

ECE606: Solid State Devices

Lecture 9

Recombination Processes and Rates

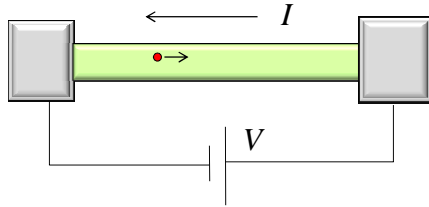
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- 1) **Non-equilibrium systems**
- 2) Recombination generation events
- 3) Steady-state and transient response
- 4) Motivation of R-G formula
- 5) Conclusion

Ref. Chapter 5, pp. 134-146





$$I = G \times V$$

$$= q \times n \times v \times A$$

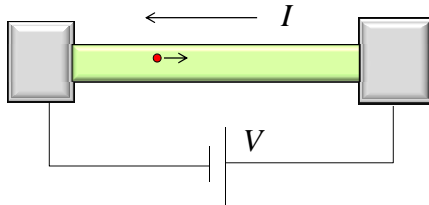
↑ Carrier Density ↑ velocity

Depends on chemical composition, crystal structure, temperature, doping, etc.

Quantum Mechanics + Equilibrium Statistical Mechanics
 ⇒ Encapsulated into concepts of effective masses and occupation factors (Ch. 1-4)

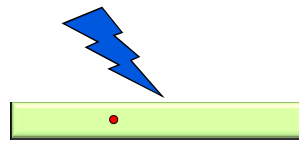
Transport with scattering, non-equilibrium Statistical Mechanics
 ⇒ Encapsulated into drift-diffusion equation with recombination-generation (Ch. 5 & 6)

Chapter 6

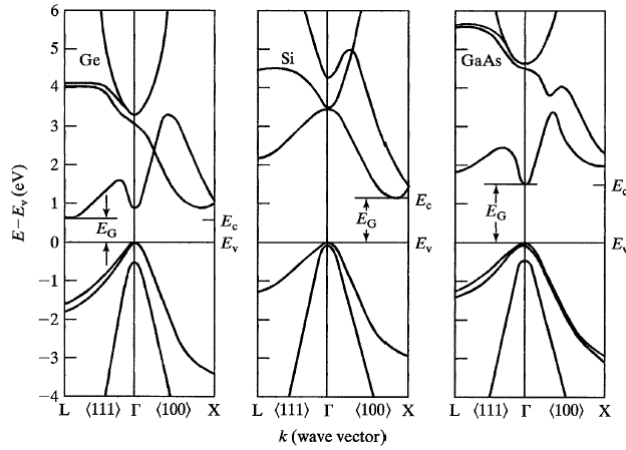


vs.

Chapter 5

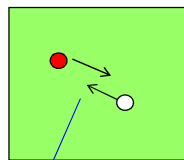


How does the system go BACK to equilibrium?



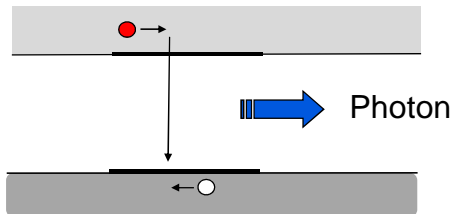
The top & bottom of bands do not align at same wavevector k for indirect bandgap material

In real space ...



Photon
 e and h must have same wavelength
 1 in 1,000,000 encounters

In energy space ...

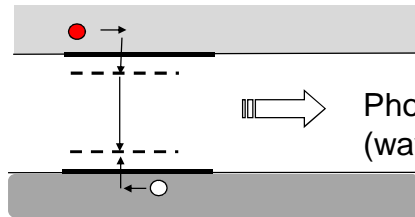


Direct transition – direct gap material

GaAs, InP, InSb (3D)

Lasers, LEDs, etc.

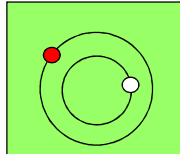
In energy space ...



