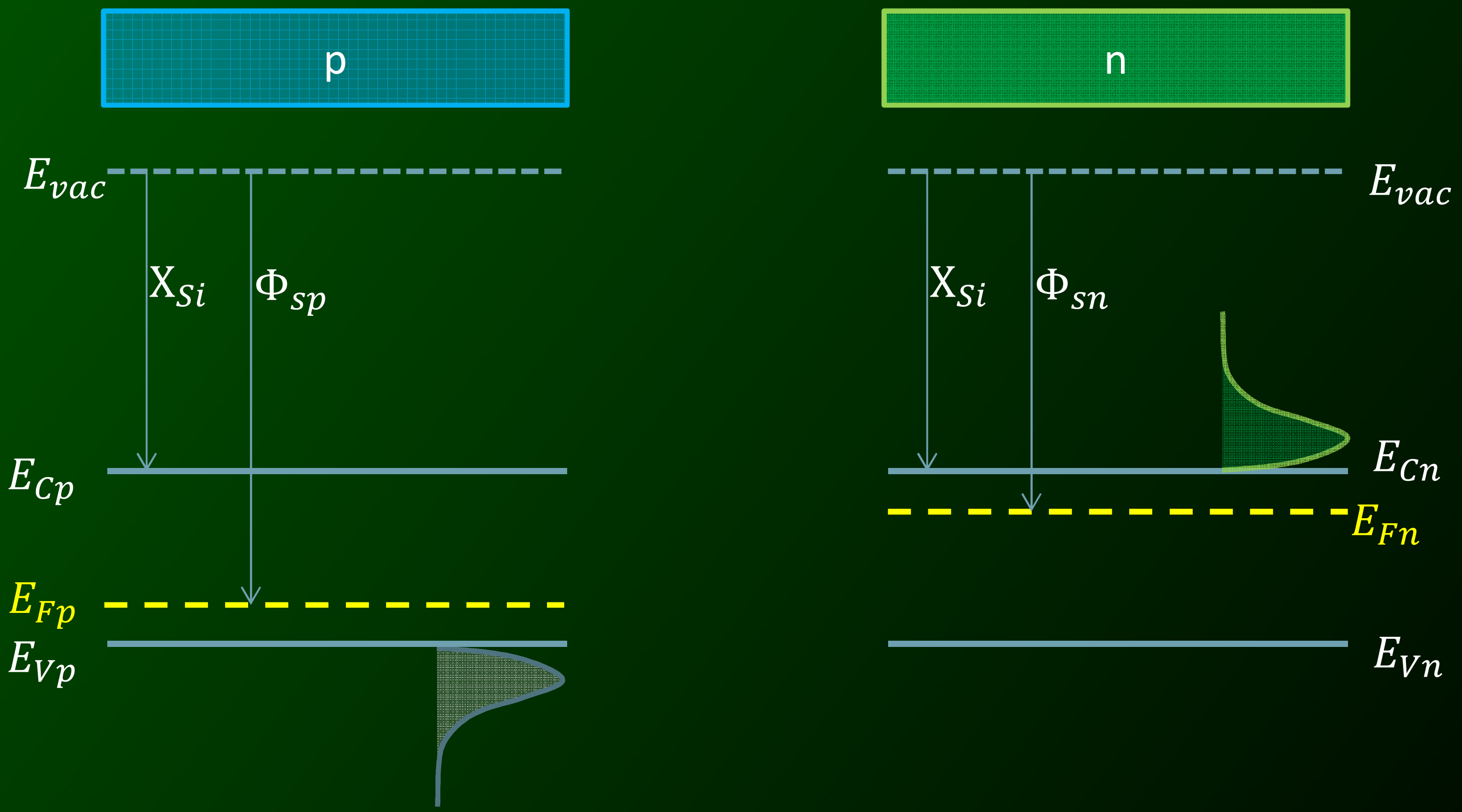
$$\frac{d}{dt}Q_P = AJ_p(x_n) - \frac{Q_P}{\tau_p}$$

$$\boxed{\frac{d}{dt}Q_P = I_p(x_n) - \frac{Q_P}{\tau_p}}$$

Steady state: $\frac{d}{dt} = 0$

$I_p(x_n) = \frac{Q_P}{\tau_p}$ similarly $I_n(-x_p) = \frac{Q_P}{\tau_n}$

PN junctions – Before Being Joined



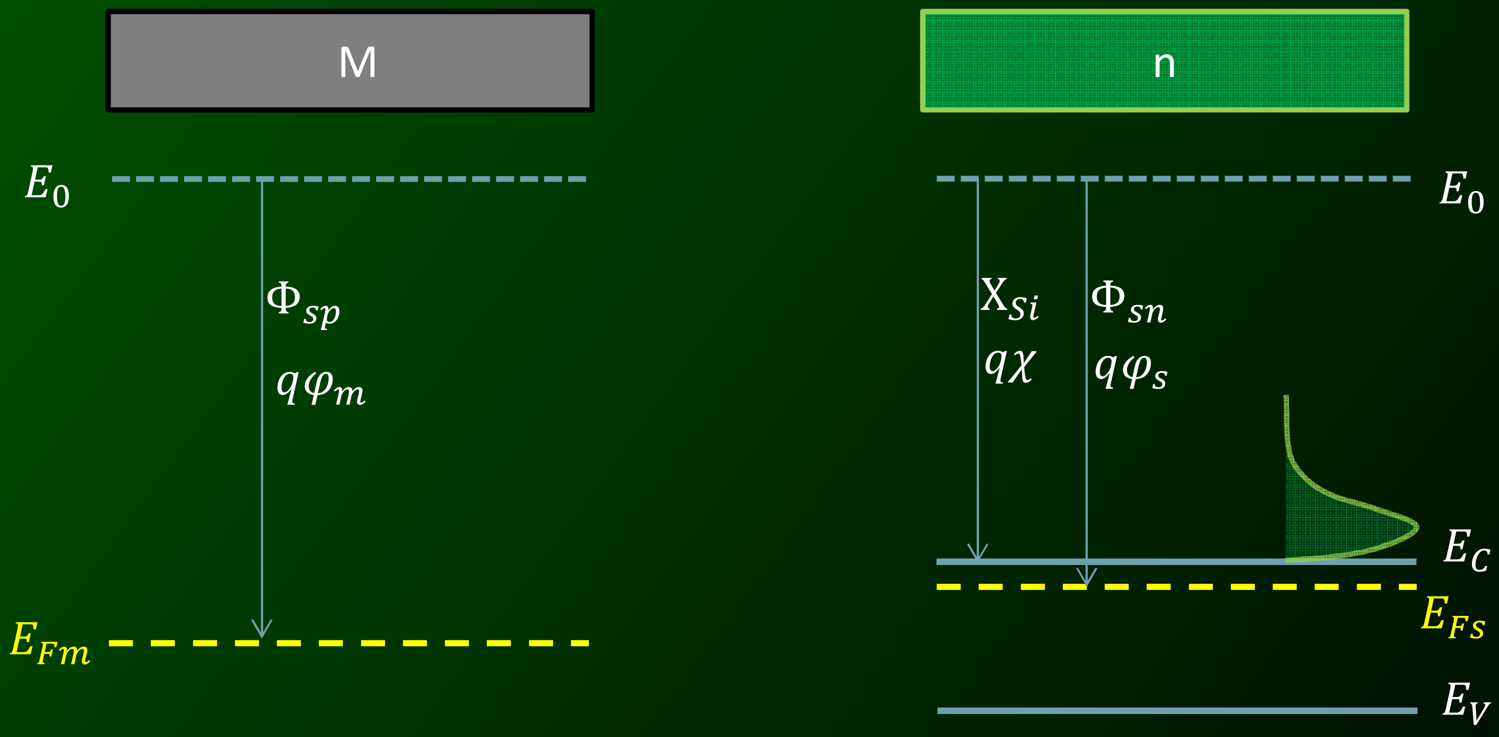
electrically neutral in every region

electron affinity : X_{Si}

work function Φ: $\Phi = E_{vac} - E_F$

$$\Phi_n \neq \Phi_p$$

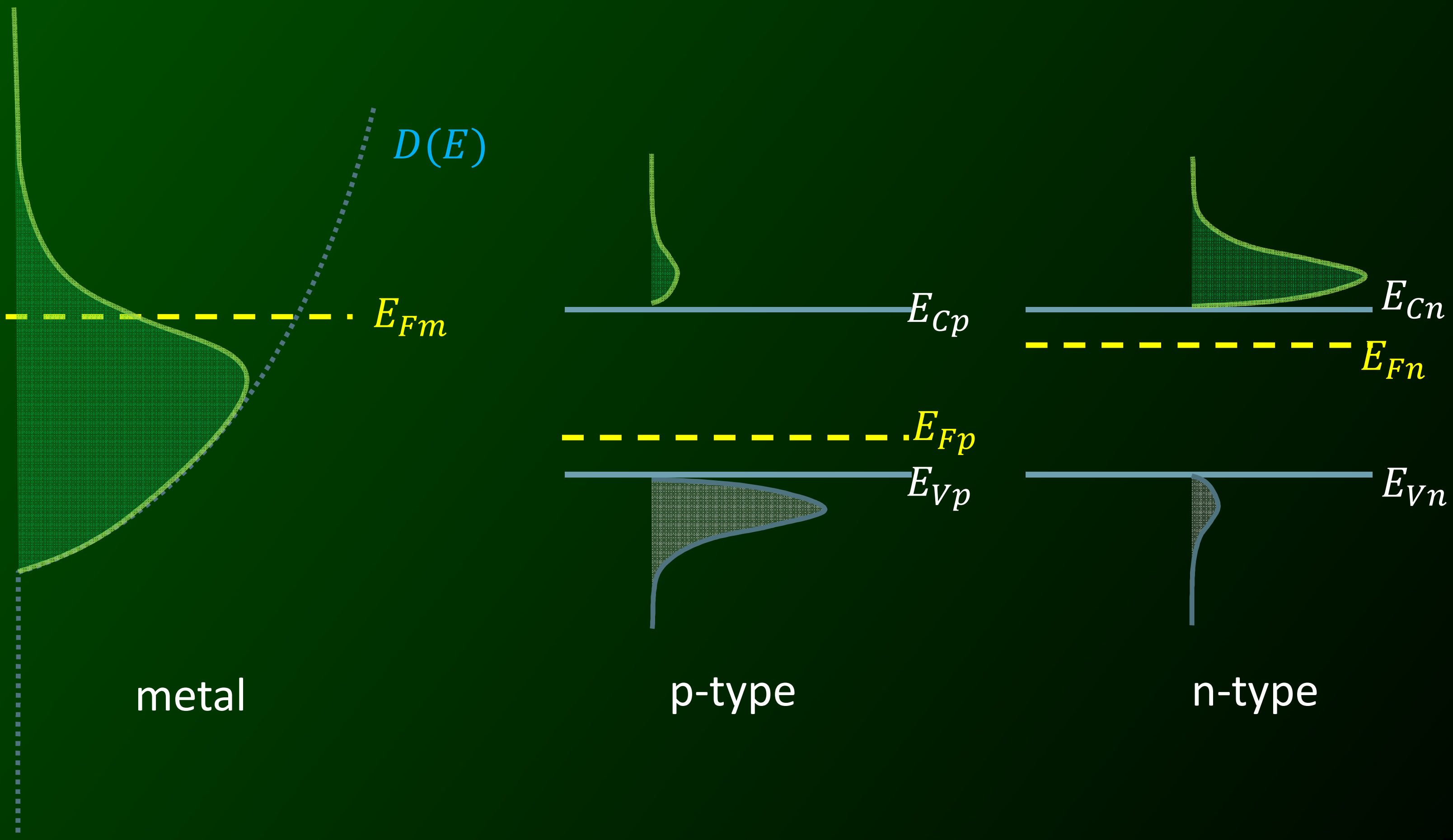
MS junctions – Before Being Joined



$q\varphi_m$ work function $\varphi_{Au} = 4.75eV$, $\varphi_{Cu} = 4.5eV$, $\varphi_{Al} = 4.28eV$

$q\chi$ electron affinity $\chi_{Si} = 4.05eV$, $\chi_{Ge} = 4eV$, $\varphi_{GaAs} = 4.07eV$

Reminder



Plotting Energy Bands for MS Junction

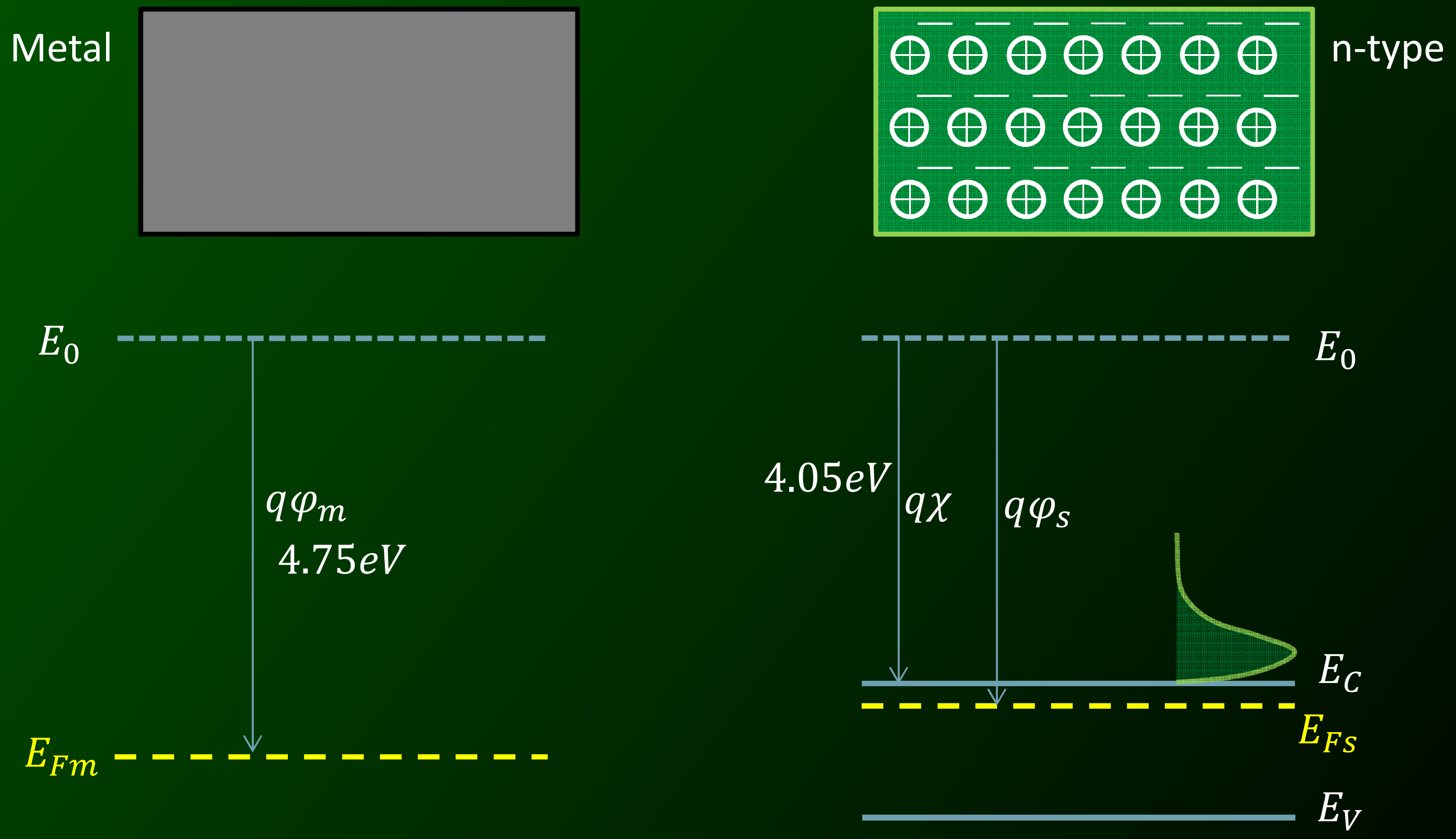
Step by step:

1. Vacuum energy (E_0) is continuous.

2. E_G and χ are intrinsic properties of materials and should remain constant. (which means E_C, E_V, and E_0 are all parallel)

3. At equilibrium E_F is constant while by applying voltage $\Delta E_F = -qV$.

MS junctions – Before Being Joined



MS junctions – Qualitative

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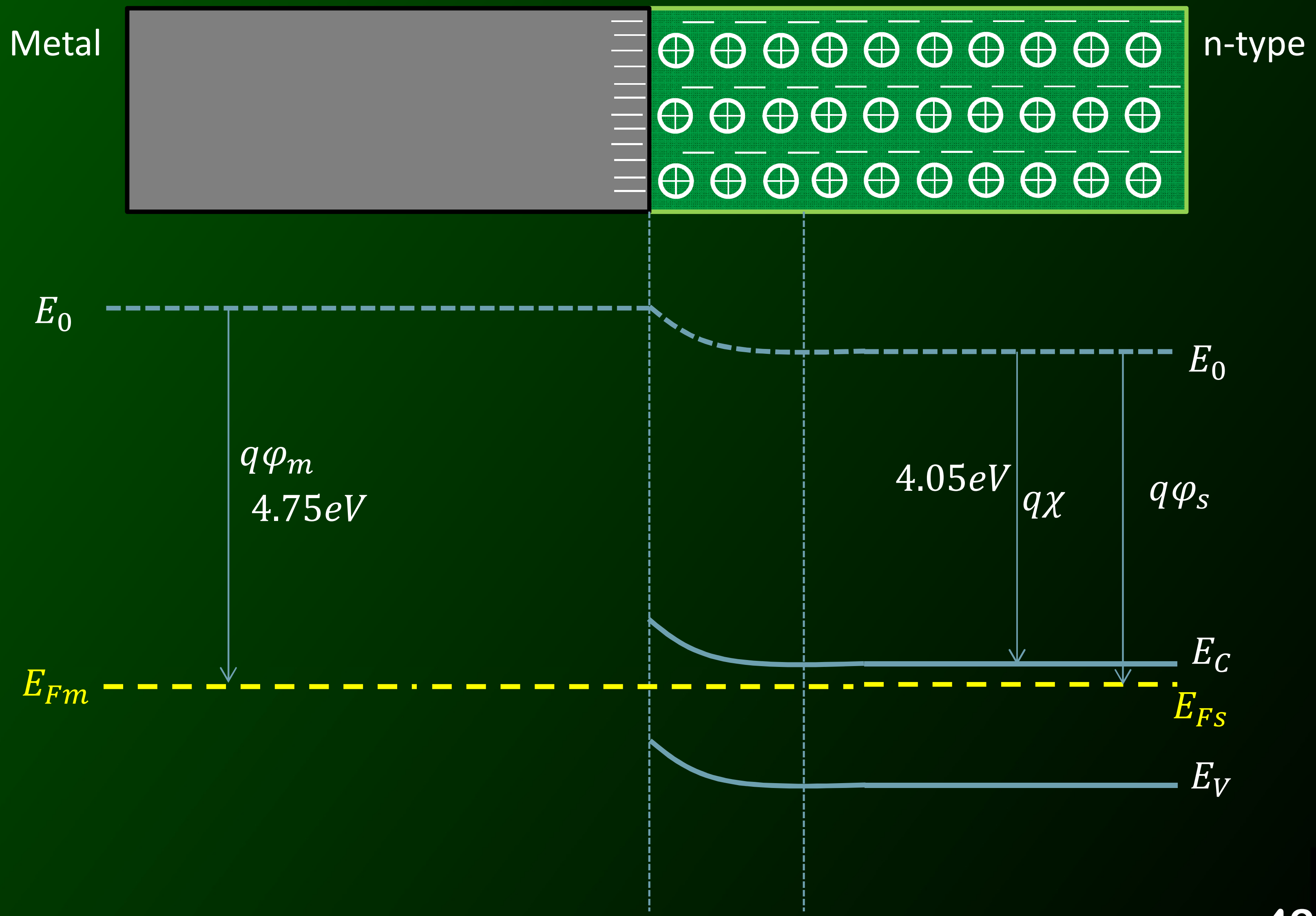
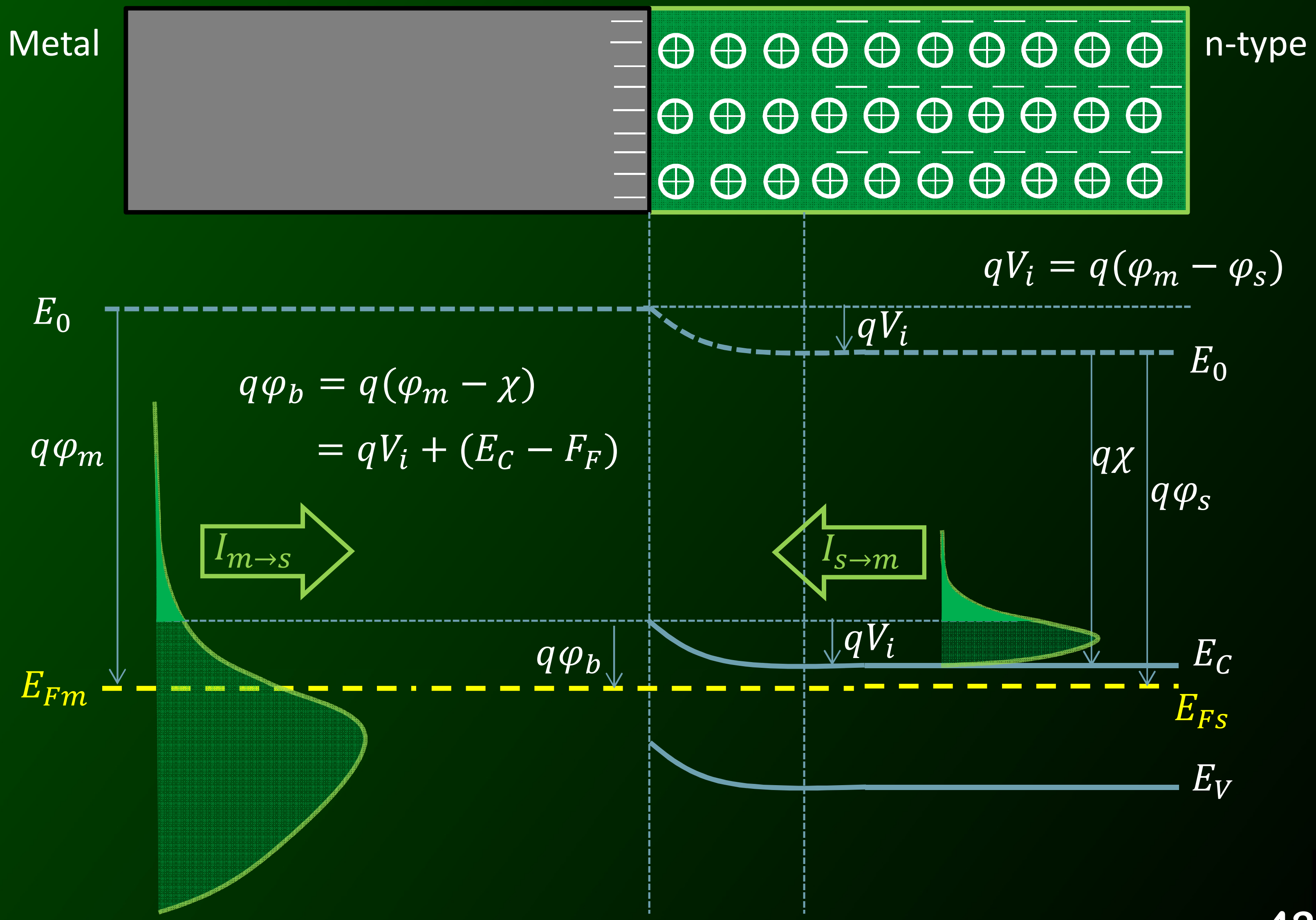


