Assignment 1

EE, Signals and Systems 1392-1

Due: 1392.07.06

Problem 1.

(a) Let $z = r\mathrm{e}^{\mathrm{j}\theta}$. Express in polar form (i.e., determine the magnitude and angle for) the following functions of z:

(i) z^*
(ii) z^2
(iii) $\mathrm{j}z$
(iv) $z\,z^*$
(v) $\frac{z}{z^*}$
(vi) $\frac{1}{z}$

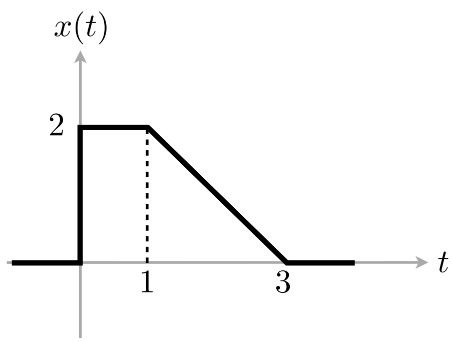
(b) Plot in the complex plane the vectors corresponding to your answers in part (a) for $r = \frac{4}{3}$ and $\theta = \text{-}5\frac{\pi}{6}$.

Problem 2. Prove that

$$1 - \mathrm{e}^{\mathrm{j}\theta} = 2\sin\left(\frac{\theta}{2}\right)\mathrm{e}^{\mathrm{j}\frac{\theta-\pi}{2}}$$

Problem 3. For $x(t)$ indicated in the below, sketch the following:

(a) $x(-t)$
(b) $x(t+2)$
(c) $x(2t+2)$
(d) $x(1-3t)$



Problem 4. Let $x(t) = \cos\left(\omega_x(t+\tau_x) + \theta_x\right)$.

(a) Determine the frequency in hertz and the period of $x(t)$ for each of the following three cases:

	ω_x	τ_x	θ_x
(i)	$\frac{\pi}{3}$	0	2π
(ii)	$\frac{3\pi}{4}$	$\frac{1}{2}$	$\frac{\pi}{4}$
(iii)	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(b) With $x(t) = cos\,(\omega_x(t+\tau_x)+\theta_x)$ and $y(t) = sin\,(\omega_y(t+\tau_y)+\theta_y)$, determine for which of the following combinations $x(t)$ and $y(t)$ are identically equal for all t.

	ω_x	τ_x	θ_x	ω_y	τ_y	θ_y
(i)	$\frac{\pi}{3}$	0	2π	$\frac{\pi}{3}$	1	-$\frac{\pi}{3}$
(ii)	$\frac{3\pi}{4}$	$\frac{1}{2}$	$\frac{\pi}{4}$	$\frac{11\pi}{4}$	1	$\frac{3\pi}{8}$
(iii)	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	1	$\frac{3}{8}$

Problem 5. Let $x[n] = \cos\,(\omega_x(n+m_x)+\theta_x)$.

(a) Determine the frequency in hertz and the period of $x[n]$ for each of the following three cases:

	ω_x	m_x	θ_x
(i)	$\frac{\pi}{3}$	0	2π
(ii)	$\frac{3\pi}{4}$	2	$\frac{\pi}{4}$
(iii)	$\frac{3}{4}$	1	$\frac{1}{4}$

(b) With $x[n] = cos\,(\omega_x(n+m_x)+\theta_x)$ and $y[n] = sin\,(\omega_y(n+m_y)+\theta_y)$, determine for which of the following combinations $x[n]$ and $y[n]$ are identically equal.

	ω_x	m_x	θ_x	ω_y	m_y	θ_y
(i)	$\frac{\pi}{3}$	0	2π	$\frac{8\pi}{3}$	0	0
(ii)	$\frac{3\pi}{4}$	2	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	1	-π
(iii)	$\frac{3}{4}$	1	$\frac{1}{4}$	$\frac{3}{4}$	0	1

Problem 6. Let $x(t) = \sqrt{2}(1+\mathrm{j})\mathrm{e}^{\mathrm{j}\frac{\pi}{4}}\mathrm{e}^{(-1+\mathrm{j}2\pi)t}$. Sketch and label the following:

(i) $\Re\{x(t)\}$

(ii) $\Im\{x(t)\}$

(iii) $x(t+2)+x^*(t+2)$

Problem 7. Let $x(t)$ be the continuous-domain complex exponential signal $x(t) = \mathrm{e}^{\mathrm{j}\omega_0 t}$ with fundamental frequency ω_0 and fundamental period $T_0 = \frac{2\pi}{\omega_0}$. Consider the discrete-domain signal obtained by taking equally spaced samples of $x(t)$. That is, $x[n] = x(nT) = \mathrm{e}^{\mathrm{j}\omega_0 nT}$.

(a) Show that $x[n]$ is periodic if and only if $\frac{T}{T_0}$ is a rational number, that is, if and only if some multiple of the sampling interval *exactly equals* a multiple of the period $x(t)$.

(b) Suppose that $x[n]$ is periodic, that is, $\frac{T}{T_0}$, where p and q are integers. What are the fundamental period and fundamental frequency of $x[n]$? Express the fundamental frequency as a fraction of $\omega_0 T$.

(c) Again assuming that $\frac{T}{T_0} = \frac{p}{q}$, determine precisely how many periods of $x(t)$ are needed to obtain the samples that form a single period of $x[n]$.