Mostly in 1D systems
Requires strong
coulomb interactions

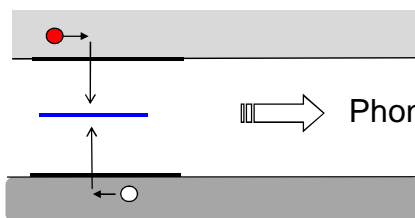
Photon
(wavelength reduced from bulk)

In real space ...



CNT, InP, ID-systems

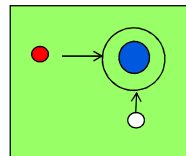
Transistors, Lasers, Solar cells, etc.



Trap needs to
be mid-gap to
be effective.
Cu or Au in Si

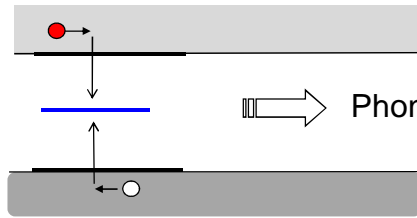
States close to
Ec or Ev do not
help efficiently.
Ge, Si,

Transistors, Solar cells, etc.



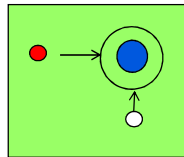
Cu in Si is extremely efficient in providing recombination
Cu in Si was avoided completely for many years
Intel introduced Cu for interconnects – special
precautions so Cu does not enter the Si.

Indirect Recombination (Trap-assisted)



Trap needs to be mid-gap to be effective.
Cu or Au in Si

States close to E_c or E_v do not



Recombination rate is in general slower than direct via photon.

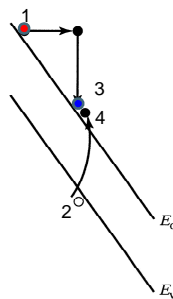
Why?

1 electron and one hole need to localize AND find a trap... less likely event.

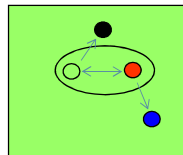
Cu in Si is extremely efficient in providing recombination
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Intel introduced Cu for interconnects – special precautions so Cu does not enter the Si.

Impact Ionization - A Generation Mechanism

Requires very high electric field

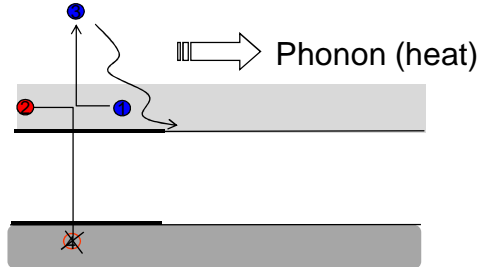


Inverse of the Auger recombination

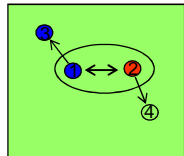


Si, Ge, InP

Lasers, Transistors, etc.

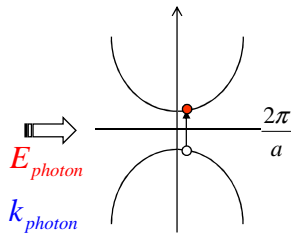


Requires very high electron density



InP, GaAs, ...

Lasers, etc.



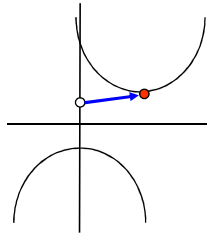
$$E_V + E_{\text{photon}} = E_C$$

$$\hbar k_V + \hbar k_{\text{photon}} = \hbar k_C$$

$$k_{\text{photon}} = \frac{2\pi}{\lambda \text{ in } \mu\text{m}} = \frac{2\pi}{1.21/E_{\text{photon}} \text{ in eV}}$$

$$\ll \frac{2\pi}{a} = \frac{2\pi}{5 \times 10^{-4} \mu\text{m}}$$

Photon has large energy for excitation through bandgap, but its wavevector is negligible compared to size of BZ



$$E_V + E_{\text{phonon}} = E_C$$

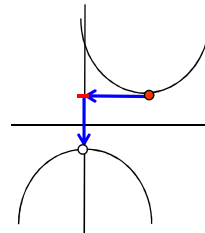
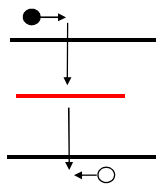
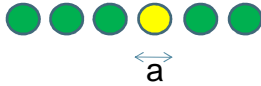
$$\hbar k_V + \hbar k_{\text{phonon}} = \hbar k_C$$