MS junctions – Qualitative



$$qV_i = q(\varphi_m - \varphi_s)$$

$$q\varphi_b = q(\varphi_m - \chi)$$
$$= qV_i + (E_C - F_F)$$

MS junctions – Qualitative

1. I
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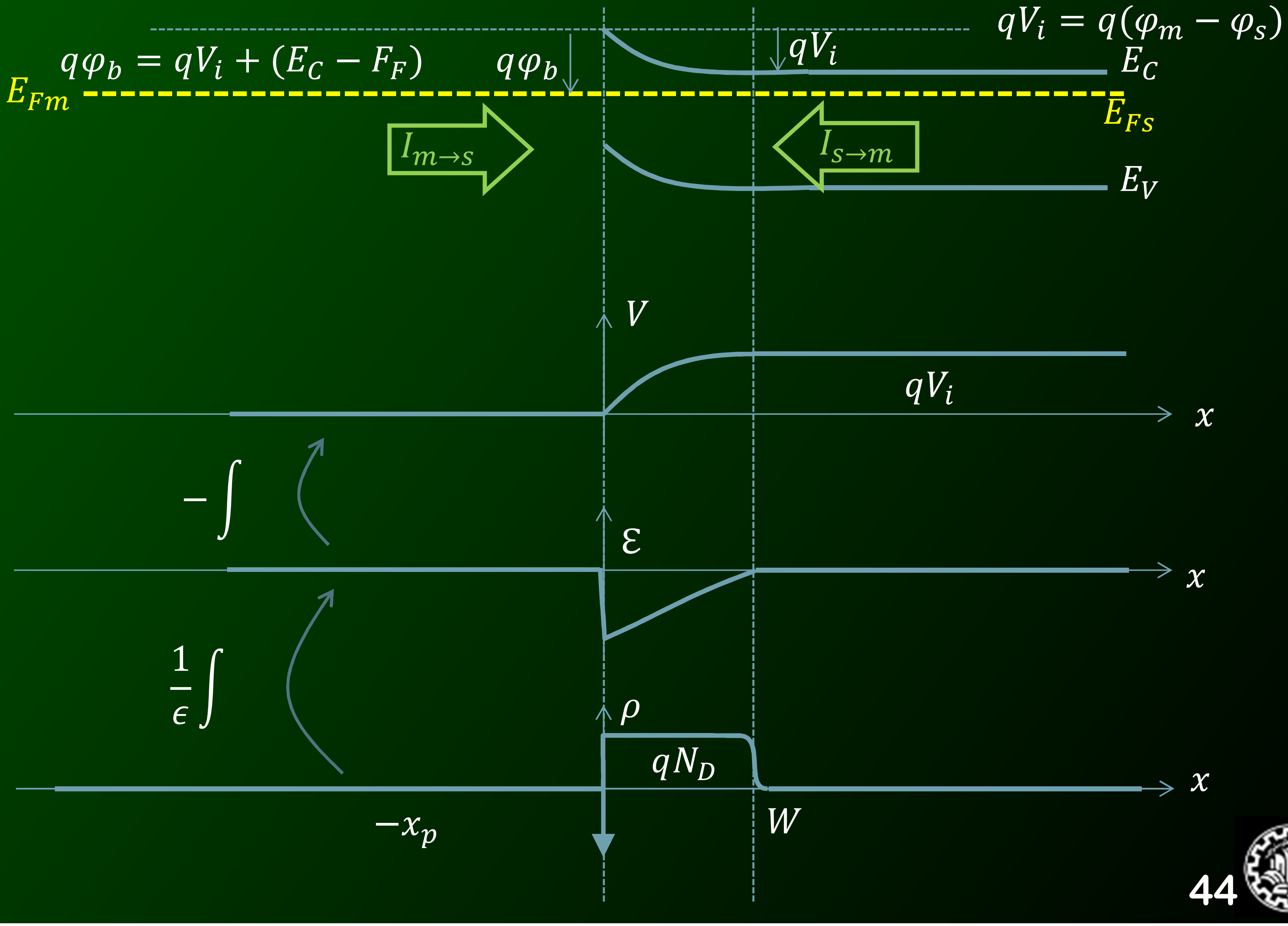


$$qV_i = q(\varphi_m - \varphi_s)$$

$$q\varphi_b = qV_i + (E_C - F_F)$$

MS junctions - Schottky Effect

$$\mathcal{E}(0) = -qN_DW/\epsilon$$

$$V_i = -\tfrac{1}{2}W\mathcal{E}(0) = qN_DW^2/2\epsilon$$

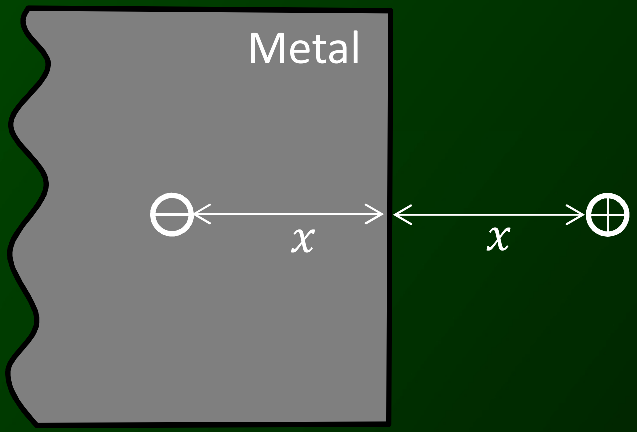
$$W = \sqrt{\frac{2\epsilon}{qN_D}(V_i - V_a)}$$

$$\mathcal{E}(0) = -\sqrt{\frac{2qN_D}{\epsilon}(V_i - V_a)}$$

as $qV_i = q(\varphi_m - \varphi_s)$ seems that V_i is independent of the applied voltage

But it is not! This is known as “Schottky Effect” This will lower $V_i(\varphi_b)$ a little bit.

Image method



$$F(x) = \frac{-q^2}{16\pi\epsilon x^2}$$

$$\rightarrow \Phi(x) = -qV(x) = \frac{-q^2}{16\pi\epsilon x}$$

MS junctions, I-V Curve

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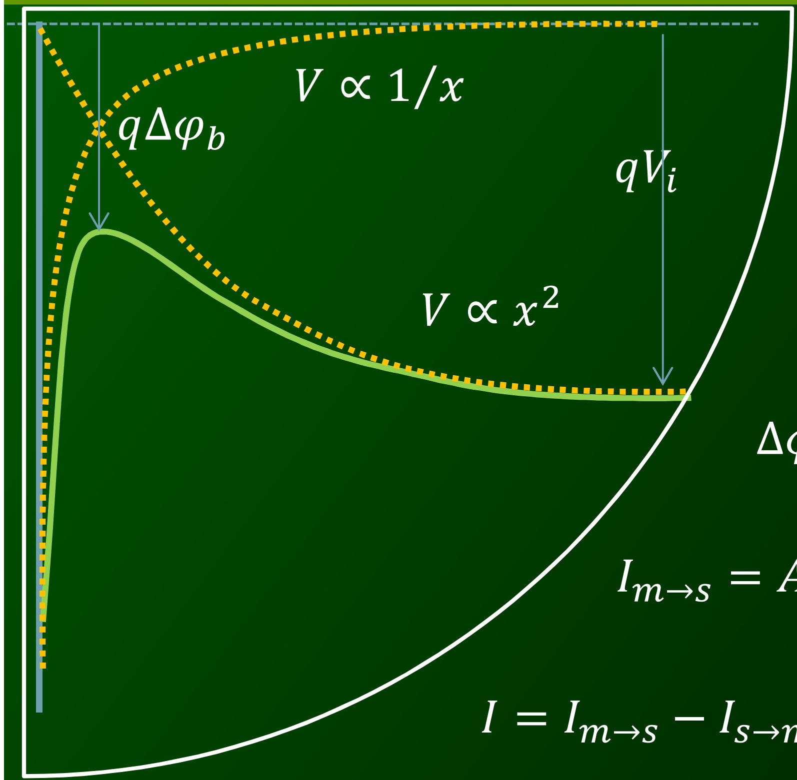
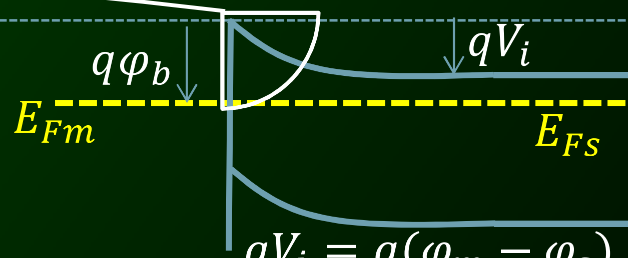




$$q\varphi_b = qV_i + (E_C - F_F)$$

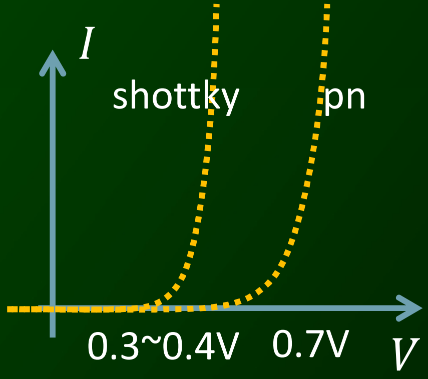
$$qV_i = q(\varphi_m - \varphi_s)$$

$$\varphi_b' = \varphi_b - \Delta\varphi_b$$

$$\Delta\varphi_b = \sqrt[4]{\frac{q^3 N_D}{8\pi^2\epsilon^2}(V_i - V_a)}$$

$$I_{m\to s} = AR^*T^2 e^{-q(\varphi_b' - V_a)/kT} \qquad I_{s\to m} = -AR^*T^2 e^{-q\varphi_b'/kT}$$

$$I = I_{m\to s} - I_{s\to m} = AR^*T^2 e^{-q\varphi_b'/kT}\left[e^{qV_a/kT} - 1\right] = I_s\left(e^{qV_a/kT} - 1\right)$$



$$I_{s_{schottky}} \cong 100 - 1000 I_{0_{pn}}$$

MS junctions – Ohmic Contact

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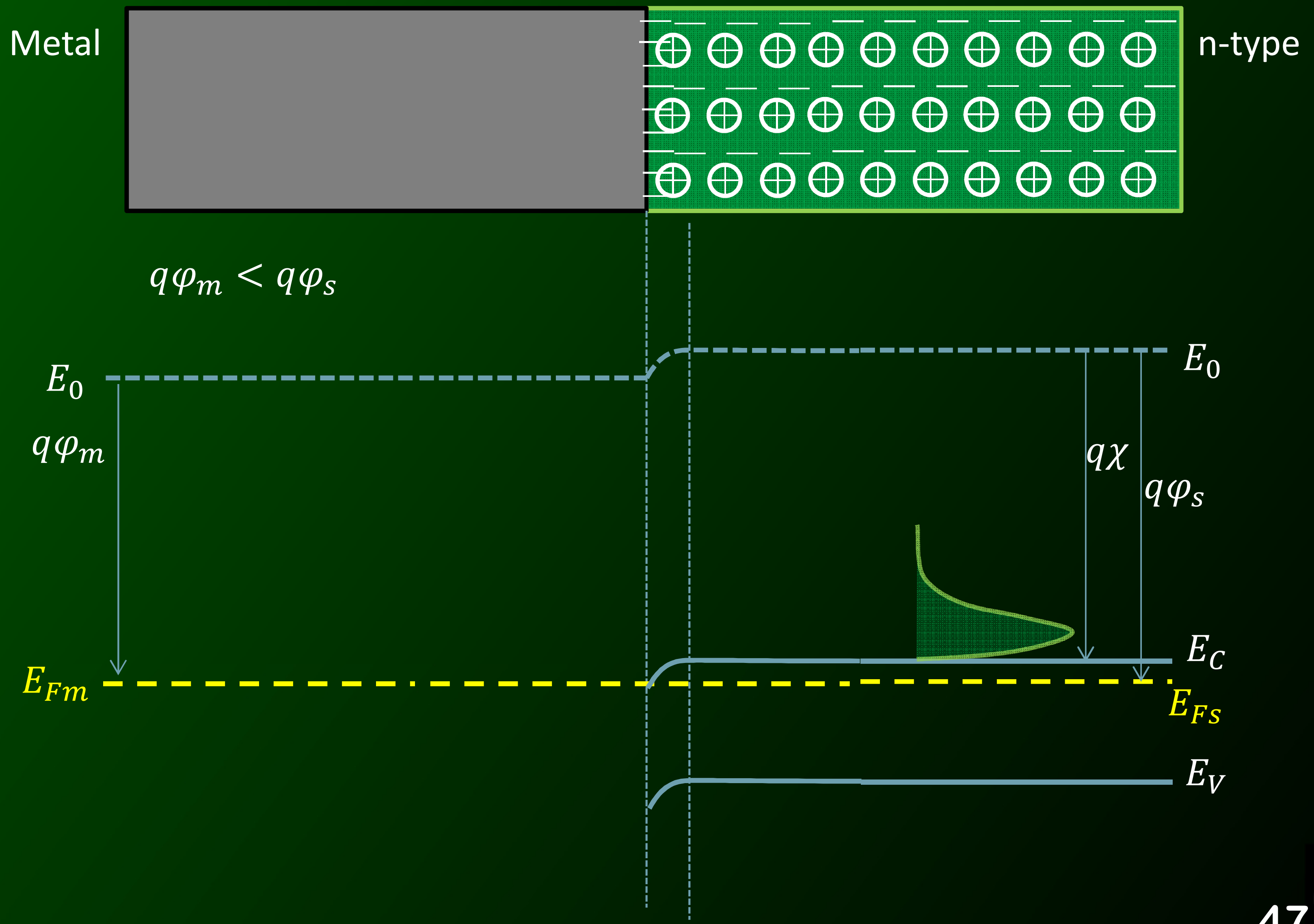

$$q\varphi_m < q\varphi_s$$

MS junctions – Ohmic Contact , Tunneling

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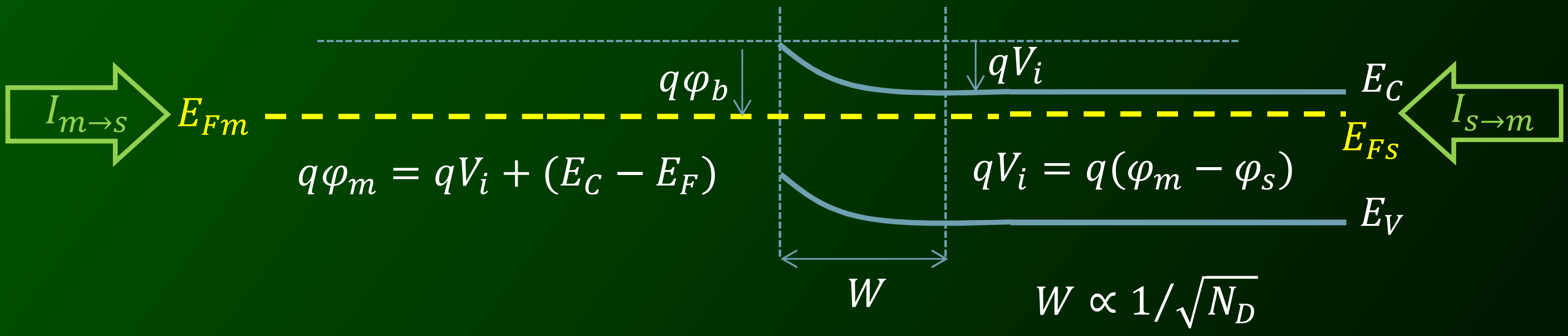
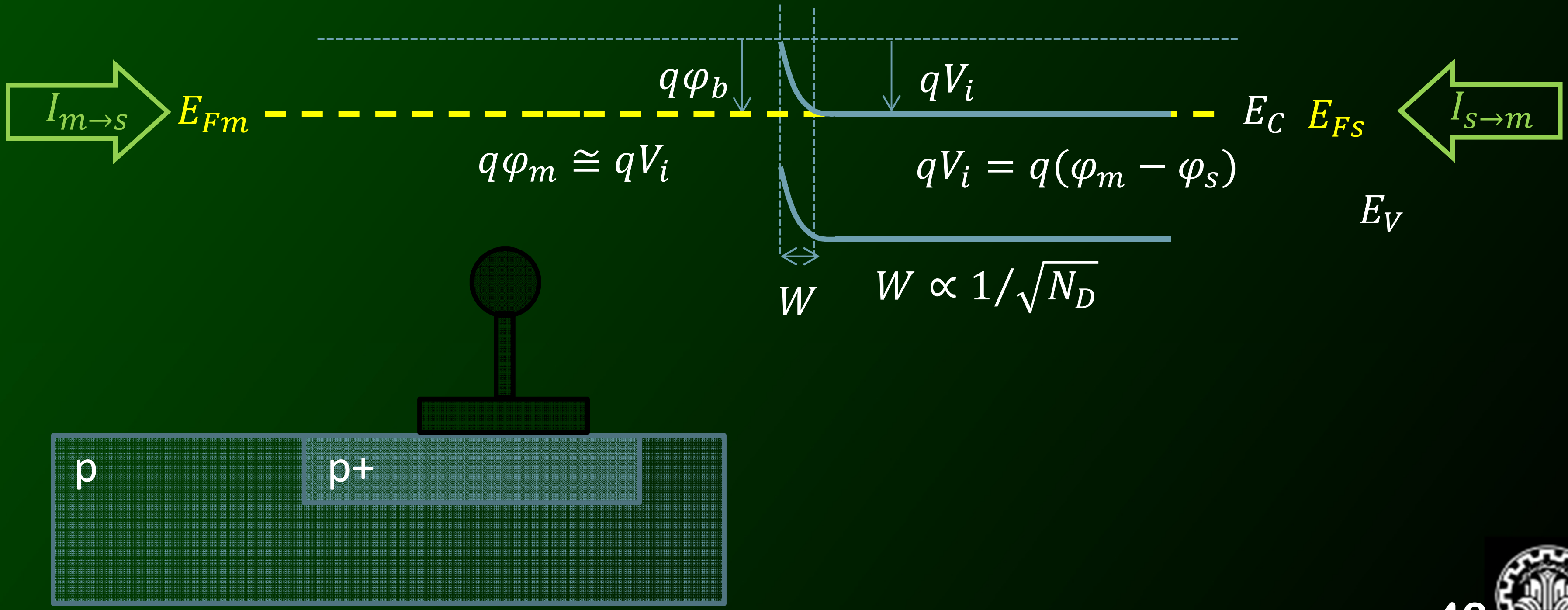


$q\varphi_m = qV_i + (E_C - E_F)$

$qV_i = q(\varphi_m - \varphi_s)$

$W \propto 1/\sqrt{N_D}$



$q\varphi_m \cong qV_i$

$qV_i = q(\varphi_m - \varphi_s)$

$W \propto 1/\sqrt{N_D}$

End of Review!

Hetro-junction!

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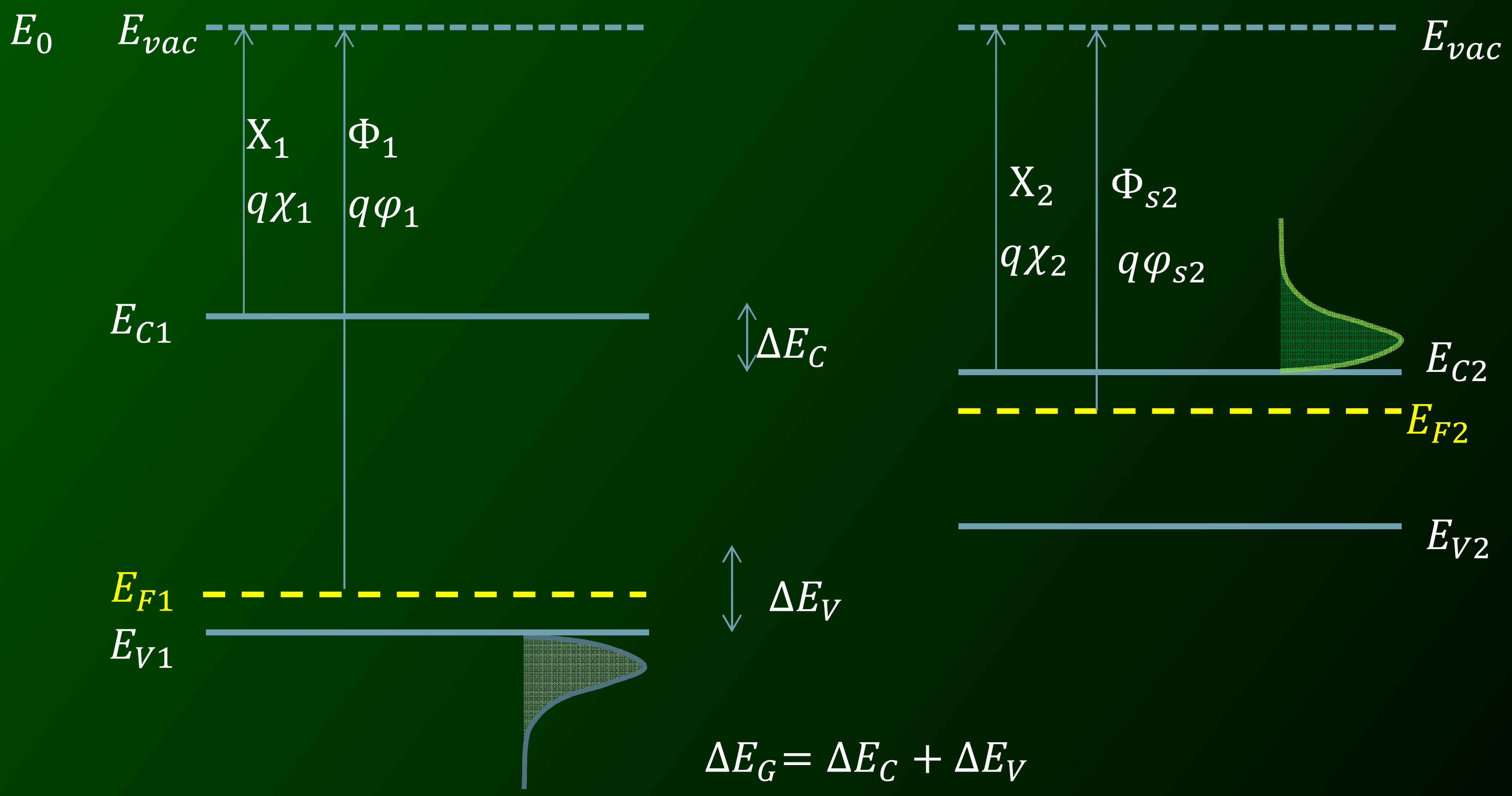

$$\Delta E_G = \Delta E_C + \Delta E_V$$

We expect discontinuities in the energy bands

Plotting Energy Bands for HetroJunction

1. Vacuum energy (E_0) is continuous.

2. E_G and χ are intrinsic properties of materials and should remain constant. (which means E_C, E_V, and E_0 are all parallel)

3. At equilibrium E_F is constant while by applying voltage $\Delta E_F = -qV$.

1. Align the Fermi level with 2 semiconductors separated (leave enough room for transition region).
2. Indicate ΔE_C, ΔE_V at the metallurgical junction .
3. Connect conduction and valance band regions, keeping the band gap constant in each region.

Plotting!

1. I [illegible]
2. [illegible]
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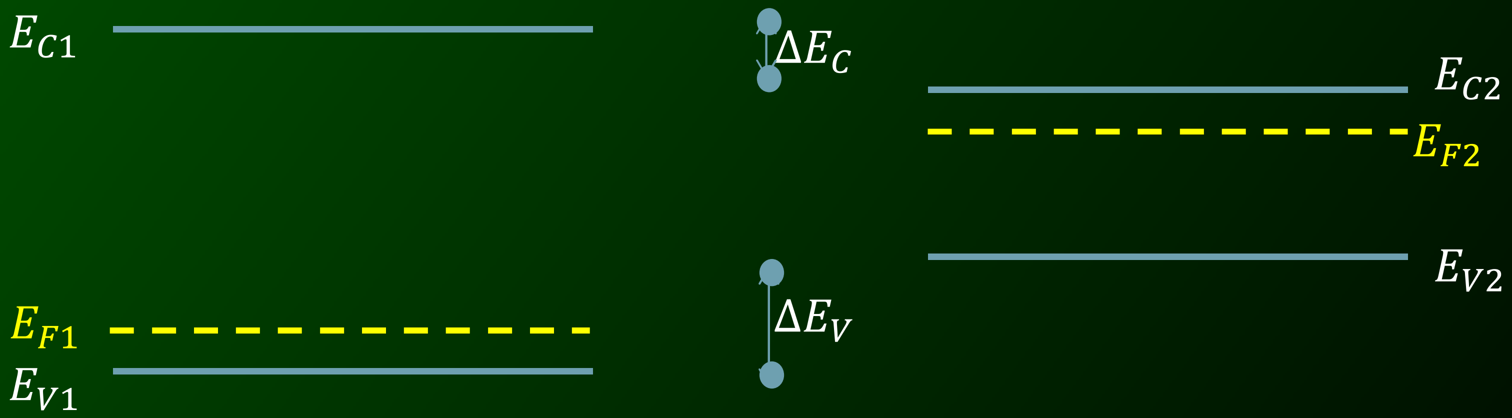

Plotting!

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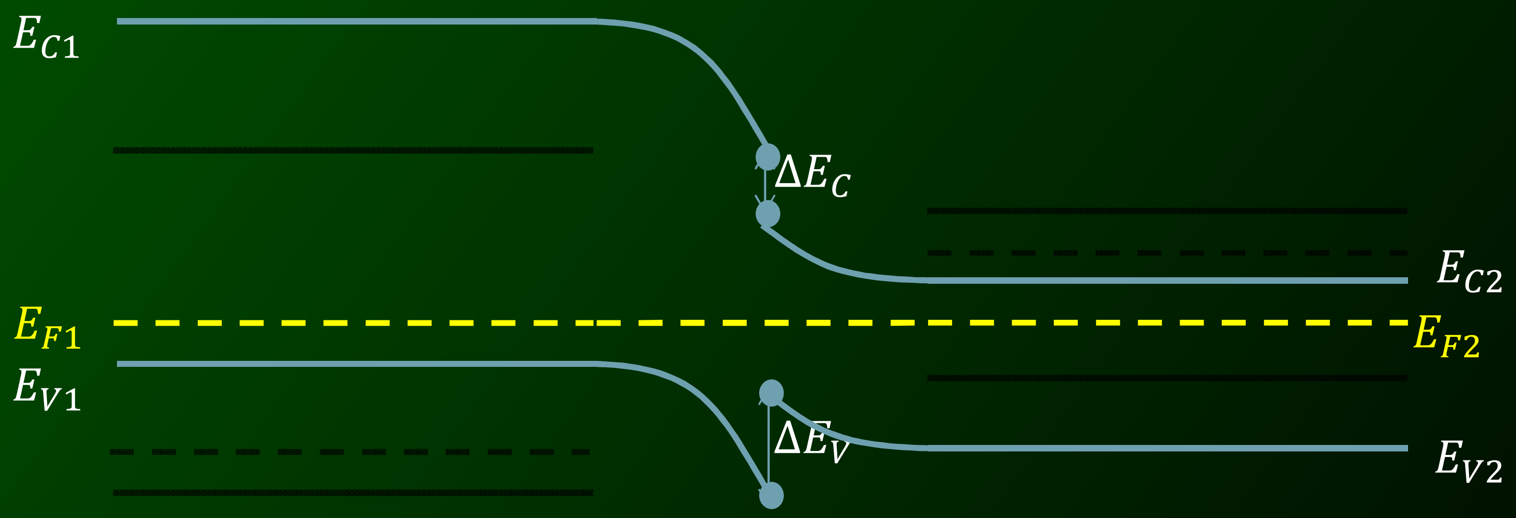
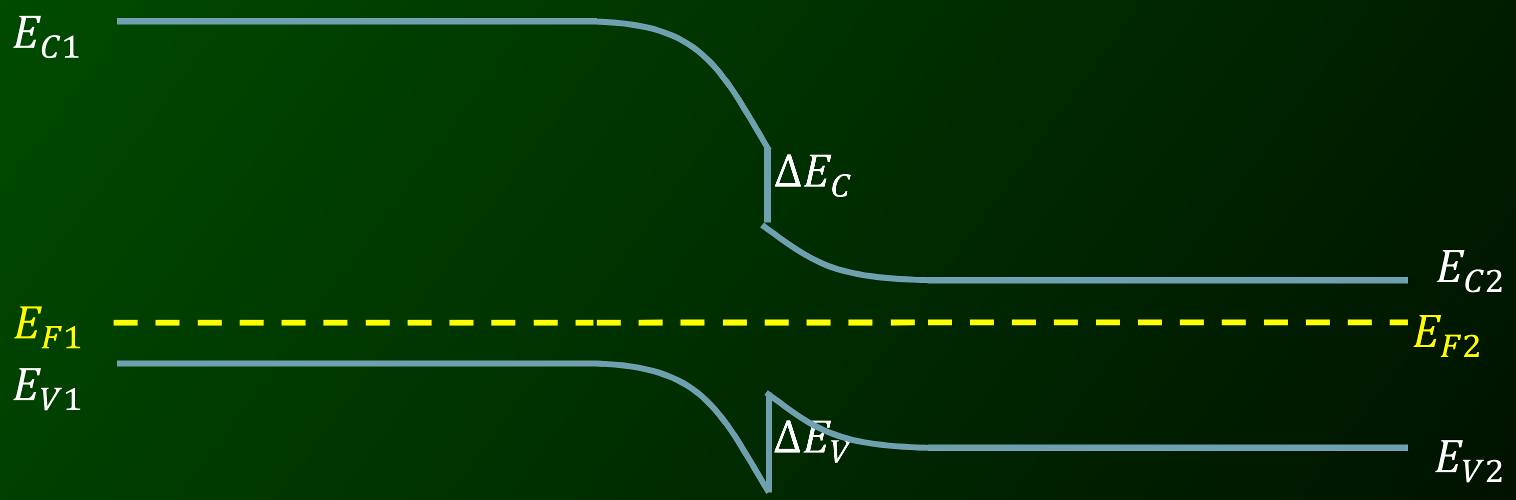
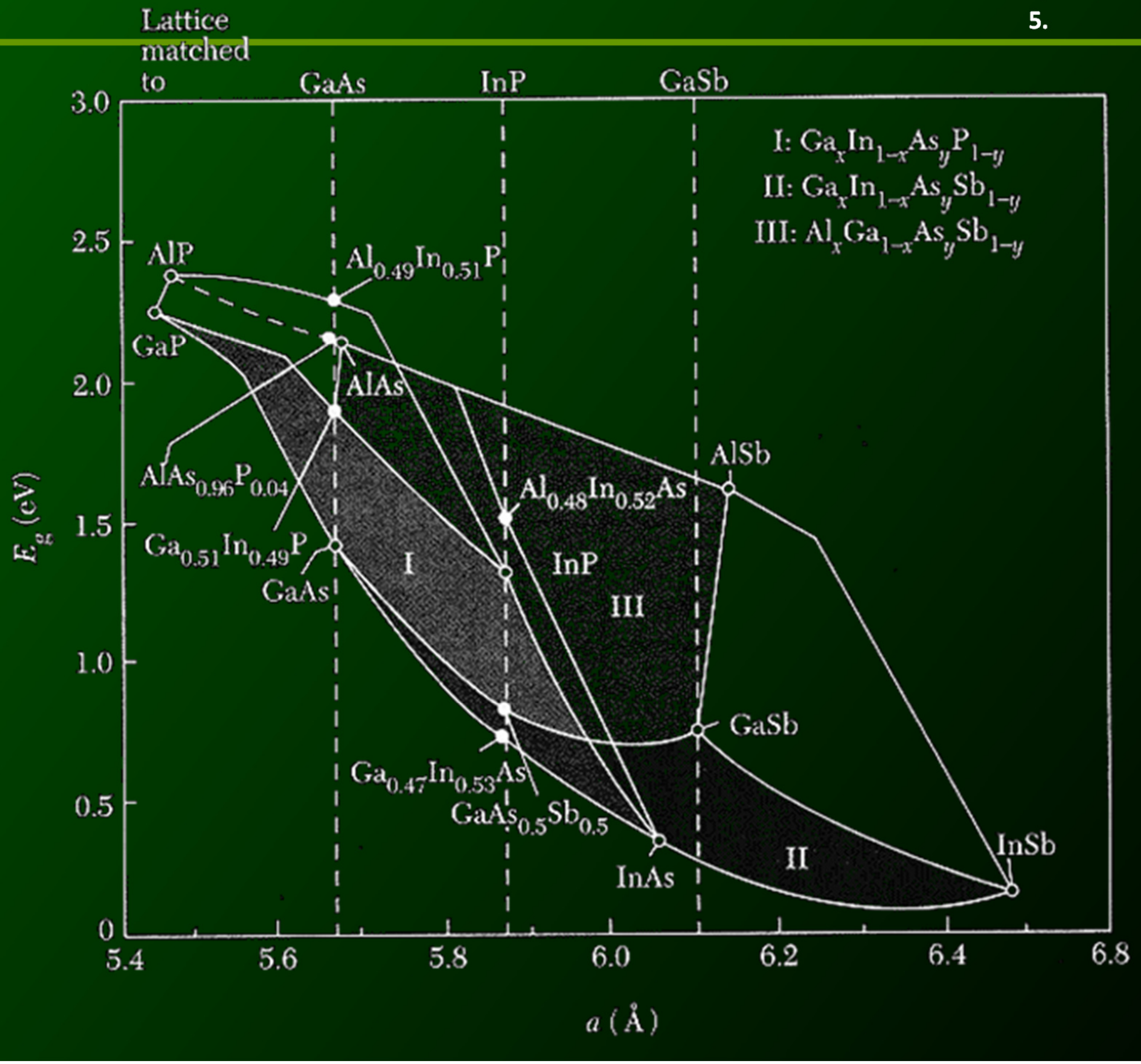


Plotting!



Interface structure



Interface structure

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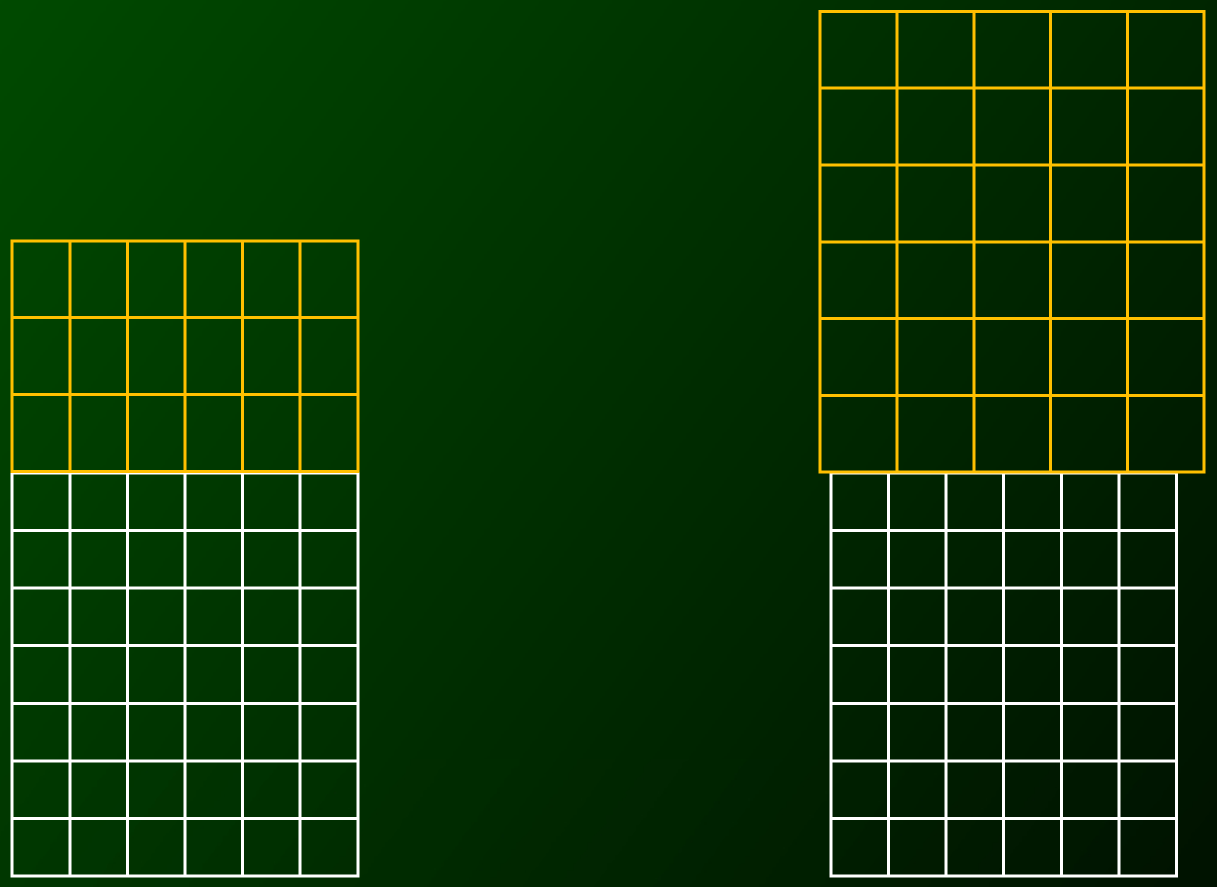
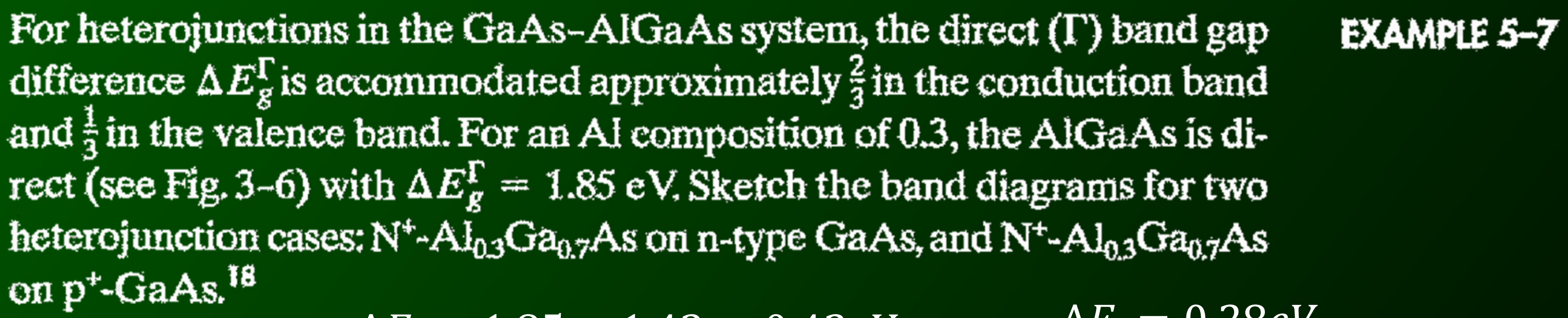