$$v_{\text{sound}} \sim 10^3 \text{ m/s} \ll v_{\text{light}} = c \sim 10^8 \text{ m/s}$$

$$\lambda_{\text{sound}} \gg \lambda_{\text{light}}$$

$$k_{\text{phonon}} = \frac{2\pi}{\lambda} = \frac{2\pi}{\hbar v_{\text{sound}} / E_{\text{phonon}}} \approx \frac{2\pi}{a} = \frac{2\pi}{5 \times 10^{-4} \text{ } \mu\text{m}}$$

Phonon has large wavevector comparable to BZ,
but negligible energy compared to bandgap

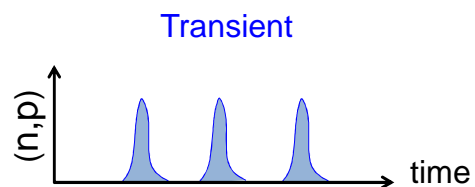
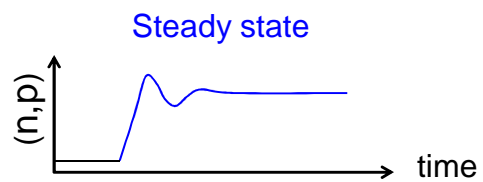
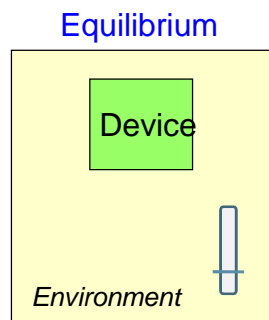


$$k_{\text{trap}} \approx \frac{2\pi}{a} \sim \frac{2\pi}{5 \times 10^{-4} \text{ } \mu\text{m}}$$

Trap provides the wavevector
necessary for indirect transition

- 1) Non-equilibrium systems
- 2) Recombination generation events
- 3) Steady-state and transient response**
- 4) Derivation of R-G formula
- 5) Conclusion

Ref. Chapter 5, pp. 134-146



Equilibrium is a very active place

In a typical semiconductor device there are $1e17$ to $1e20$ electrons in the conduction band. All electrons carry charge and are occupying their respective DOS.

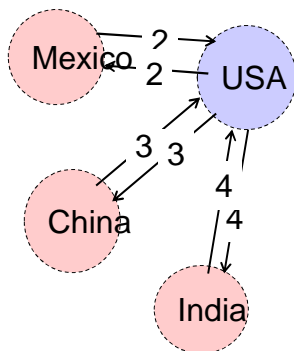
Fermi-Dirac distribution demands exploration of allowed states

In equilibrium each process is balanced by its counter process

=> Detailed Balance

=> Externally it looks as if nothing is happening.

This is different from steady state!!!



The rates of exchange of people (particles) between every pair of countries (energy levels) is balanced. Hence the name "Detailed Balance".

Detailed balance is the property of equilibrium

The population of each of the countries (energy levels) remains constant under detailed balance.

The concept of detailed balance is powerful, because it can be used for many things (e.g. reduce the number of unknown rate constants by half, and derive particle distributions like Fermi-Dirac, Bose-Einstein distributions, etc.)

Equilibrium is a very active place

- 9 in & 9 out
- All numbers are people/unit time.

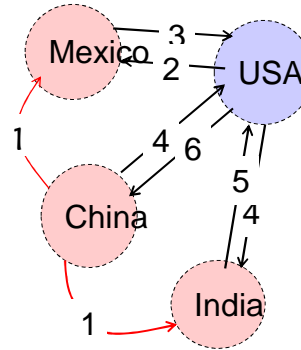
Fermi-Dirac distribution demands exploration of allowed states

Disturbing the detailed balance requires non-equilibrium conditions (needs energy). Unidirectional forces (red lines) can create such Non-equilibrium conditions.