Streetman's Example

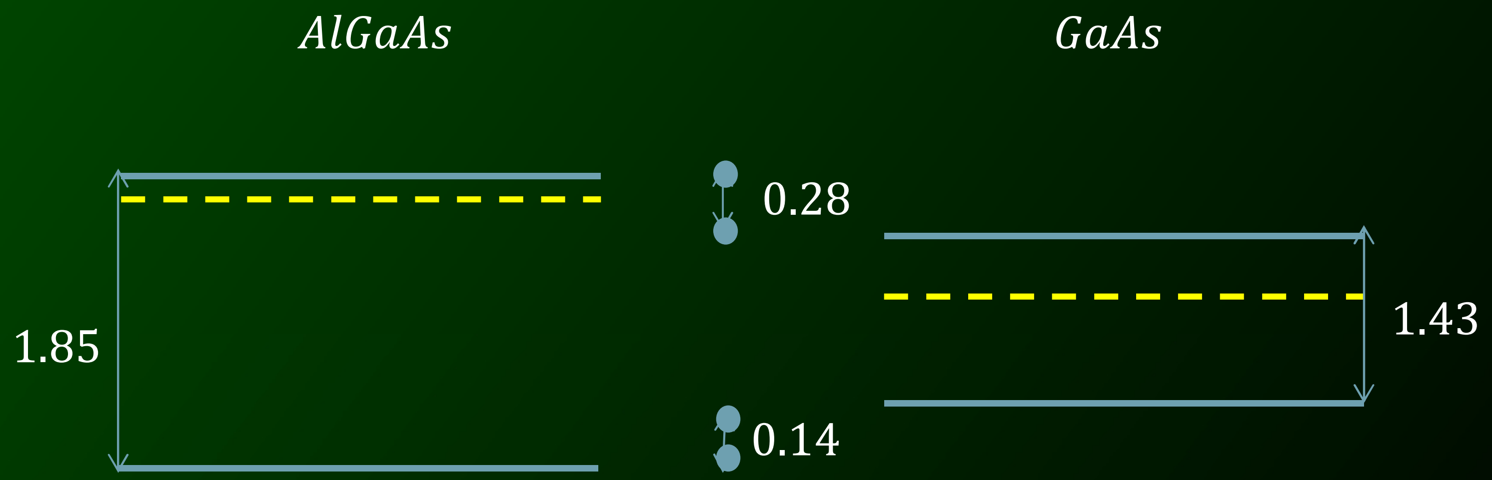


For heterojunctions in the GaAs–AlGaAs system, the direct (Γ) band gap difference ΔE_g^Γ is accommodated approximately $\frac{2}{3}$ in the conduction band and $\frac{1}{3}$ in the valence band. For an Al composition of 0.3, the AlGaAs is direct (see Fig. 3–6) with $\Delta E_g^\Gamma = 1.85$ eV. Sketch the band diagrams for two heterojunction cases: N$^+$-Al$_{0.3}$Ga$_{0.7}$As on n-type GaAs, and N$^+$-Al$_{0.3}$Ga$_{0.7}$As on p$^+$-GaAs.[18]

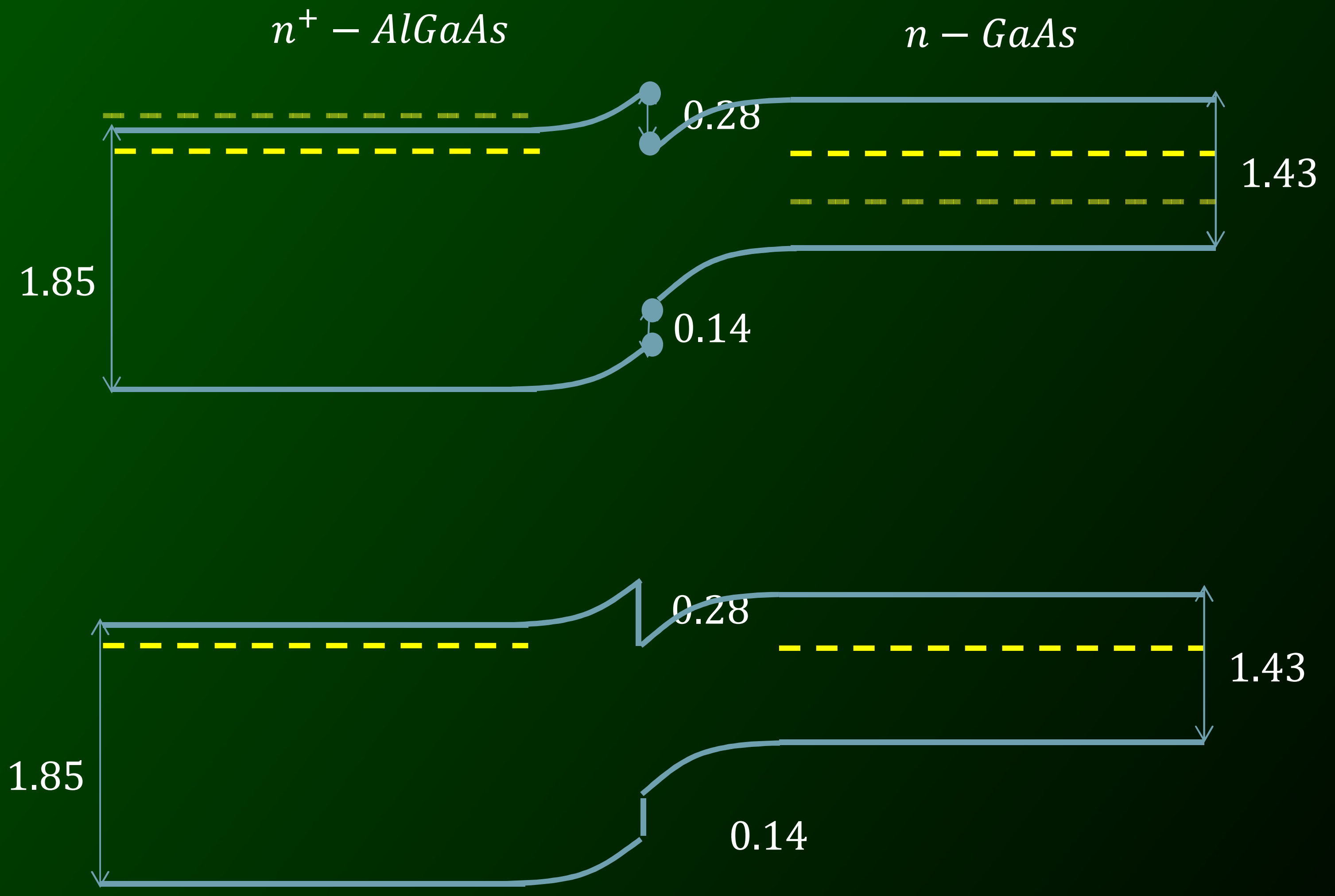
EXAMPLE 5–7

$$\Delta E_G = 1.85 - 1.43 = 0.42eV$$

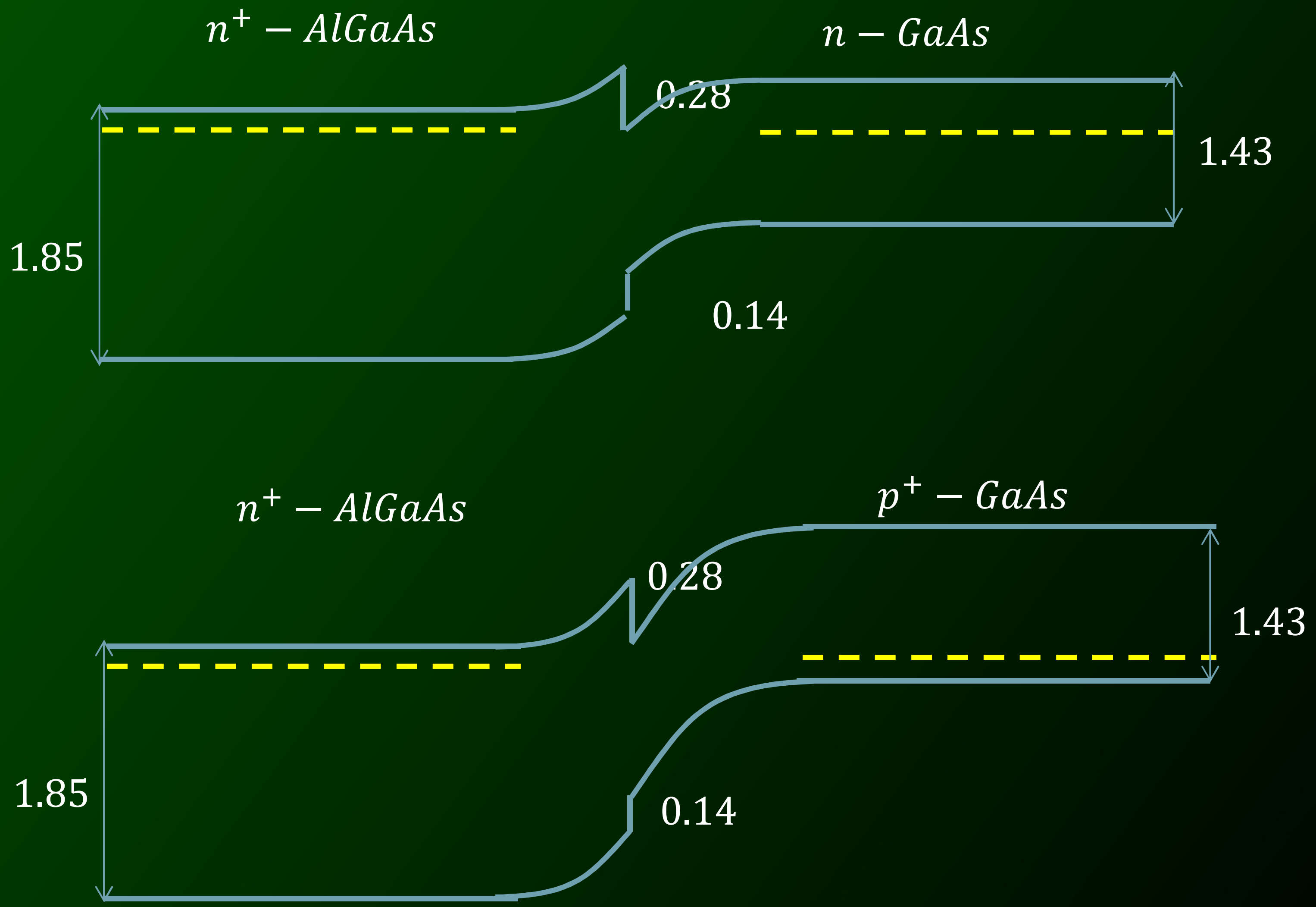
$$\Delta E_C = 0.28eV$$
$$\Delta E_V = 0.14eV$$



Streetman’s Example



Streetman's Example



Why it is important?

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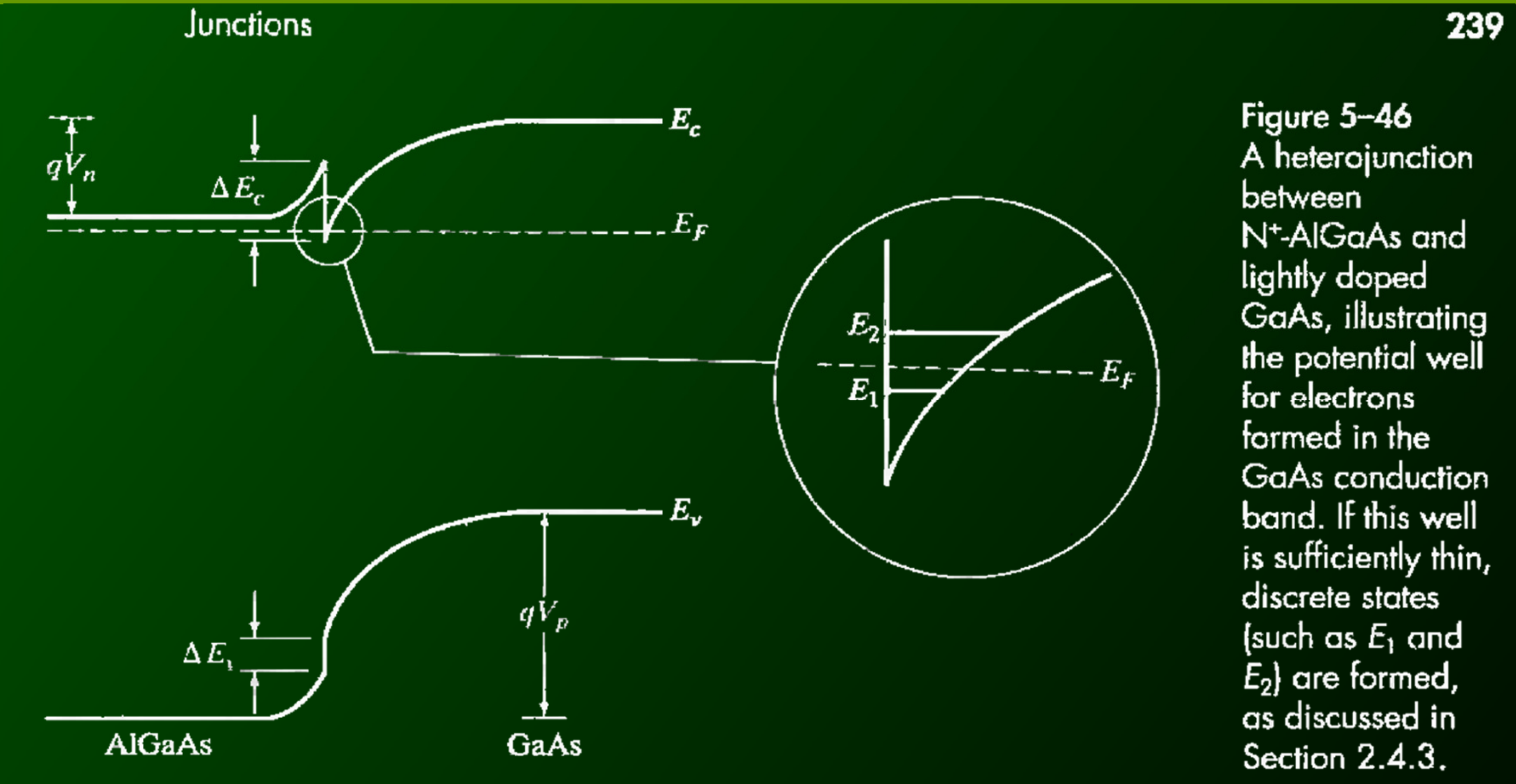

Junctions

239

Figure 5–46
A heterojunction between N$^+$-AlGaAs and lightly doped GaAs, illustrating the potential well for electrons formed in the GaAs conduction band. If this well is sufficiently thin, discrete states (such as E_1 and E_2) are formed, as discussed in Section 2.4.3.

carrier transport

MODFET: along the heterostructure
HBT: perpendicular to the heterojunction

3 types of Heterostructures

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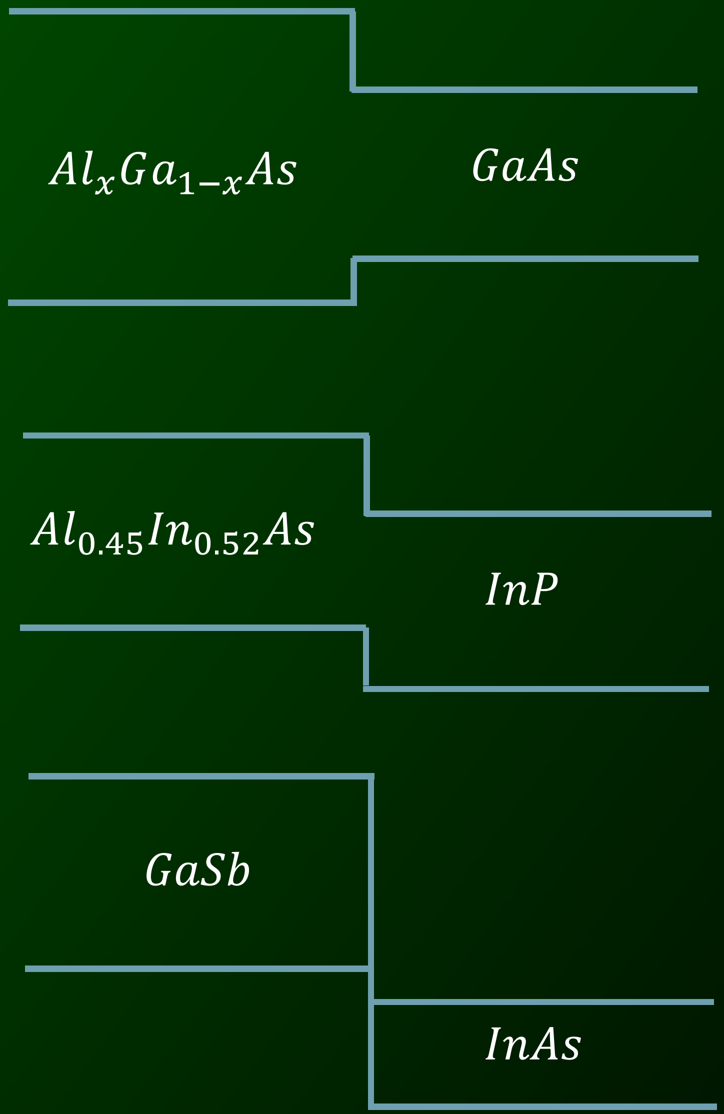


Type I

$Al_xGa_{1-x}As$

$GaAs$

Type II

$Al_{0.45}In_{0.52}As$

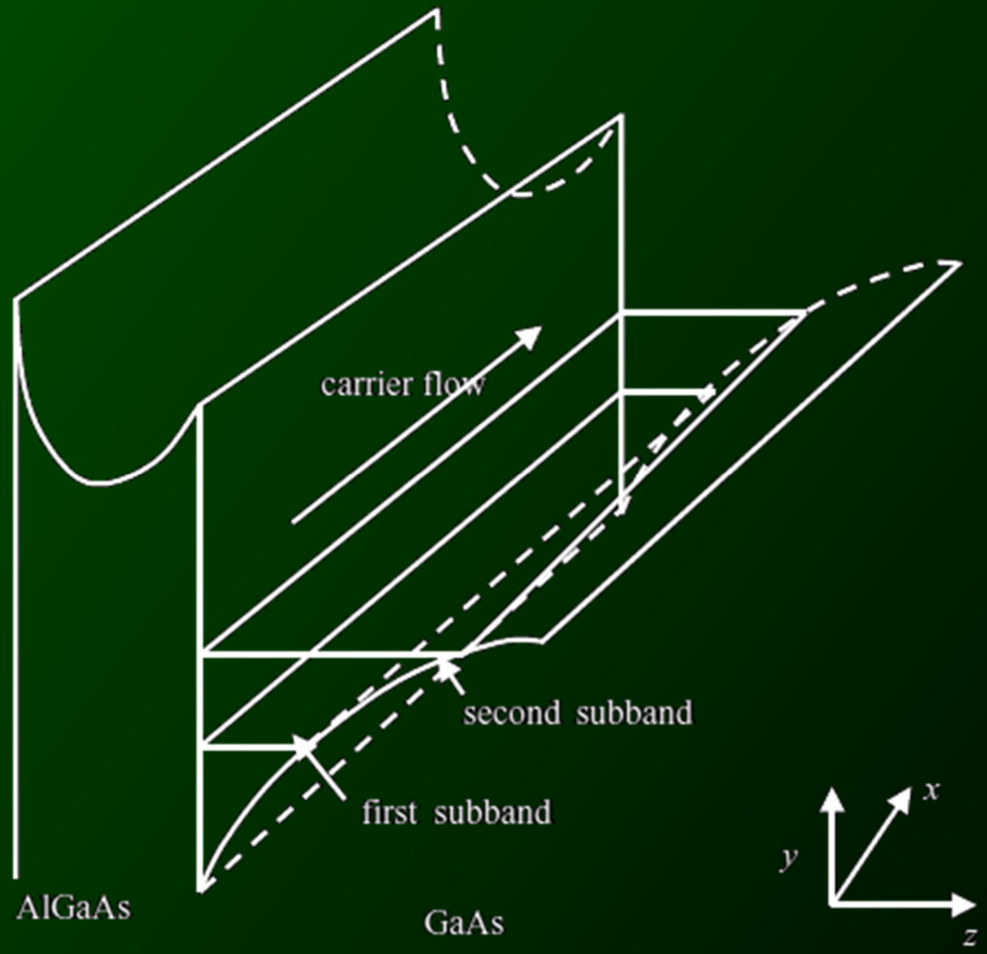
InP

Type III

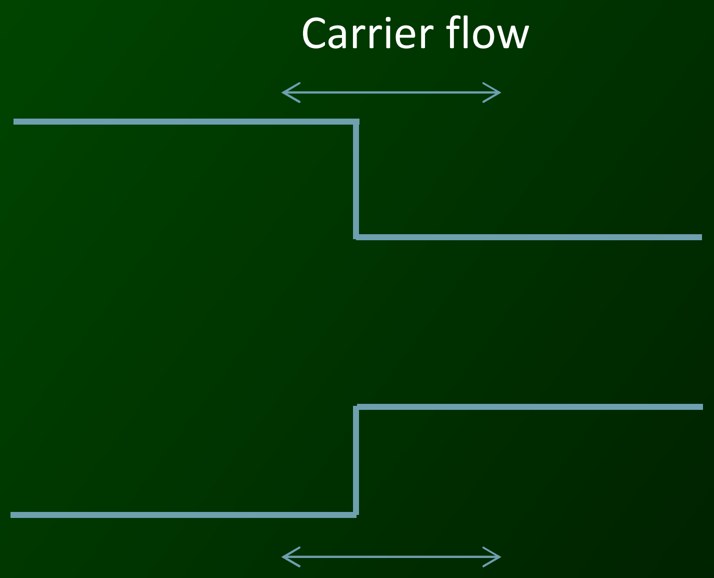
$GaSb$

$InAs$

Carrier Flow Parallel to the Heterointerface



carrier flow perpendicula to the heterointerface



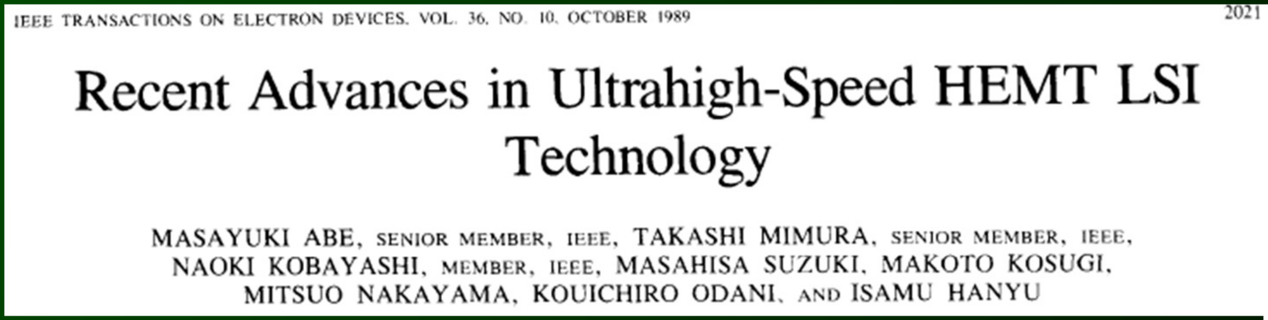
Carrier flow in a heterostructure from the narrow gap semiconductor towards the wider gap semiconductor. In this case, the carriers encounter a potential barrier at the interface that arises from the band edge discontinuity. The electrons can overcome the barrier and enter the wide gap material provided they have sufficient kinetic energy. (b) Carrier flow in a heterostructure from the wide gap semiconductor towards the narrow gap semiconductor. In this case, the carriers gain energy from crossing the potential step.

MODFET

modulation doping provides a means of increasing the free carrier concentration without introducing donor atoms into the channel.

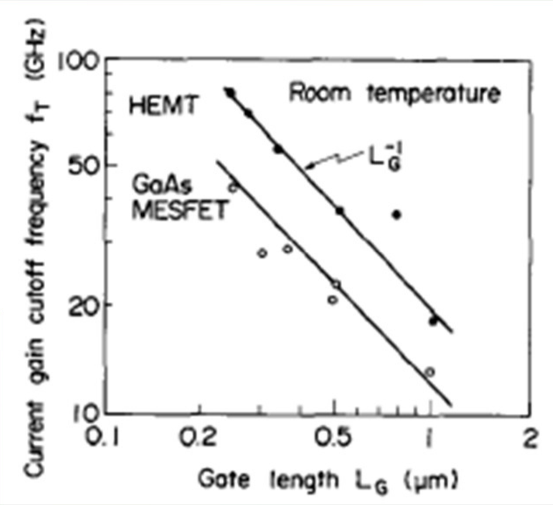
MODFETs, HEMTs (high electron mobility transistors)

80s, GaAs (channel) and AlGaAs (n-doped layer) are lattice matched

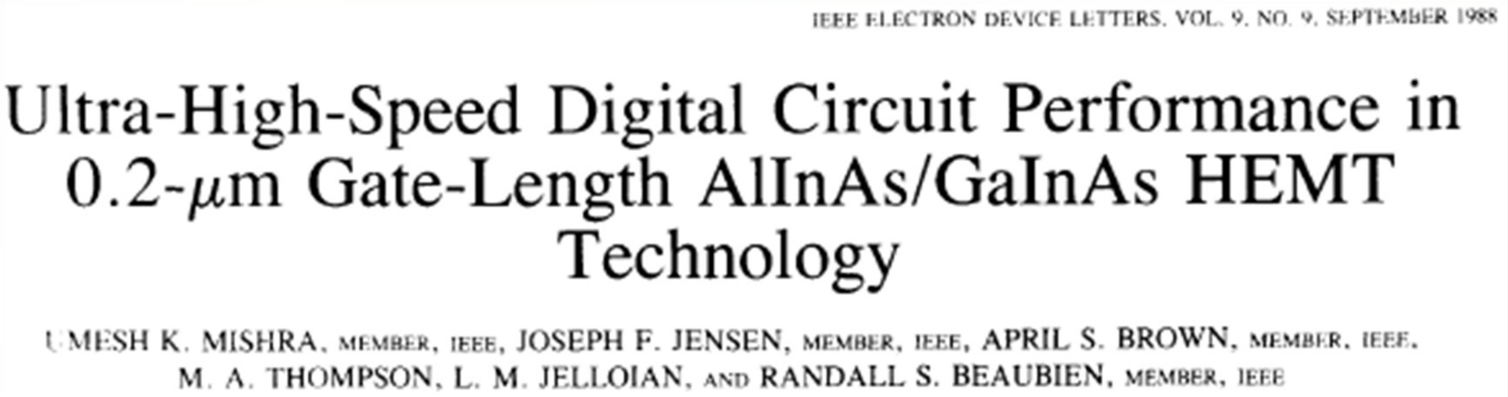

IEEE TRANSACTIONS ON ELECTRON DEVICES, VOL. 36, NO. 10, OCTOBER 1989 2021

Recent Advances in Ultrahigh-Speed HEMT LSI Technology

MASAYUKI ABE, SENIOR MEMBER, IEEE, TAKASHI MIMURA, SENIOR MEMBER, IEEE, NAOKI KOBAYASHI, MEMBER, IEEE, MASAHISA SUZUKI, MAKOTO KOSUGI, MITSUO NAKAYAMA, KOUICHIRO ODANI, AND ISAMU HANYU



HEMT



IEEE ELECTRON DEVICE LETTERS, VOL. 9, NO. 9, SEPTEMBER 1988

Ultra-High-Speed Digital Circuit Performance in 0.2-μm Gate-Length AlInAs/GaInAs HEMT Technology

UMESH K. MISHRA, MEMBER, IEEE, JOSEPH F. JENSEN, MEMBER, IEEE, APRIL S. BROWN, MEMBER, IEEE, M. A. THOMPSON, L. M. JELLOIAN, AND RANDALL S. BEAUBIEN, MEMBER, IEEE

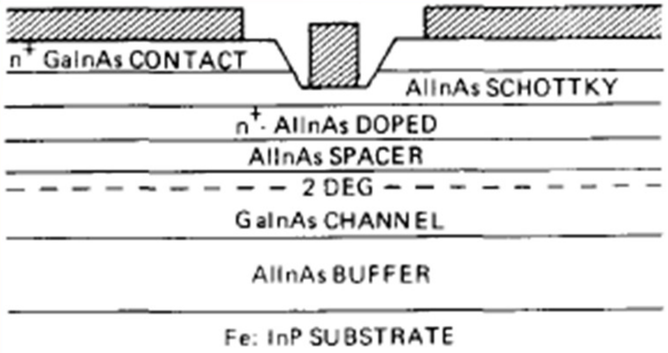


Fig. 1. AlInAs/GaInAs HEMT device structure.

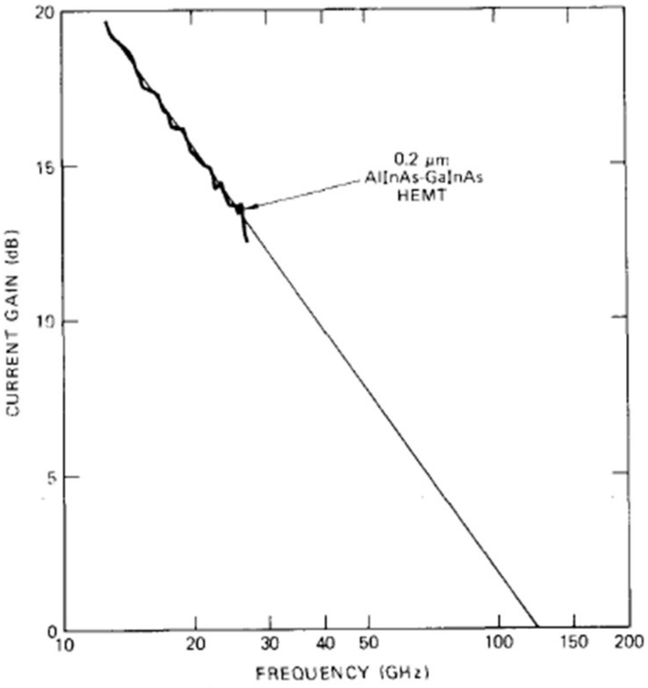
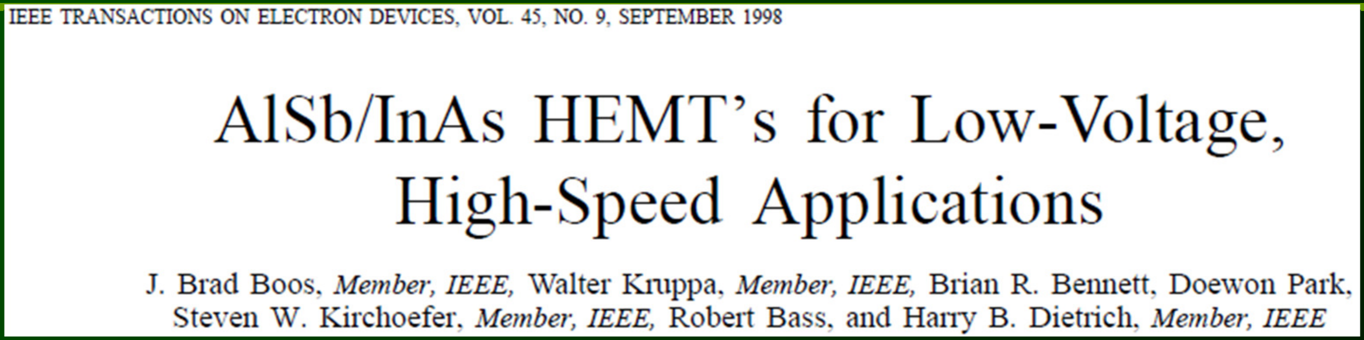


Fig. 5. The measured f_T of a 0.2-μm AlInAs/GaInAs HEMT extrapolated from the current gain measurement.

HEMT



IEEE TRANSACTIONS ON ELECTRON DEVICES, VOL. 45, NO. 9, SEPTEMBER 1998

AlSb/InAs HEMT's for Low-Voltage, High-Speed Applications

J. Brad Boos, *Member, IEEE,* Walter Kruppa, *Member, IEEE,* Brian R. Bennett, Doewon Park, Steven W. Kirchoefer, *Member, IEEE,* Robert Bass, and Harry B. Dietrich, *Member, IEEE*

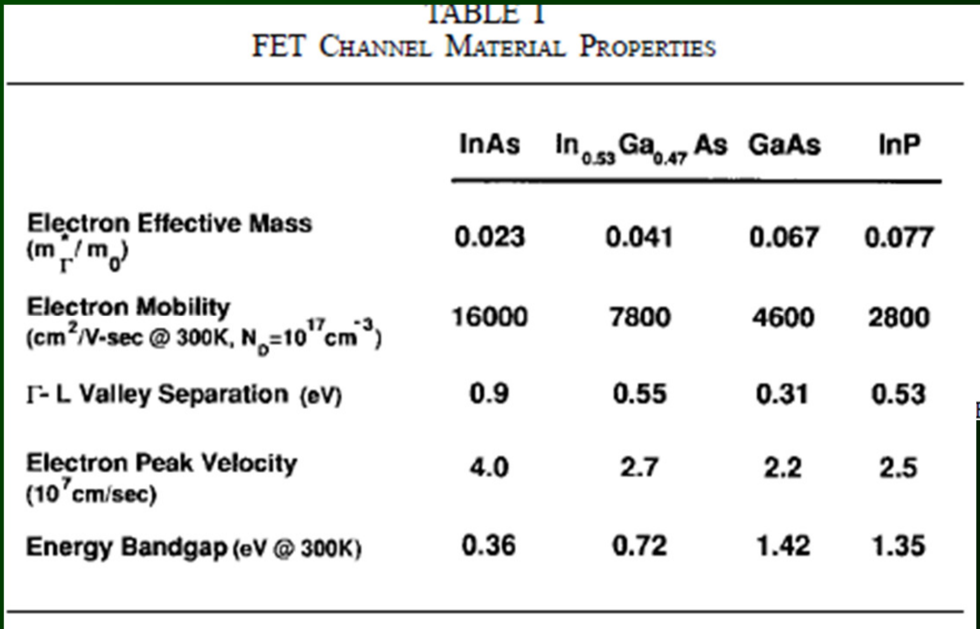


TABLE I
FET CHANNEL MATERIAL PROPERTIES

	InAs	$In_{0.53}Ga_{0.47}As$	GaAs	InP
Electron Effective Mass (m^*_Γ / m_0)	0.023	0.041	0.067	0.077
Electron Mobility (cm^2/V-sec @ 300K, $N_D=10^{17} cm^{-3}$)	16000	7800	4600	2800
Γ- L Valley Separation (eV)	0.9	0.55	0.31	0.53
Electron Peak Velocity (10^7cm/sec)	4.0	2.7	2.2	2.5
Energy Bandgap (eV @ 300K)	0.36	0.72	1.42	1.35

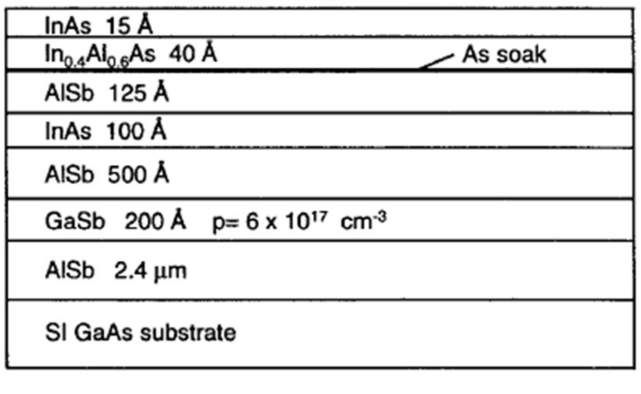


Fig. 1. HEMT starting material.

Performance Metric

$$f_T = \frac{g_m}{2\pi C_{GS}} = \frac{1}{2\pi\tau_r}$$

$$\mu = \frac{q\tau}{m_e^*}$$

τ_r transit time of electrons in the device depends on the length of the channel

$L = 1um$ (1450)

Long Channel: $\tau_r = \frac{L^2}{\mu V_{DS}}$

$$\mu_{Si} = 300\,\frac{\mathrm{cm}^2}{\mathrm{Vs}} \rightarrow f_T = 4GHz$$

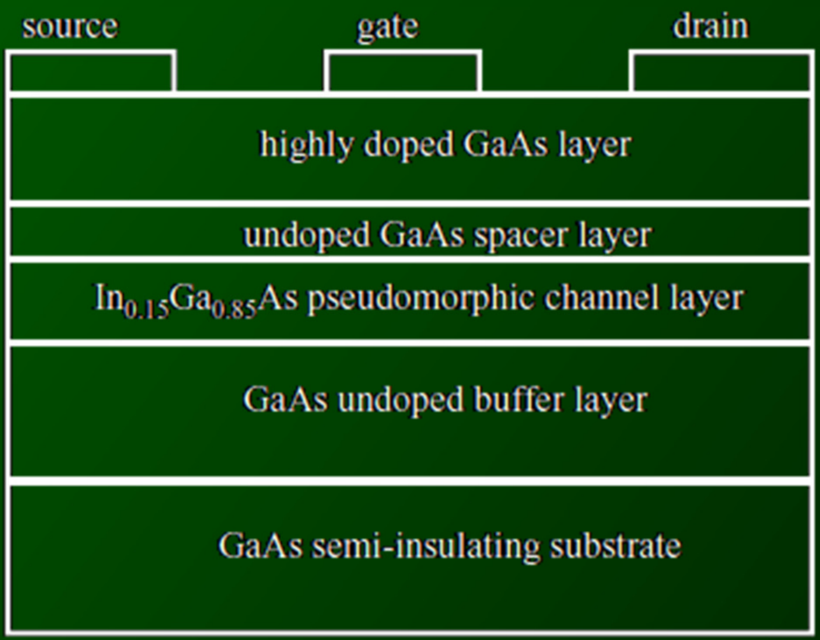
$$\mu_{2D} = 4000\,\frac{\mathrm{cm}^2}{\mathrm{Vs}} \rightarrow f_T = 62GHz$$

(8500)

Short Channel: $\tau_r = \frac{L}{v_{sat}}$

tradeoff between f_T and Power dissipation

InGaAs Channel (PHEMT)



highly doped GaAs layer

undoped GaAs spacer layer

$In_{0.15}Ga_{0.85}As$ pseudomorphic channel layer

GaAs undoped buffer layer

GaAs semi-insulating substrate

channel layer is formed with $In_{0.15}Ga_{0.85}As$ and the doped layer is GaAs. (Rosenberg *et al*. 1985)

The InGaAs layer is pseudomorphic.

critical thickness of about 20.0 nm

Though the GaAs and InGaAs are not lattice matched, if the InGaAs layer is grown sufficiently thin it will adopt the lattice constant of the underlying GaAs layer.

$$\mu_{Si} = 1450\,\frac{\mathrm{cm}^2}{\mathrm{Vs}}$$

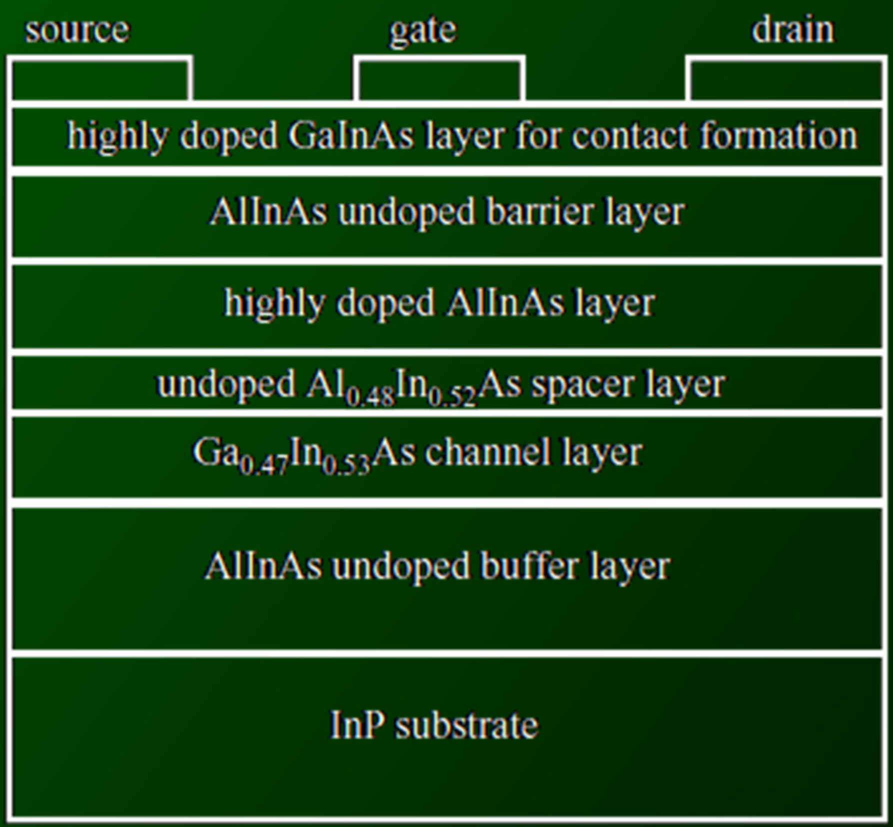
$$\mu_{GaAs} = 8500\,\frac{\mathrm{cm}^2}{\mathrm{Vs}}$$

$$\mu_{InAs} = 33000\,\frac{\mathrm{cm}^2}{\mathrm{Vs}}$$

$$E_{G\,GaAs} = 1.42\mathrm{eV}$$

$$E_{G\,InAs} = 0.35\mathrm{eV}$$

HEMT



(Brown et al., 1989).