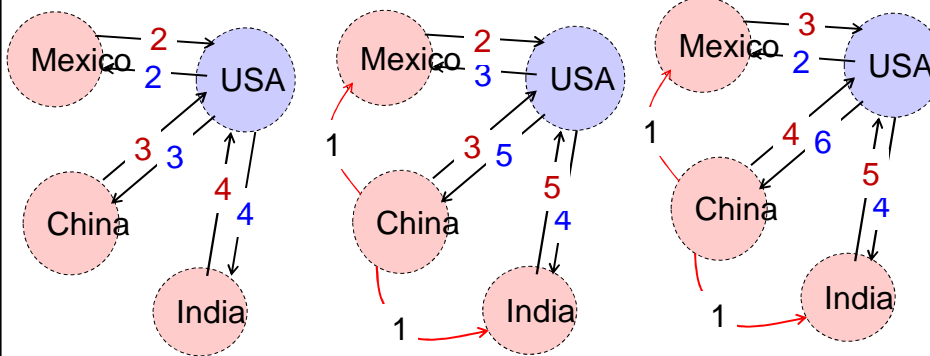
The rates of exchange of people (particles) between every pair of countries (energy levels) is NOT balanced, but the sum of all arrival and departures to all countries is zero.

The flux at steady state is balanced overall, but the flux is NOT the same as in detailed balance (e.g. 12 in and 12 out in SS vs. 9 in and 9 out for Detailed Balance, for example).

The population of a country (energy level) remains constant with time after steady state is reached.



One can use the requirement that net flux at steady state be zero to calculate steady state population of a country (Eq. 5.21)



9 in 9 out
Population conserved

Equilibrium =
Detailed balance

Forced unidirectional connections (red lines) disturbs equilibrium (e.g. 10 in/12 out at time t1 local populations not conserved, but global population is

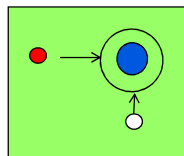
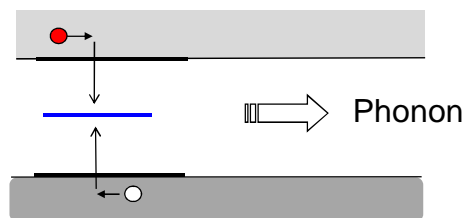
Transient populations

12 in 12 out
Population stabilized

Steady State
But NOT Equilibrium

- 1) Non-equilibrium systems
- 2) Recombination generation events
- 3) Steady-state and transient response
- 4) Motivation of R-G formula**
- 5) Conclusion

Ref. Chapter 5, pp. 134-146



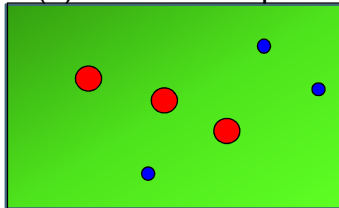
Ge, Si,

Transistors, Solar cells, etc.

total traps electron-filled traps empty traps

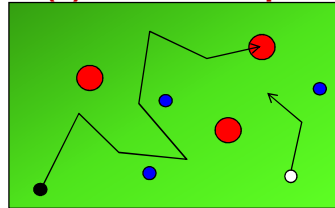
$$N_T = n_T + p_T$$

Traps have destroyed one electron-hole pair
No change in n_T and p_T
(3) After hole capture

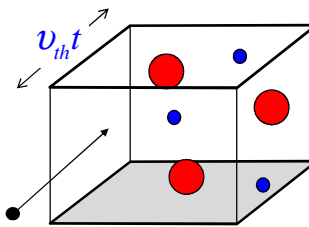
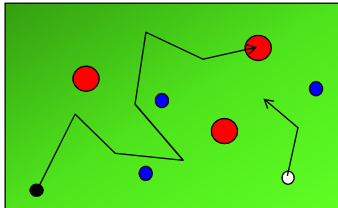
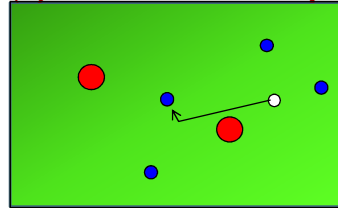