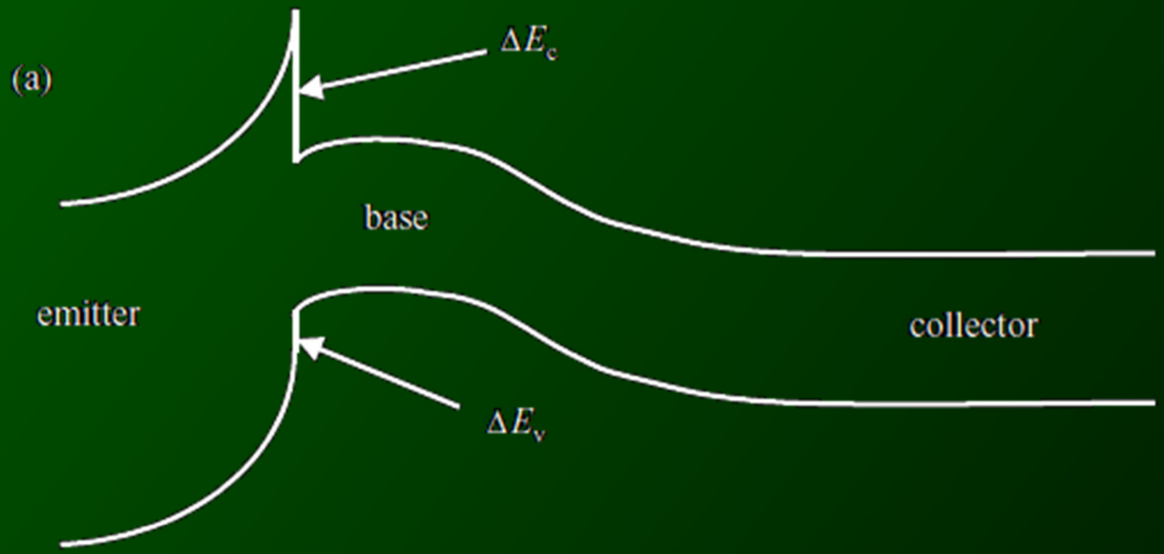
deep well!

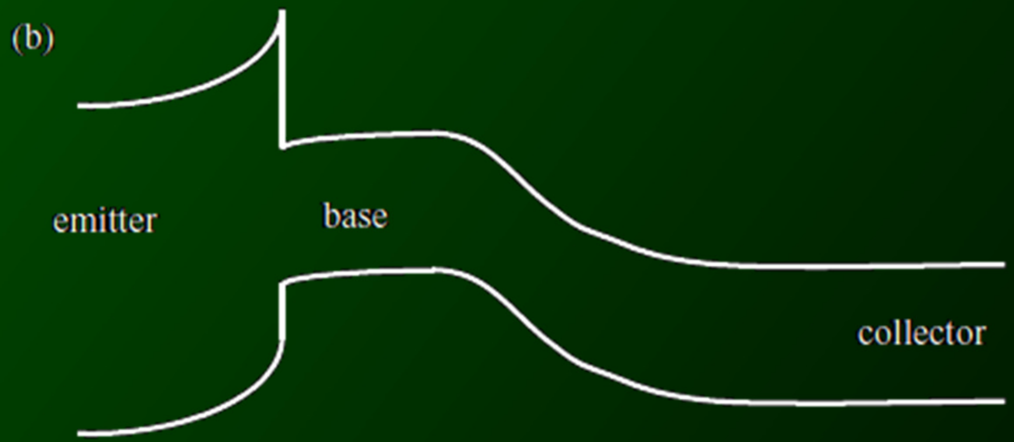
$$\mu_{2D} = 12000\,\frac{\mathrm{cm}^2}{\mathrm{Vs}}$$

$$v_{sat} = 3 \times 10^7 cm/s$$

$$f_T = 170\ GHz$$

HBT

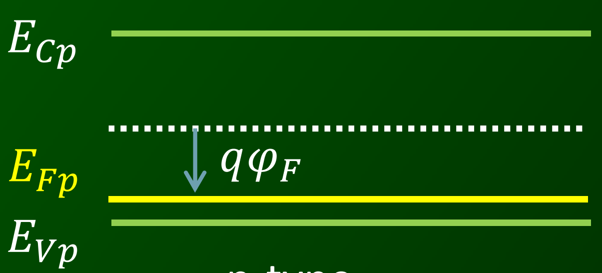

(a)
ΔE_c
base
emitter
collector
ΔE_v


(b)
emitter
base
collector

Bulk Semiconductor Potential , φ_F

Definition:

$$q\varphi_F \equiv E_i - E_F = E_{i(bulk)} - E_F$$

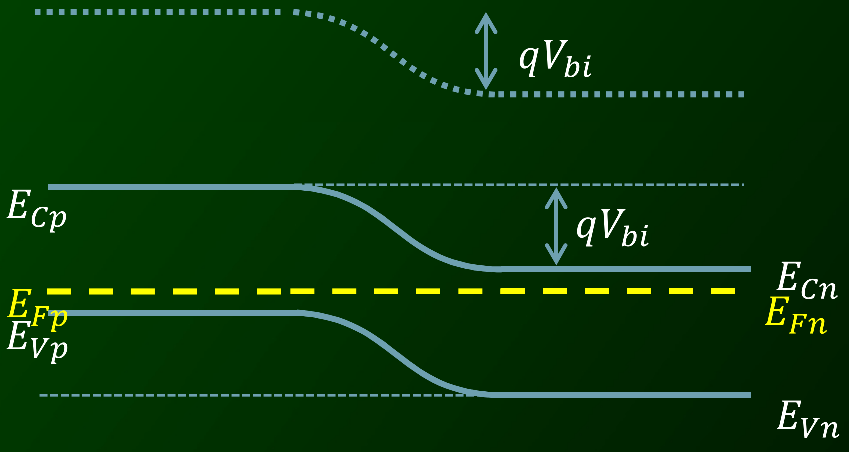


p-type

n-type

$$\varphi_F = \frac{kT}{q}\ln\left(\frac{N_A}{n_i}\right) > 0$$

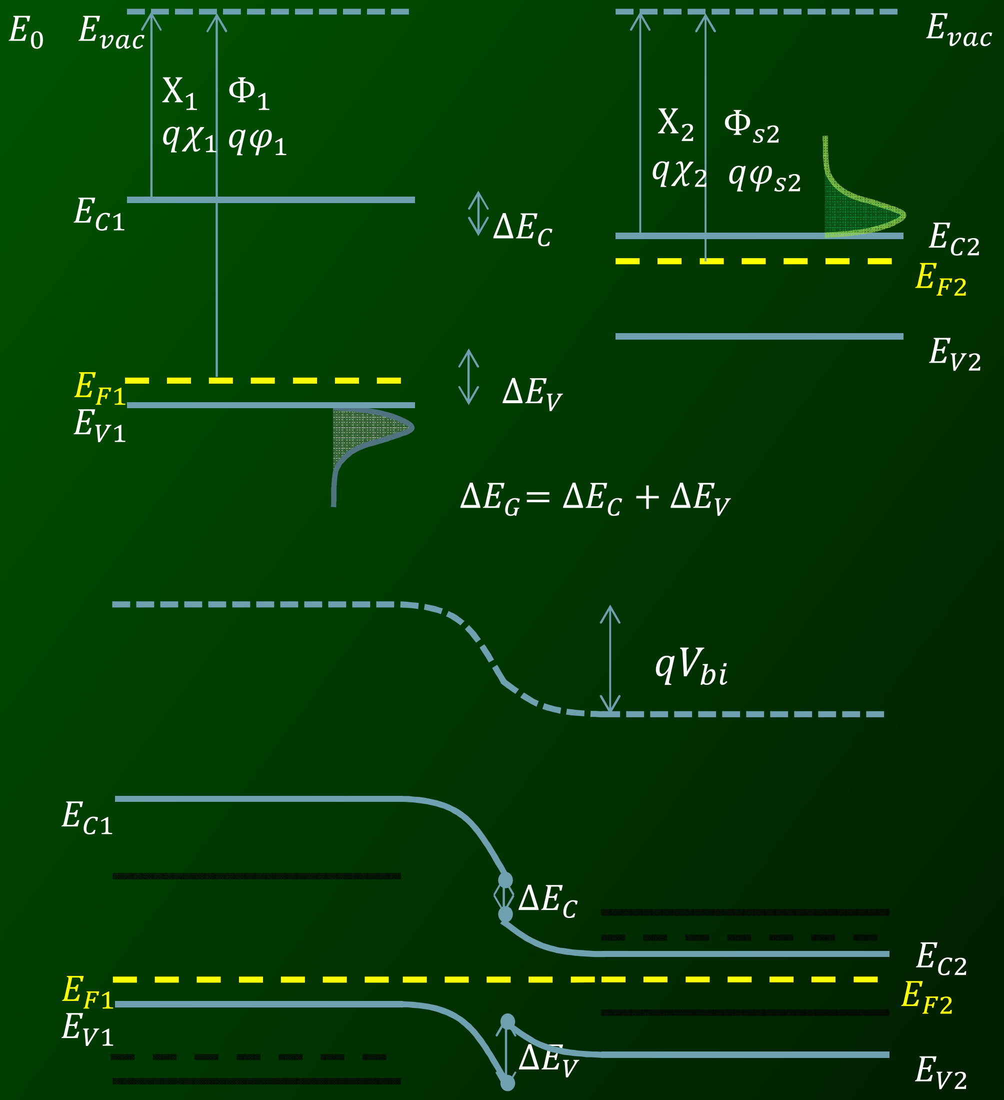
$$\varphi_F = -\frac{kT}{q}\ln\left(\frac{N_D}{n_i}\right) < 0$$



$$V_{bi} = \varphi_{Fp} - \varphi_{Fn} = \frac{kT}{q}\ln\left(\frac{N_D N_A}{n_i^2}\right)$$

$$= \frac{kT}{q}\ln\left(\frac{n_n}{n_p}\right) = \frac{kT}{q}\ln\left(\frac{p_p}{p_n}\right)$$

Built in Voltage



$$\Delta E_G = \Delta E_C + \Delta E_V$$

$$V_{bi} = \varphi_1 - \varphi_2$$

$$= \frac{\Delta E_C}{q} + \frac{(E_{C1} - E_f) - (E_{C2} - E_F)}{q}$$

$$n = N_C\, e^{-(E_C - E_F)/kT}$$

$$p = N_V\, e^{-(E_F - E_V)/kT}$$

effective density of states

$$n = n_i\, e^{(E_F - E_i)/kT}$$

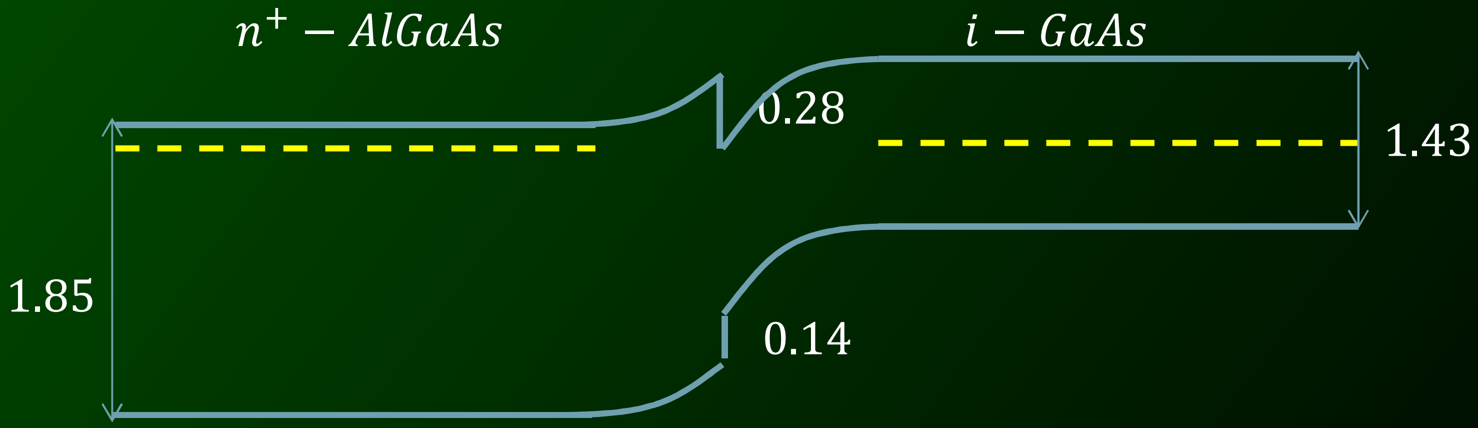
$$p = n_i\, e^{(E_i - E_F)/kT}$$

$$V_{bi} = \frac{\Delta E_C}{q} + \frac{kT}{q} \ln\left(\frac{n_{10} N_{C2}}{n_{20} N_{C1}}\right)$$

Modulation Doping

Modulation doping : free carrier concentration (within semiconductor layer) can be increased significantly without the introduction of dopant impurities.

$N_D \nearrow \Longrightarrow$ ionized impurity scattering $\nearrow \Longrightarrow$ carrier mobility $\searrow$



Typically, an undoped AlGaAs spacer layer is formed between the doped AlGaAs and undoped GaAs layers to increase the spatial separation of the electrons from the ionized donors, further reducing the ionized impurity scattering.

Free Electron

$$-\frac{\hbar^2}{2m_0}\frac{d^2\psi}{dx^2} + V(r)\psi = E\psi$$

time-independent Schrodinger equation

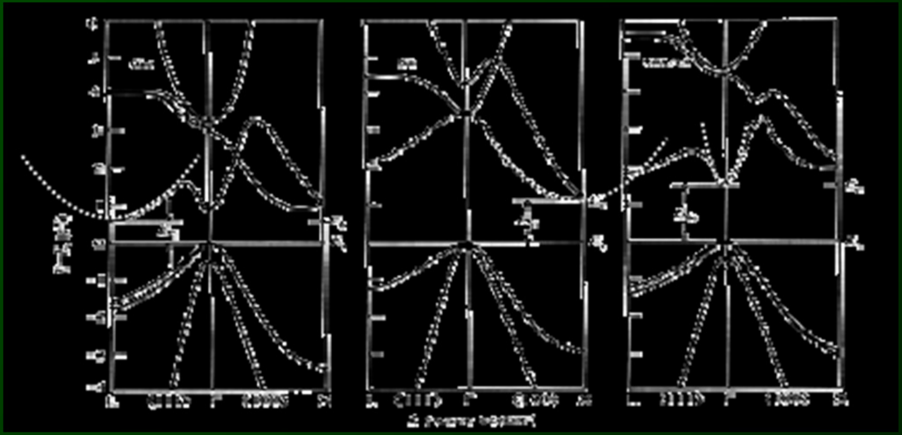
Free electron $V(x) = 0$, with energy E

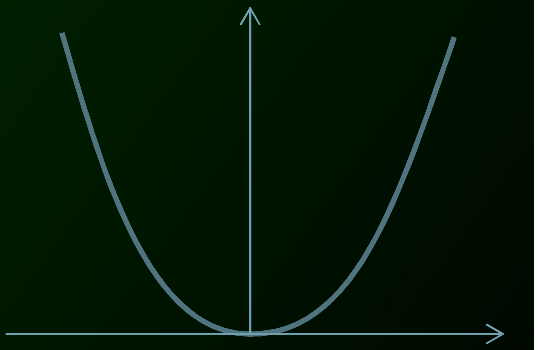
$$\frac{d^2\psi}{dx^2} = -\frac{2m_0 E}{\hbar^2}\psi = -k^2\psi$$

$$\psi = A_+ e^{-ikx} + A_- e^{+ikx}$$

$$k = \frac{\sqrt{2m_0 E}}{\hbar} \qquad E = \frac{\hbar^2 k^2}{2m_0}$$

$$\Psi(x,t) = A_+ e^{-i(kx-\omega t)} + A_- e^{+i(kx-\omega t)}$$

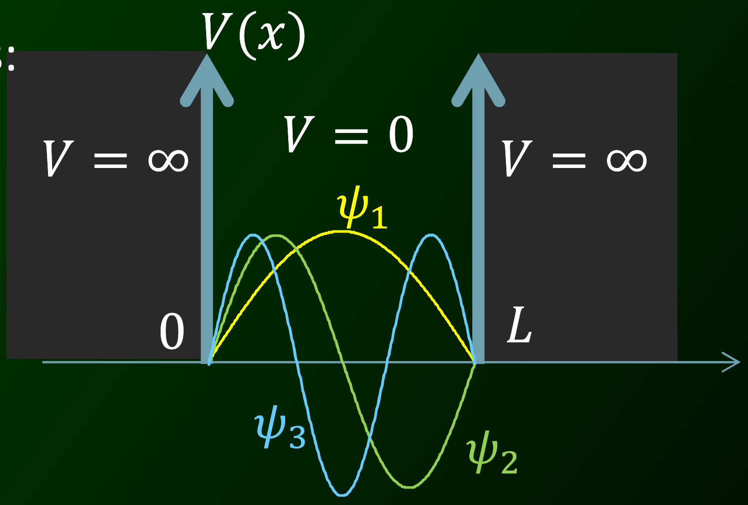




$$E = \frac{\hbar^2 k^2}{2m_0} \quad \rightarrow \quad m^* = \frac{\hbar^2}{d^2E/dk^2}$$

1-D Quantum Well (Box)

Consider a particle with mass m under potential as:



Outside the box: $V = \infty \rightarrow \psi = 0$

Inside the box: $\dfrac{d^2\psi}{dx^2} + \dfrac{2mE}{\hbar^2}\psi = 0$

$\sin kx$, $\cos kx$ as $k = \sqrt{2mE}/\hbar$

continuity of ψ at 0 and L:

n is the quantum number

$$\psi(0) = \psi(L) = 0 \rightarrow \psi = A \sin kx \quad ; \quad k = \frac{n\pi}{L} \ , n = 1,2,3,\cdots$$

$$\frac{n\pi}{L} = \frac{\sqrt{2mE}}{\hbar} \rightarrow E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

normalization:

$$\int_{-\infty}^{+\infty} \psi^* \psi dx = \int_0^L A^2 \left(\sin \tfrac{n\pi x}{L}\right)^2 dx = 1 \rightarrow A = \sqrt{2/L}$$

$$\psi_n = \sqrt{\tfrac{2}{L}} \sin(\tfrac{n\pi}{L} x)$$

1-D Quantum Well (Box)

1. I
2.
3.
4.
5.

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

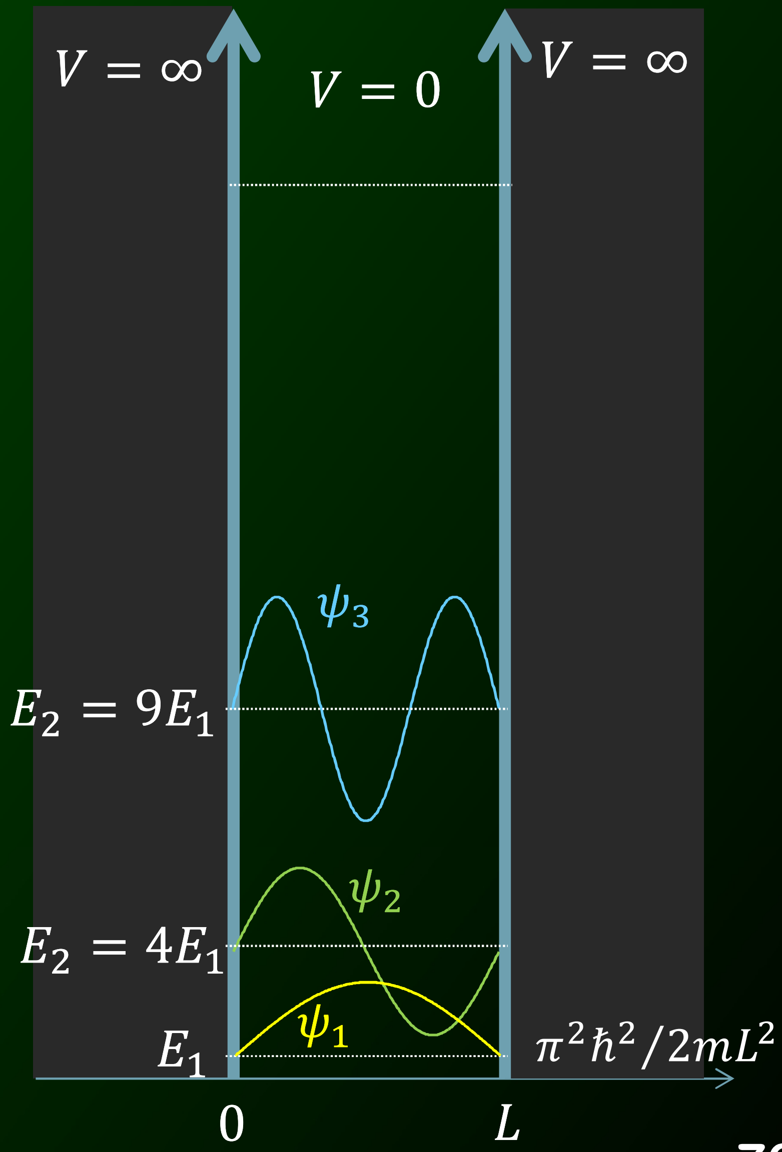
$$\psi_n = \sqrt{\frac{2}{L}}\sin(\frac{n\pi}{L}x)$$

$L \nearrow \infty$ → free electron

$L{\sim}0.5\ nm$ (atom)

$E_1 = 1.5\ eV$

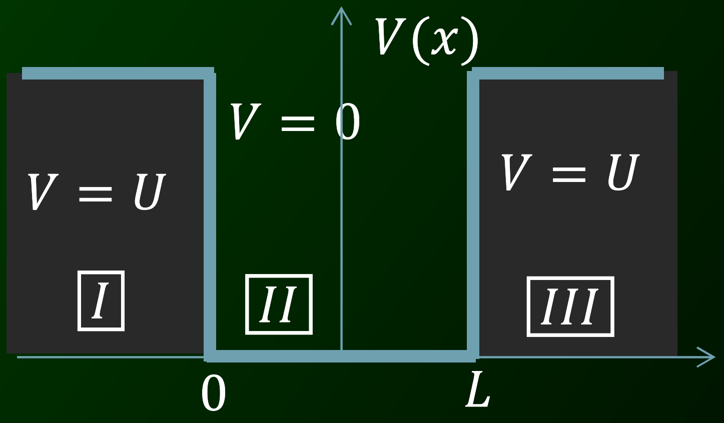
$E_2 - E_1 = 4.5\ eV$



1-D Finite Well

We first need to find the values of the energy for which there are solutions to the Schrödinger equation, then deduce the corresponding wavefunctions.

Boundary conditions are given by continuity of the wavefunction and its first derivative.



assume $E < U$

Region $\boxed{II}$

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi = -k^2\psi$$

$$k = \frac{\sqrt{2mE}}{\hbar}$$

$$\psi_{II} = F\sin kx + G\cos kx$$

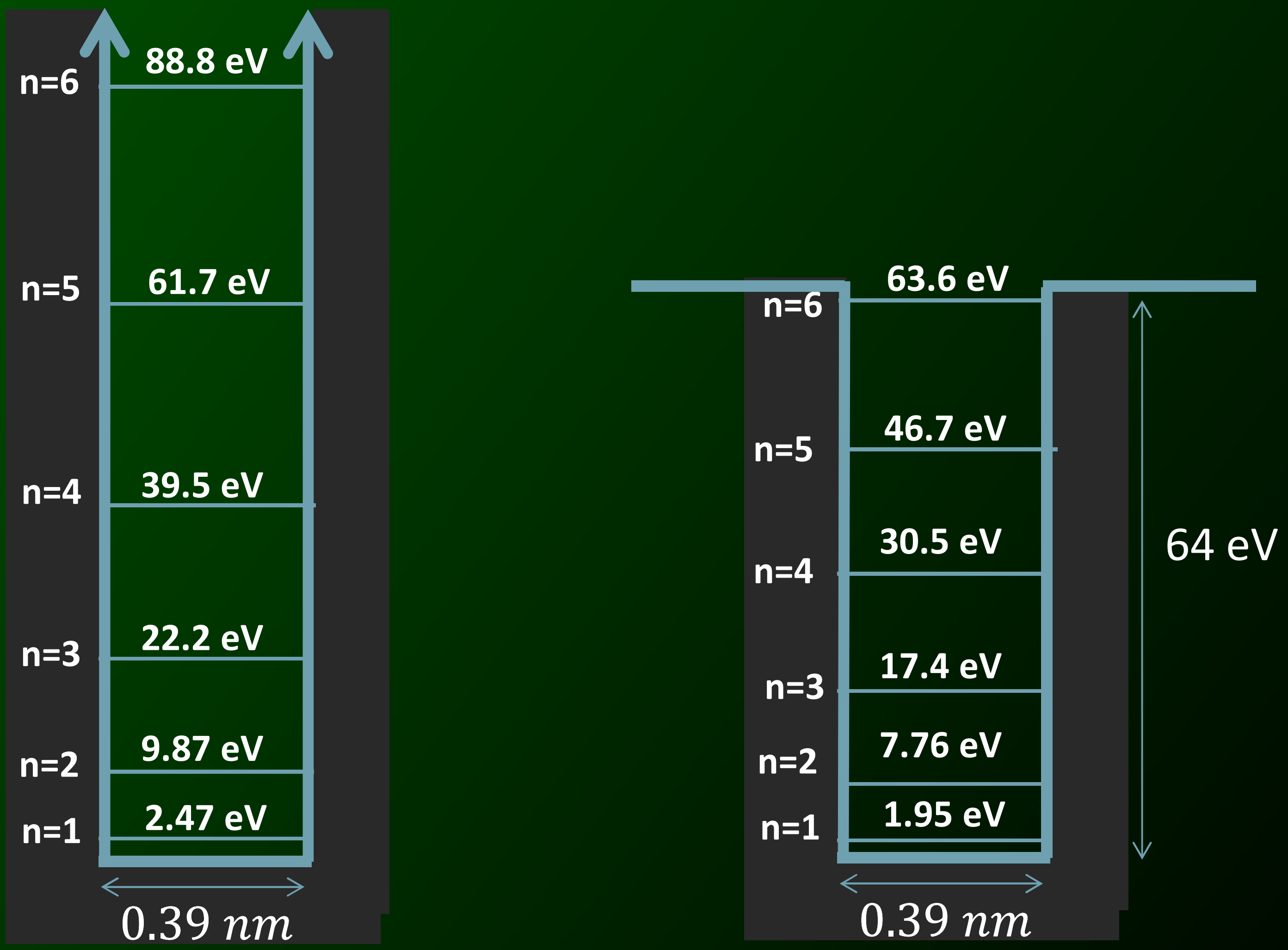
Region $\boxed{I}$ and $\boxed{III}$

$$\frac{d^2\psi}{dx^2} = \frac{2m(U-E)}{\hbar^2}\psi = \alpha^2\psi \qquad \alpha = \frac{\sqrt{2m(U-E)}}{\hbar}$$

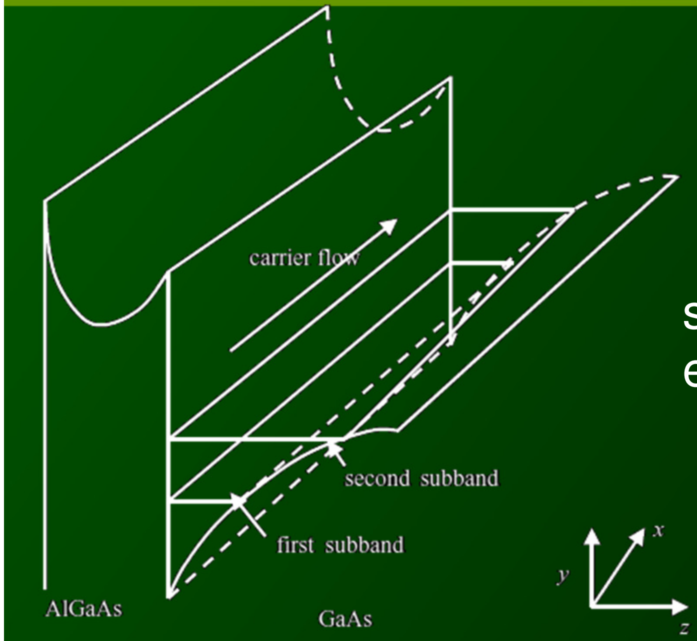
$$\Rightarrow \psi = Ae^{\alpha x} + Be^{-\alpha x}\ , \qquad x < 0 \text{ and } x > L$$

Finite ψ: $\Rightarrow \begin{cases} \psi_I = Ae^{\alpha x} & , \quad x < 0 \\ \psi_{III} = Be^{-\alpha x} & , \quad x < L \end{cases}$

Finite vs. Infinite Well



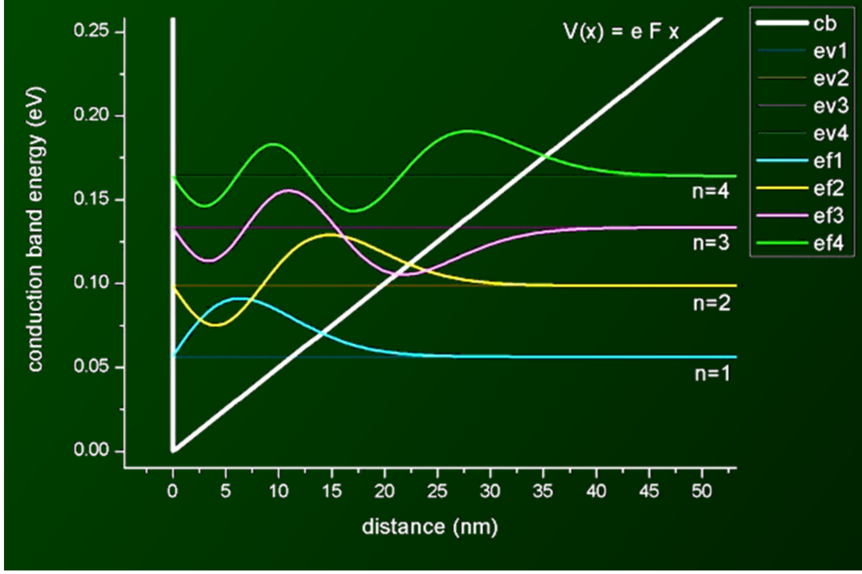
Bulk Semiconductor Potential , φ_F



$$E = \frac{\hbar^2 k_x^2}{2m_0} + \frac{\hbar^2 k_y^2}{2m_0} + E_i$$

self-consistent solution of the Schrodinger and Poisson equations

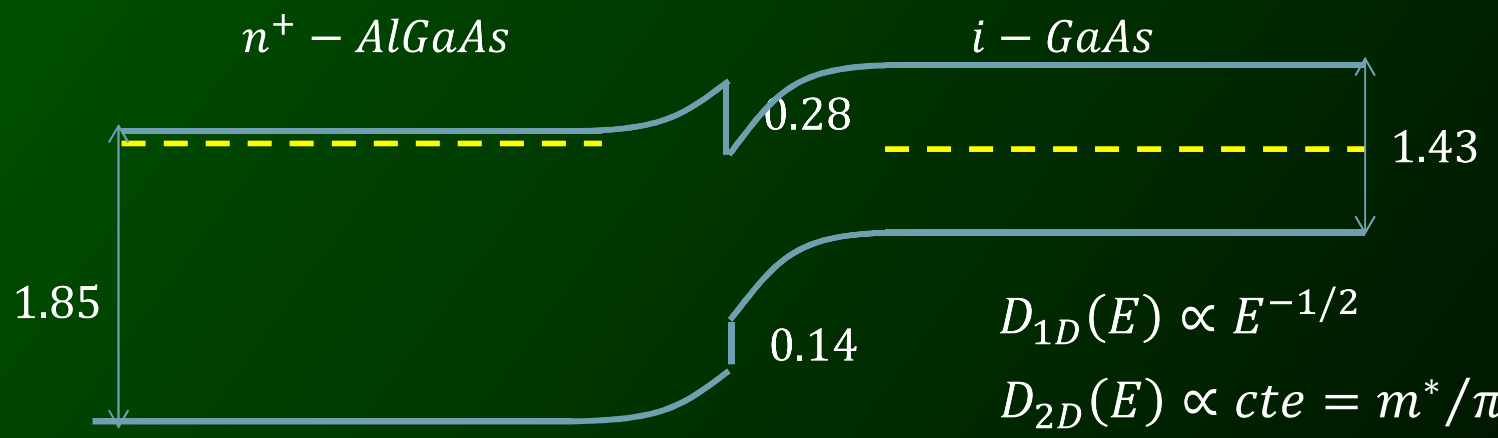
Infinite triangular potential well



$$E_i = \left(\frac{\hbar^2}{2m_0}\right)^{\frac{1}{3}} \left(\frac{3}{2}\pi q \mathcal{E}\right)^{\frac{2}{3}} \left(i + \frac{3}{4}\right)^{\frac{2}{3}}$$

$\mathcal{E}$: electric field strength corresponding to the slope of the energy band

Electron Concentration



$$D_{1D}(E) \propto E^{-1/2}$$

$$D_{2D}(E) \propto cte = m^*/\pi\hbar^2$$

$$D_{3D}(E) \propto E^{1/2}$$

$$E_F = \Delta E_C - E_d - qV_{dep}$$

$$V_{dep} = -\int_{0.}^{-W} \mathcal{E} dz = \frac{qN_D W^2}{2\epsilon_0 \epsilon_{AlGaAs}}$$

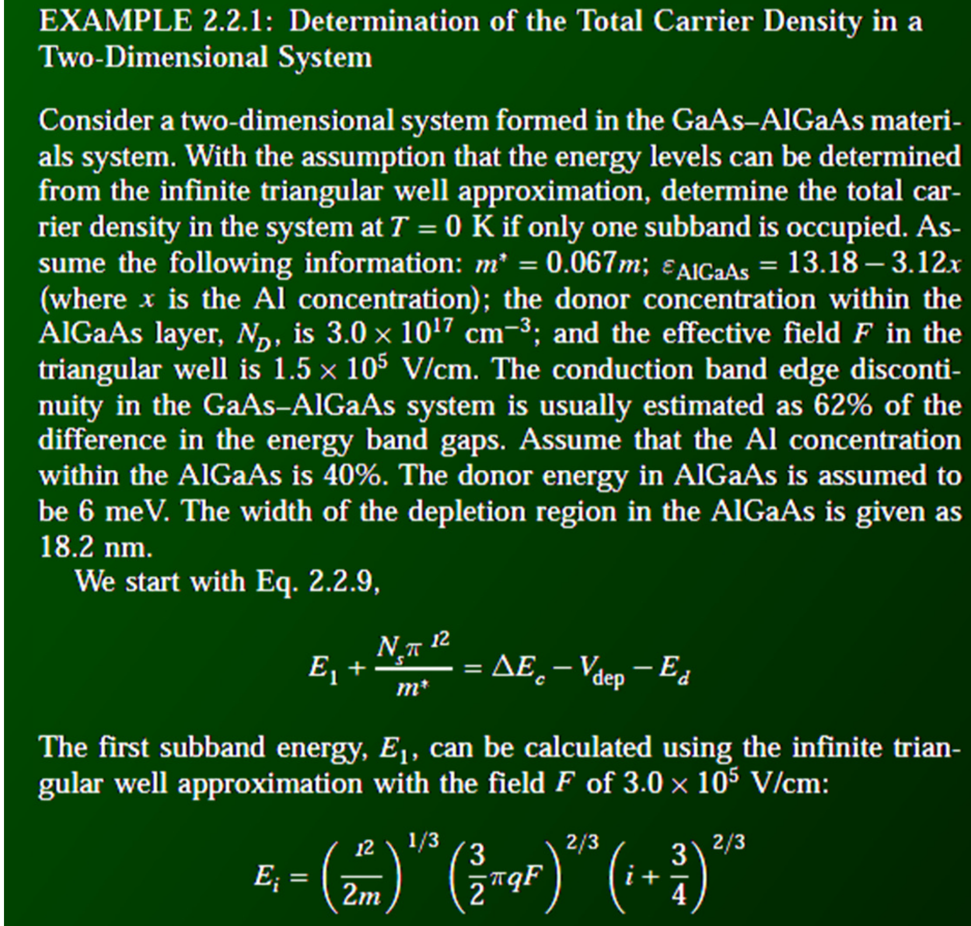
$$N_s = \int_{E_1}^{E_1+E_f} f(E) D(E) dE$$

$$E_F = \frac{\pi\hbar^2 N_S}{m^*}$$

$$E_F = E_1 + \frac{\pi\hbar^2 N_S}{m^*}$$

$$E_1 + \frac{\pi\hbar^2 N_S}{m^*} = \Delta E_C - E_d - qV_{dep}$$

Electron Concentration - Example



EXAMPLE 2.2.1: Determination of the Total Carrier Density in a Two-Dimensional System

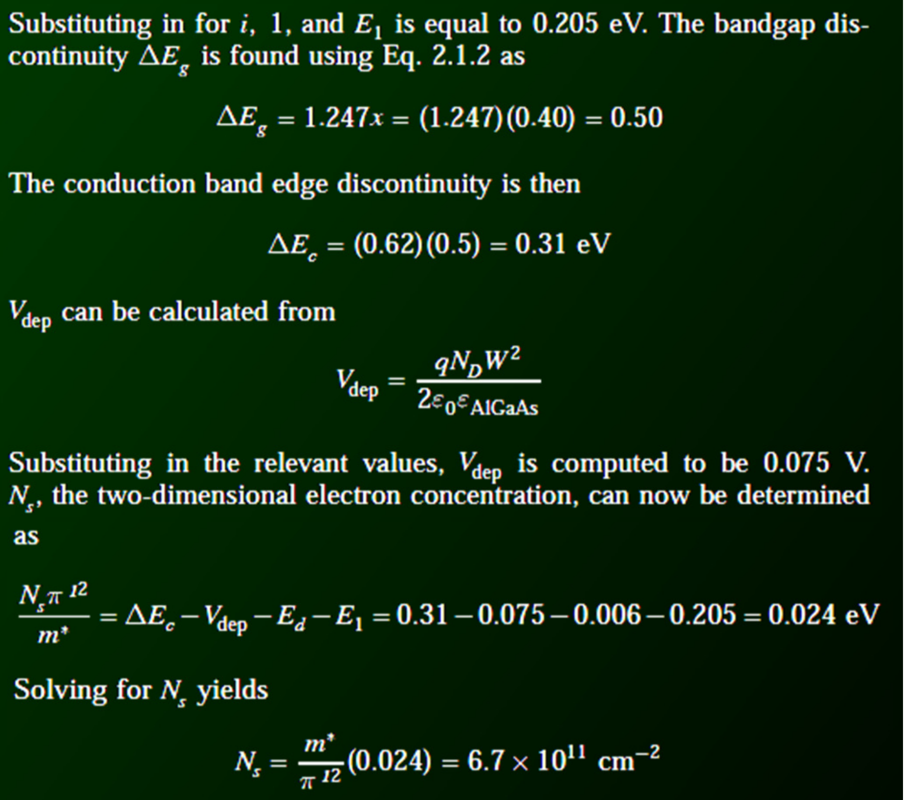
Consider a two-dimensional system formed in the GaAs–AlGaAs materials system. With the assumption that the energy levels can be determined from the infinite triangular well approximation, determine the total carrier density in the system at $T = 0$ K if only one subband is occupied. Assume the following information: $m^* = 0.067m$; $\varepsilon_{\text{AlGaAs}} = 13.18 - 3.12x$ (where x is the Al concentration); the donor concentration within the AlGaAs layer, N_D, is 3.0×10^{17} cm^{-3}; and the effective field F in the triangular well is 1.5×10^5 V/cm. The conduction band edge discontinuity in the GaAs–AlGaAs system is usually estimated as 62% of the difference in the energy band gaps. Assume that the Al concentration within the AlGaAs is 40%. The donor energy in AlGaAs is assumed to be 6 meV. The width of the depletion region in the AlGaAs is given as 18.2 nm.