- electron
- hole
- Crystal / atoms with vibrations

(1) Before a capture



(2) After electron capture



$$\frac{1}{2} m^* v_{th}^2 = \frac{3}{2} kT$$

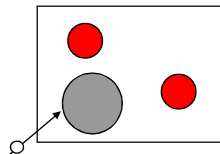
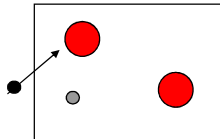
$$v_{th} \approx 10^7 \frac{cm}{s}$$

$$\frac{dn}{dt} = -n \times \left[\frac{Volume \times p_T \times RelArea}{TotalArea \times t} \right]$$

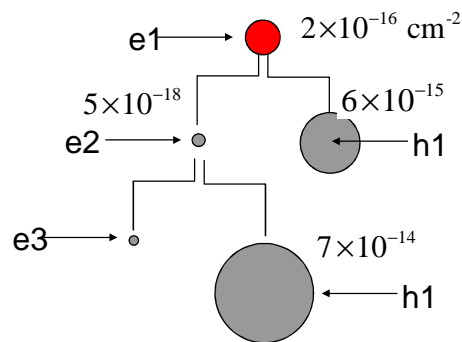
$$\frac{dn}{dt} = -n \times \left[\frac{A \times v_{th} t \times p_T \times \sigma_n}{A \times t} \right]$$

$$\equiv -c_n p_T n \quad c_n \equiv \sigma_n v_{th}$$

$$\sigma_n = \pi r_0^2$$



Zn capture model ...

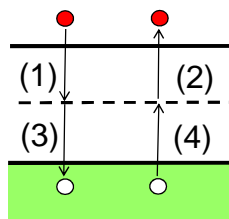


Cascade model for capture

- 1) There are wide variety of generation-recombination events that allow restoration of equilibrium once the stimulus is removed.
- 2) Direct recombination is photon-assisted, indirect recombination phonon assisted.
- 3) Concepts of equilibrium, steady state, and transient dynamics should be clearly understood.

- 1) **Derivation of SRH formula**
- 2) Application of SRH formula for special cases
- 3) Direct and Auger recombination
- 4) Conclusion

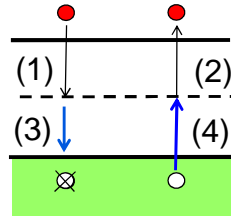
Ref. ADF, Chapter 5, pp. 141-154



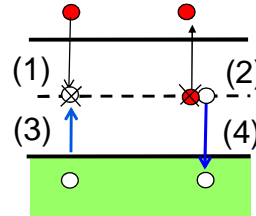
(1)+(3): one electron reduced from Conduction-band & one-hole reduced from valence-band

(2)+(4): one hole created in valence band and one electron created in conduction band

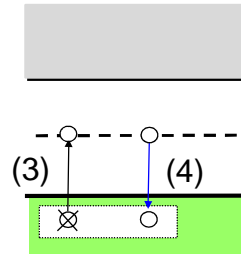
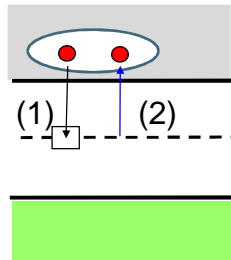
Physical picture



Equivalent picture

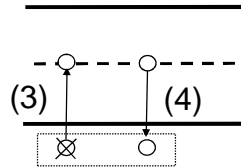
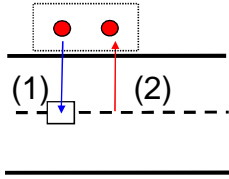


- (1)+(3): one electron reduced from C-band & one-hole reduced from valence-band
- (2)+(4): one hole created in valence band & one electron created in conduction band



$$\left. \frac{\partial n}{\partial t} \right|_{1,2} = -c_n n p_T + e_n n_T (1 - f_c)$$

$$\left. \frac{\partial p}{\partial t} \right|_{3,4} = -c_p p n_T + e_p p_T f_v$$



$(1 - f_c) \approx 1$ Assume non-degenerate

$$\left. \frac{\partial n}{\partial t} \right|_{1,2} = -c_n n_0 p_{T0} + e_n n_{T0}$$

$$0 = -c_n n_0 p_{T0} + e_n n_{T0}$$

$$e_n = c_n \frac{n_0 p_{T0}}{n_{T0}} \equiv c_n n_1$$

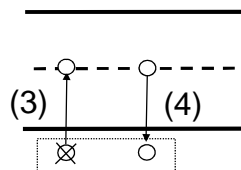
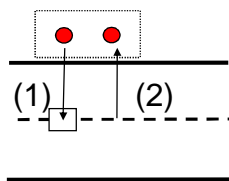
$$0 = -c_n (n_0 p_{T0} - n_{T0} n_1)$$

$$\left. \frac{\partial p}{\partial t} \right|_{3,4} = -c_p p n_T + e_p p_T$$

$$0 = -c_p p_0 n_{T0} + p_{T0} e_p$$

$$e_p \equiv \frac{c_p p_0 n_{T0}}{p_{T0}} = c_p p_1$$

$$0 = -c_p (p_0 n_{T0} - p_{T0} p_1)$$



$$n_1 = \frac{n_0 p_{T0}}{n_{T0}}$$

$$p_1 = \frac{p_0 n_{T0}}{p_{T0}}$$

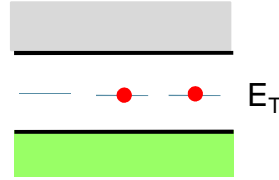
$$n_1 p_1 = \frac{n_0 p_{T0}}{n_{T0}} \times \frac{p_0 n_{T0}}{p_{T0}} = n_0 p_0 = n_i^2$$

Trap is like a donor!

$$n_{T0} = N_T (1 - f_{00}) = \frac{N_T}{1 + g_D e^{\beta(E_T - E_F)}}$$

$$(1 - f_{00}) = \frac{1}{1 + g \exp} \quad f_{00} = 1 - \frac{1}{1 + g \exp}$$

$$f_{00} = \frac{g \exp}{1 + g \exp} \quad f_{00} / (1 - f_{00}) = \frac{g \exp}{1 + g \exp} / \frac{1}{1 + g \exp} = g \exp$$



$$n_1 = \frac{n_0 p_{T0}}{n_{T0}} = n_0 \frac{(N_T f_{00})}{N_T (1 - f_{00})}$$

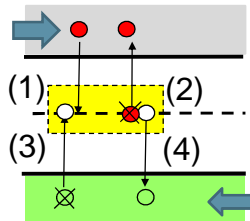
$$n_1 = n_i e^{\beta(E_F - E_i)} g_D e^{\beta(E_T - E_F)}$$

$$= n_i g_D e^{\beta(E_T - E_i)}$$

$$p_1 n_1 = n_i^2$$

$$p_1 = n_i^2 / n_1$$

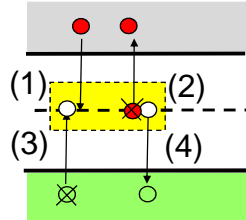
$$= n_i g_D^{-1} e^{\beta(E_i - E_T)}$$



$$\frac{\partial n_T}{\partial t} = -\frac{\partial n}{\partial t} \Big|_{1,2} + \frac{\partial p}{\partial t} \Big|_{3,4}$$

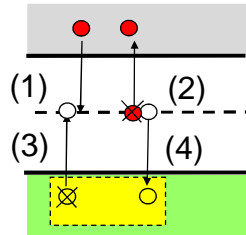
$$= c_n n p_T - e_n n_T - c_p p n_T + e_p p_T$$

$$= c_n (n p_T - n_T n_1) - c_p (p n_T - p_T p_1)$$



$$\frac{\partial n_T}{\partial t} = 0 = c_n (np_T - n_T n_1) - c_p (p n_T - p_T p_1)$$

$$n_T = \frac{c_n N_T n + c_p N_T p_1}{c_n (n + n_1) + c_p (p + p_1)} = c_n (np_T - n_T n_1)$$