We start with Eq. 2.2.9,

$$E_1 + \frac{N_s \pi \hbar^2}{m^*} = \Delta E_c - V_{\text{dep}} - E_d$$

The first subband energy, E_1, can be calculated using the infinite triangular well approximation with the field F of 3.0×10^5 V/cm:

$$E_i = \left(\frac{\hbar^2}{2m}\right)^{1/3} \left(\frac{3}{2}\pi q F\right)^{2/3} \left(i + \frac{3}{4}\right)^{2/3}$$



Substituting in for i, 1, and E_1 is equal to 0.205 eV. The bandgap discontinuity ΔE_g is found using Eq. 2.1.2 as

$$\Delta E_g = 1.247x = (1.247)(0.40) = 0.50$$

The conduction band edge discontinuity is then

$$\Delta E_c = (0.62)(0.5) = 0.31 \text{ eV}$$

V_{dep} can be calculated from

$$V_{\text{dep}} = \frac{q N_D W^2}{2\varepsilon_0 \varepsilon_{\text{AlGaAs}}}$$

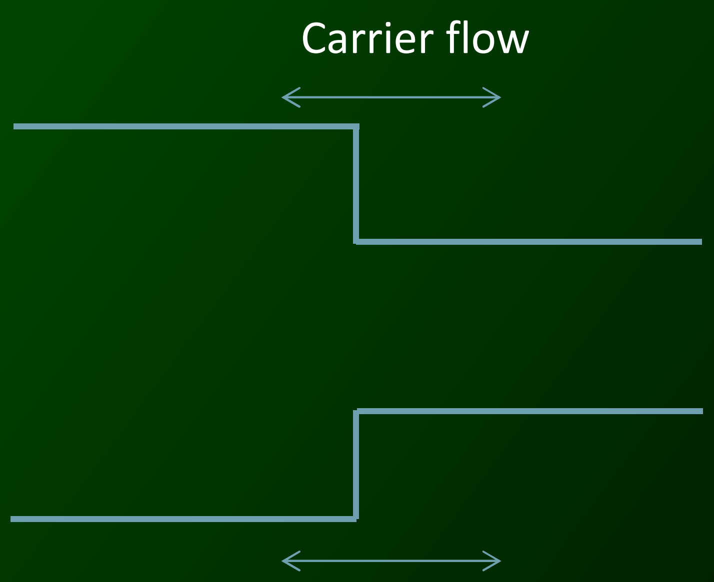
Substituting in the relevant values, V_{dep} is computed to be 0.075 V. N_s, the two-dimensional electron concentration, can now be determined as

$$\frac{N_s \pi \hbar^2}{m^*} = \Delta E_c - V_{\text{dep}} - E_d - E_1 = 0.31 - 0.075 - 0.006 - 0.205 = 0.024 \text{ eV}$$

Solving for N_s yields

$$N_s = \frac{m^*}{\pi \hbar^2}(0.024) = 6.7 \times 10^{11} \text{ cm}^{-2}$$

carrier flow perpendicula to the heterointerface



Carrier flow in a heterostructure from the narrow gap semiconductor towards the wider gap semiconductor. In this case, the carriers encounter a potential barrier at the interface that arises from the band edge discontinuity. The electrons can overcome the barrier and enter the wide gap material provided they have sufficient kinetic energy. (b) Carrier flow in a heterostructure from the wide gap semiconductor towards the narrow gap semiconductor. In this case, the carriers gain energy from crossing the potential step.

Potential Well

Region $\boxed{I}$ $$\frac{-\hbar^2}{2m}\frac{d^2\psi_I}{dx^2} = E\psi_I$$

$$\frac{d^2\psi_I}{dx^2} + k^2\psi_I = 0 \quad \text{where} \quad k = \sqrt{2mE}/\hbar$$

Region $\boxed{II}$ $$\frac{-\hbar^2}{2m}\frac{d^2\psi_{II}}{dx^2} + V_0\psi_{II} = E\psi_{II}$$

$$\frac{d^2\psi_{II}}{dx^2} - \alpha^2\psi_{II} = 0 \quad \text{where} \quad \alpha = \sqrt{2m(E - V_0)}/\hbar$$

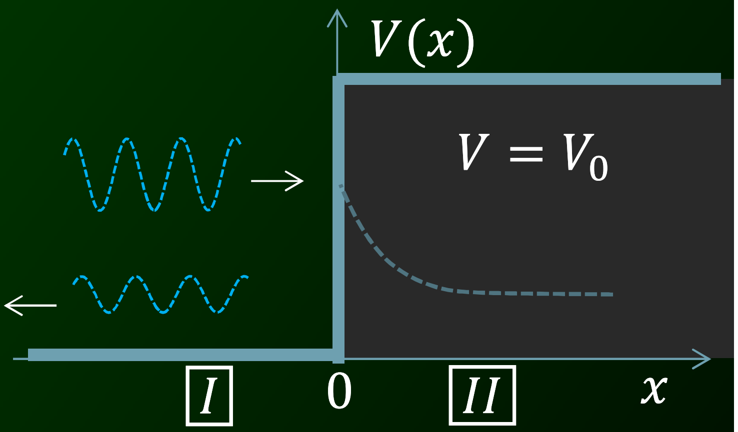
$$\psi_I = Ae^{ikx} + Be^{-ikx} \qquad (x < 0)$$

$$\psi_{II} = Ce^{\alpha x} + De^{-\alpha x} \qquad (x > 0)$$

B.C. → A, B, D

$$\Psi_I(x,t) = Ae^{i(kx - Et/\hbar)} + Be^{i(kx + Et/\hbar)}$$

$$\Psi_{II}(x,t) = De^{-\alpha x - iEt/\hbar}$$



$$\frac{D}{A} = 2\frac{E - i\sqrt{(V_0 - E)E}}{V_0}$$

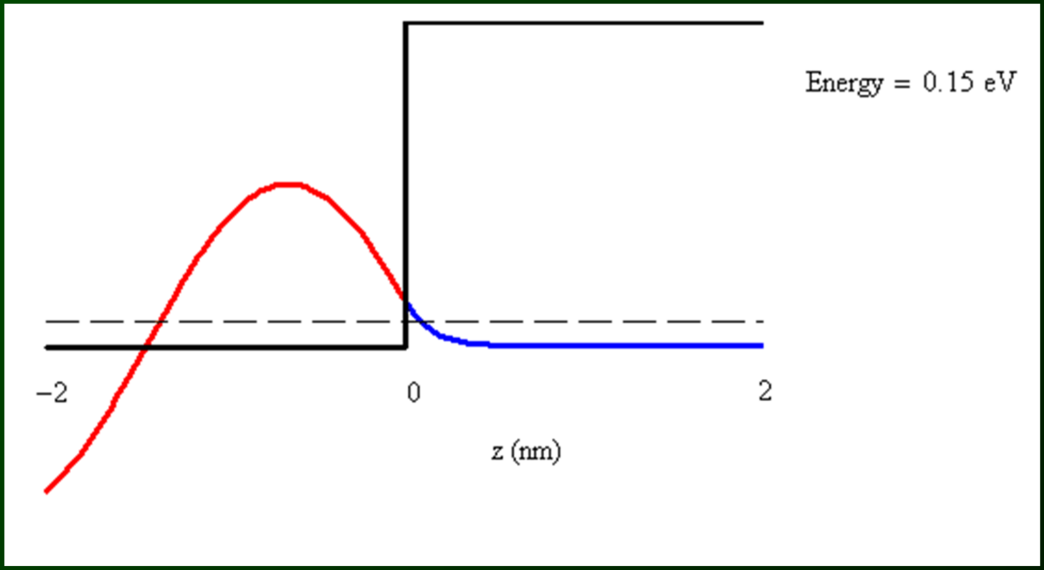
$$\frac{B}{A} = \frac{2E - V_0 - 2i\sqrt{(V_0 - E)E}}{V_0}$$

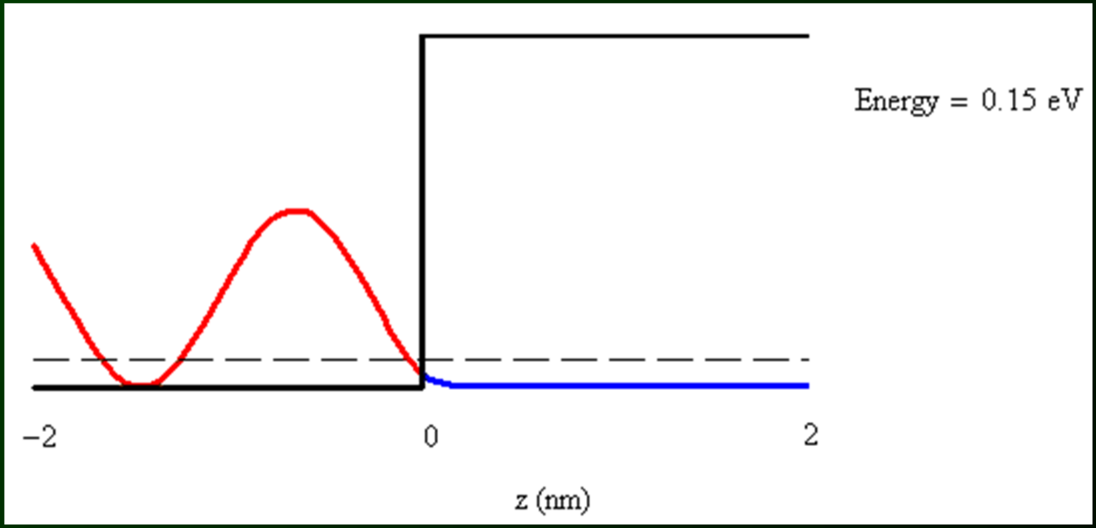
$$E = 1eV, V_0 = 2\ eV$$

$$\rightarrow \frac{1}{\alpha} = 0.2nm$$

penetration depth

Potential Well





Bulk Semiconductor Potential , φ_F

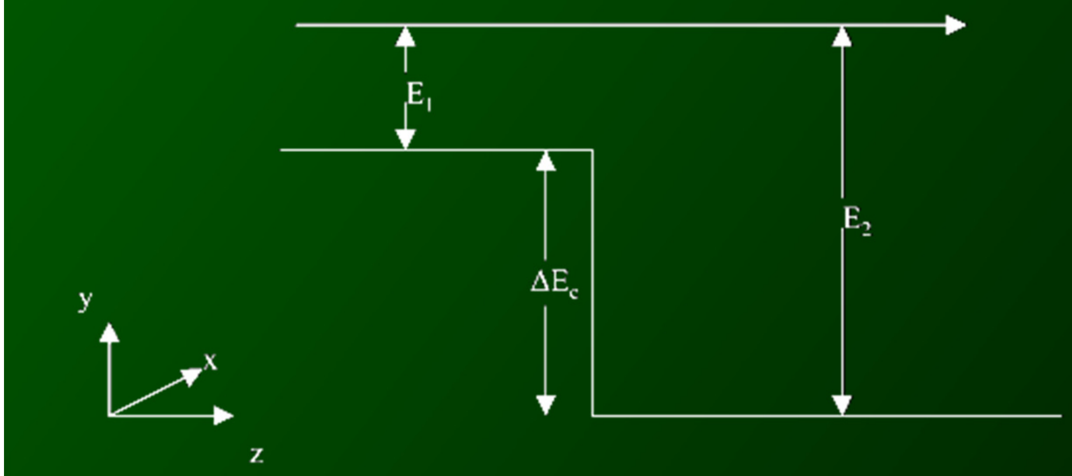


FIGURE 2.5.2 Heterostructure system for the above-the-barrier injection condition.

$$\frac{\hbar^2(k_{1x}^2+k_{2z}^2)}{2m_2}=\frac{\hbar^2(k_{1x}^2+k_{1z}^2)}{2m_1}+\Delta E_C$$

$$T=\frac{4(k_{2z}/k_{1z})(m_1/m_2)}{[1+(k_{2z}/k_{1z})(m_1/m_2)]^2}$$

$$R=\frac{[1-(k_{2z}/k_{1z})(m_1/m_2)]^2}{[1+(k_{2z}/k_{1z})(m_1/m_2)]^2}$$

$$\frac{\hbar^2k_2^2}{2m_2}=\frac{\hbar^2k_1^2}{2m_1}+\Delta E_C$$

$$k_{1x}=k_{2x}$$

$$k_{1y}=k_{2y}$$

For simplicity, let us assume that the electron is confined completely to the x–z plane

Solving for k_{2z}

Tunneling

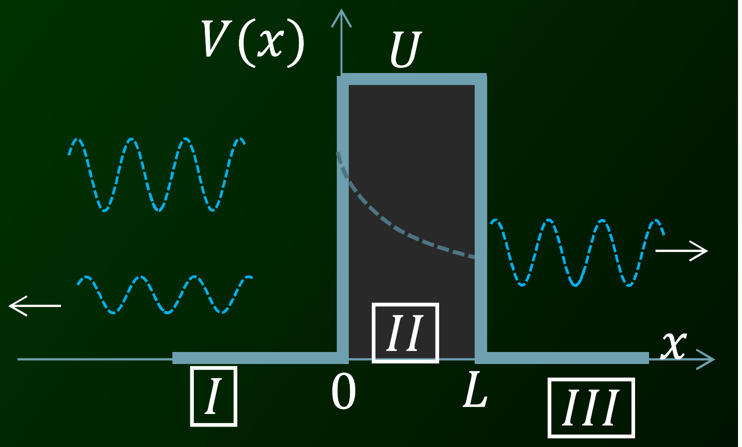
1. I [illegible]
2. [illegible]
3. [illegible]
4. [illegible]
5. [illegible]

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U\psi = E\psi$$

$$\begin{cases} \psi_I = e^{ikx} + re^{-ikx} & (x < 0) \\ \psi_{II} = Ae^{\alpha x} + Be^{-\alpha x} & (0 < x < L) \\ \psi_{III} = te^{ikx} & (x > L) \end{cases}$$

$$k = \sqrt{2mE}/\hbar \qquad \alpha = \sqrt{2m(U-E)}/\hbar$$

$$\begin{cases} \psi_I = \psi_{II} \ @ \ x = 0 \\ \dfrac{d\psi_I}{dx} = \dfrac{d\psi_{II}}{dx} \ @ \ x = 0 \\ \psi_{II} = \psi_{III} \ @ \ x = L \\ \dfrac{d\psi_{II}}{dx} = \dfrac{d\psi_{III}}{dx} \ @ \ x = L \end{cases} \Rightarrow \begin{cases} A \\ B \\ t \longrightarrow T = |t|^2 \\ r \longrightarrow R = |r|^2 \end{cases}$$

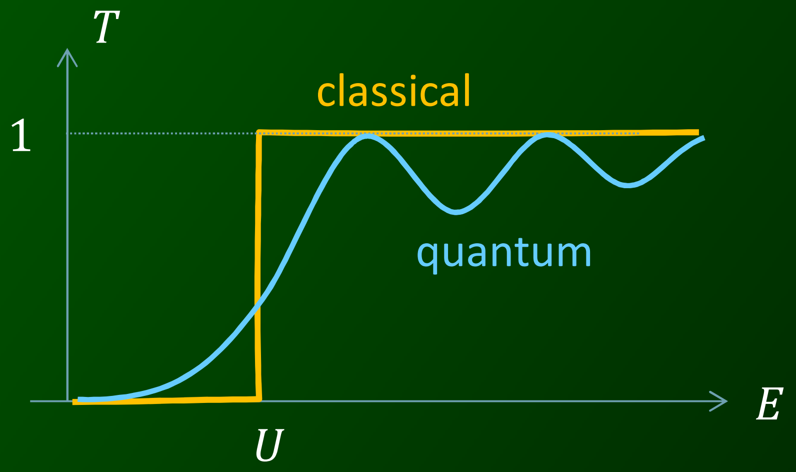


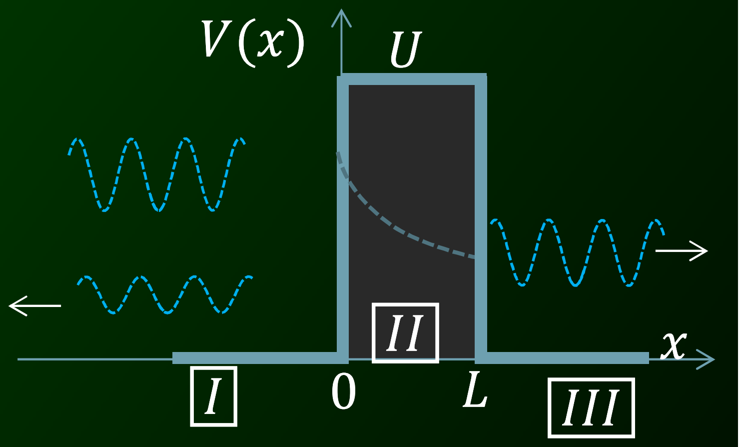
- Transmission coefficient (T): The probability that the particle penetrates the barrier.

- Reflection coefficient (R): The probability that the particle is reflected by the barrier.

- T + R = 1

Tunneling





$$T = \left[1 + \frac{U^2}{4E(E-U)} \sin^2 \alpha L\right]^{-1} \qquad \alpha = \sqrt{2m(U-E)}/\hbar$$

For $U = E$ $$T = \left[1 + \frac{E}{2}\frac{mL^2}{\hbar^2}\right]^{-1}$$

For $U \gg E$ $$T \sim \exp[-\frac{2L}{\hbar}\sqrt{2m(U-E)}]$$

Material Properties

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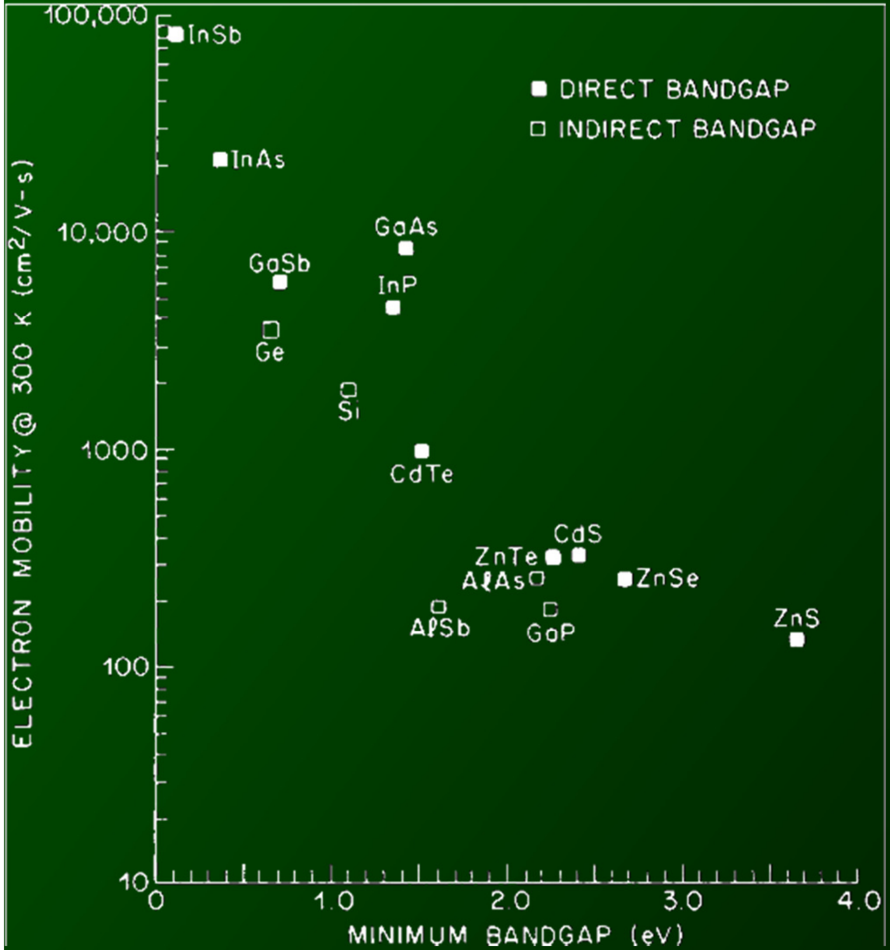

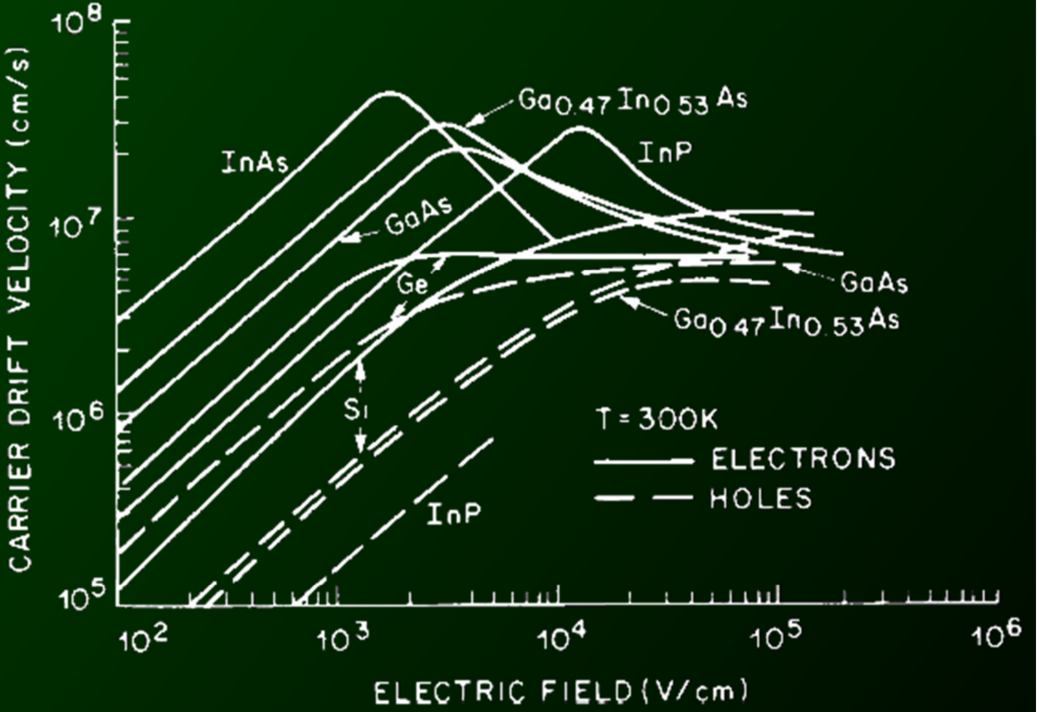

Bulk Semiconductor Potential , φ_F

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Bulk Semiconductor Potential , φ_F

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Bulk Semiconductor Potential , φ_F

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