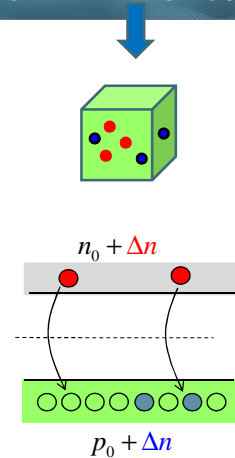
$$R = -\frac{dp}{dt} = c_p (p n_T - p_T p_1)$$

$$= \frac{np - n_i^2}{\left(\frac{1}{c_p N_T}\right)(n + n_1) + \left(\frac{1}{c_n N_T}\right)(p + p_1)}$$

τ_n τ_p

- 1) Derivation of SRH formula
- 2) **Application of SRH formula for special cases**
- 3) Direct and Auger recombination
- 4) Conclusion

$$\begin{aligned}
 R &= \frac{np - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)} \\
 &= \frac{(n_0 + \Delta n)(p_0 + \Delta n) - n_i^2}{\tau_p(n_0 + \Delta n + n_1) + \tau_n(p_0 + \Delta p + p_1)} \\
 &= \frac{\cancel{\Delta n}(n_0 + p_0) + \cancel{\Delta n}^2}{\tau_p(n_0 + \Delta n + n_1) + \tau_n(p_0 + \Delta p + p_1)} \\
 &= \frac{\Delta n(p_0)}{\tau_n(p_0)} = \frac{\Delta n}{\tau_n} \quad \Delta n^2 \approx 0 \\
 & \quad \quad \quad p_0 \gg \Delta n \gg n_0
 \end{aligned}$$



Lots of holes, few electrons => independent of holes

$$R = \frac{np - n_i^2}{\tau_p (n + n_1) + \tau_n (p + p_1)}$$

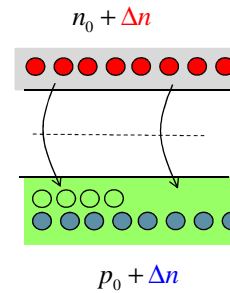
e.g. organic solar cells

$$= \frac{(n_0 + \Delta n)(p_0 + \Delta p) - n_i^2}{\tau_p (n_0 + \Delta n + n_1) + \tau_n (p_0 + \Delta p + p_1)}$$

$$= \frac{\Delta n (n_0 + p_0) + \Delta n^2}{\tau_p (n_0 + \Delta n + n_1) + \tau_n (p_0 + \Delta n + p_1)}$$

$$= \frac{\Delta n^2}{(\tau_n + \tau_p) \Delta n} = \frac{\Delta n}{(\tau_n + \tau_p)}$$

$$\Delta n \gg p_0 \gg n_0$$



Lots of holes, lots of electrons => dependent on both relaxations

$$R_{high} = \frac{\Delta n}{(\tau_n + \tau_p)}$$

$$\Delta n \gg p_0 \gg n_0$$

$$R_{low} = \frac{\Delta n}{\tau_p}$$

$$p_0 \gg \Delta n \gg n_0$$

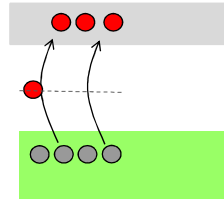
which one is larger and why?

Depletion region – in PN diode: $n=p=0$

$$n \ll n_1 \quad p \ll p_1$$

$$R = \frac{\cancel{np} - n_i^2}{\tau_p (\cancel{n} + n_1) + \tau_n (\cancel{p} + p_1)}$$

$$= \frac{-n_i^2}{\tau_p (n_1) + \tau_n (p_1)}$$



NEGATIVE Recombination => Generation

$n=p=0 \ll n_i \Rightarrow$ generation to create n, p

Equilibrium restoration!

- 1) Derivation of SRH formula
- 2) Application of SRH formula for special cases
- 3) **Direct and Auger recombination**
- 4) Conclusion

$$R = B(np - n_i^2) \quad B \text{ is a material property}$$

Direct recombination at low-level injection

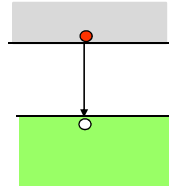
$$n_0 \ll (\Delta n = \Delta p) \ll p_0$$

$$R = B[(n_0 + \Delta n)(p_0 + \Delta p) - n_i^2] \approx Bp_0 \times \Delta n$$

Direct generation in depletion region

$$n, p \sim 0$$

$$R = B(np - n_i^2) \approx -Bn_i^2$$



2 electron & 1 hole

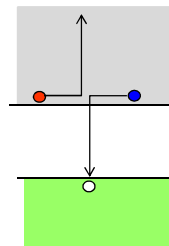
$$R = c_n(n^2 p - n_i^2 n) + c_p(np^2 - n_i^2 p)$$

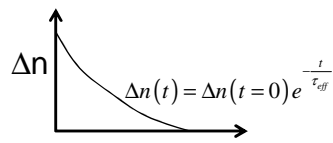
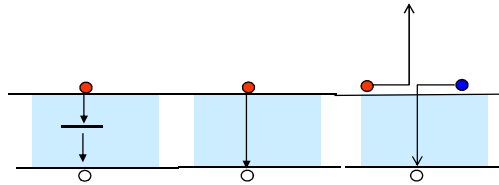
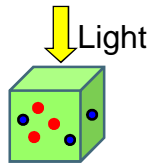
$$c_n, c_p \sim 10^{-29} \text{ cm}^6/\text{sec}$$

Auger recombination at low-level injection

$$n_0 \ll (\Delta n = \Delta p) \ll (p_0 = N_A)$$

$$R \approx c_p N_A^2 \Delta n = \frac{\Delta n}{\tau_{auger}} \quad \tau_{auger} = \frac{1}{c_p N_A^2}$$



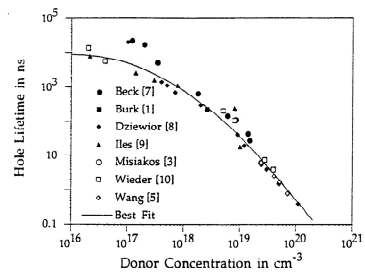
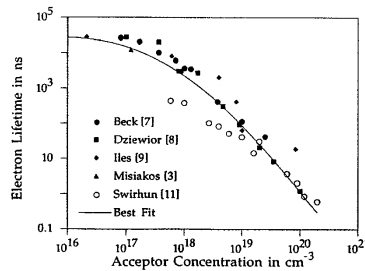
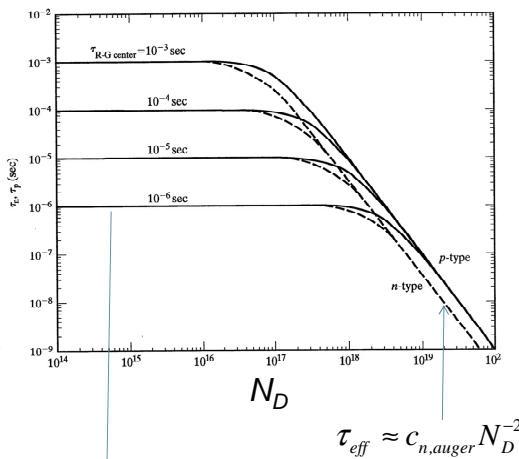


$$R = R_{SRH} + R_{direct} + R_{Auger}$$

$$= \Delta n \left(\frac{1}{\tau_{SRH}} + \frac{1}{\tau_{direct}} + \frac{1}{\tau_{Auger}} \right)$$

$$= \Delta n (c_n N_T + B N_D + c_{n, auger} N_D^2)$$

$$\tau_{eff} = (c_n N_T + B N_D + c_{n, auger} N_D^2)^{-1}$$



$$\tau_{eff} = (c_n N_T + B N_D + c_{n, auger} N_D^2)^{-1}$$

SRH is an important recombination mechanism in important semiconductors like Si and Ge.

SRH formula is complicated, therefore simplification for special cases are often desired.

Direct band-to-band and Auger recombination can also be described with simple phenomenological formula.

These expressions for recombination events have been widely validated by measurements.