Recent Results on Passivity–based Control of Mechanical Systems

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1. Robustness to External Disturbances

- Perturbed port–Hamiltonian (pH) model

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ -I_n & -K_p \end{bmatrix} \nabla H + \begin{bmatrix} 0 \\ I_n \end{bmatrix} u + \begin{bmatrix} d_1 \\ d_2 \end{bmatrix},$$

$$H(q,p) = \frac{1}{2} p^\top M^{-1}(q) p + V(q).$$

- d_1, d_2 are time–varying disturbances.
- $K_p > 0$, $q^\star = \arg\min V(q) \Rightarrow$ global asymptotic stability (GAS) if $d = 0$.
- Objective: Design a state–feedback controller that:
  - preserves asymptotic stability for constant disturbances,
  - ensures input–to–state stability (ISS).
- Main technical tools (Donaire/Junco, *Automatica*'10, Ortega/Romero, SCL'12):
  - Change of coordinates (preserving pH structure and Hamiltonian function form)
  - Addition of integral action ♡

Destabilization of Integral Action on Velocities

- Integral control on passive output

$$\begin{aligned} u &= -\eta \\ \dot{\eta} &= K_i M^{-1}(q)p, \quad K_i > 0 \end{aligned}$$

- If d_1 is a non–zero constant the system admits no constant equilibrium, and if $d_1 = 0$ and d_2 is constant there is an equilibrium set

$$\mathcal{E} = \Big\{(q, p, \eta) \,|\, p = 0,\ \nabla V(q) + \eta = d_2\Big\}.$$

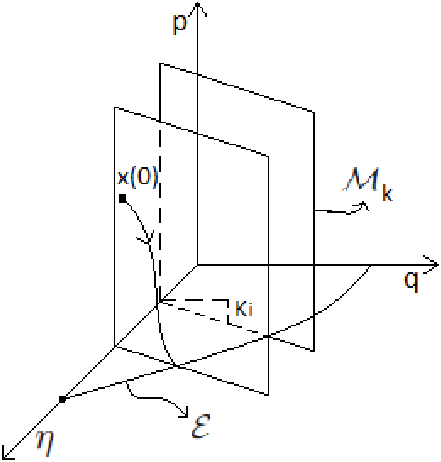
- With or without disturbances, the foliation

$$\mathcal{M}_\kappa = \Big\{(q, p, \eta) \,|\, K_i q - \eta = \kappa,\ \kappa \in \mathbb{R}\Big\},$$

is invariant.

- Convergence to $(q^\star, 0, d_2)$ is attained only for a zero measure set of initial conditions.

Invariant Foliation in the State Space



Robustness for Constant Inertia Matrix and $d(t) = \bar{d}$

Proposition Consider the PI control

$$\begin{aligned} u &= -K_p z_3 - MK_i \nabla V \\ \dot{z}_3 &= K_i \nabla V. \end{aligned}$$

(i) The closed–loop dynamics expressed in the coordinates,

$$z_1 = q, \quad z_2 = p + M(z_3 - K_p^{-1} d_2)$$

takes the pH form

$$\dot{z} = \begin{bmatrix} 0 & I_n & -K_i \\ -I_n & -K_p & 0 \\ K_i & 0 & 0 \end{bmatrix} \nabla H_z(z),$$

$$H_z(z) := H(z) + \frac{1}{2}(z_3 - z_3^*)^\top K_i^{-1}(z_3 - z_3^*).$$

(ii) $z^\star := (q^\star, 0, z_3^*)$, is GAS.

Non–constant $M(q)$: Change of Coordinates

Fact (Venkatraman, et al., TAC'10) Consider the system without damping ($K_p = 0$) and no unmatched disturbances ($d_1 = 0$). The change of coordinates

$$(q, \bar{p}) = (q, T(q)p), \quad M^{-1}(q) = T^\top(q)T(q).$$

transforms the dynamics into

$$\begin{bmatrix} \dot{q} \\ \dot{\bar{p}} \end{bmatrix} = \begin{bmatrix} 0 & T(q) \\ -T(q) & J_2(q, \bar{p}) \end{bmatrix} \nabla W + \begin{bmatrix} 0 \\ I_n \end{bmatrix} v + \begin{bmatrix} 0 \\ Td_2 \end{bmatrix},$$

with $v := T(q)u$, new Hamiltonian function

$$W(q, \bar{p}) = \frac{1}{2}|\bar{p}|^2 + V(q),$$

and the gyroscopic forces matrix

$$J_2(q, \bar{p}) := \nabla^\top(Tp)T - T\nabla(Tp)|_{p=T^{-1}\bar{p}}.$$

Robustness *vis–à–vis* $d_2(t)$

Proposition Control law

$$
\begin{aligned}
v &= -(\nabla^2 VT + J_2 + R_2 + R_3)\bar{p} - (R_2 + R_3)z_3 - (T + R_2 + R_3)\nabla V \\
\dot{z}_3 &= (T + R_3)\nabla V + R_3\bar{p}
\end{aligned}
$$

(i) Closed–loop dynamics in $z = (q, \bar{p} + \nabla V(q) + z_3, z_3)$,

$$
\dot{z} = \begin{bmatrix} -T & T & -T \\ -T & -R_2 & -R_3 \\ T & R_3 & -R_3 \end{bmatrix} \nabla U + \begin{bmatrix} 0 \\ Td_2 \\ 0 \end{bmatrix}
$$

with $2U(z) := |z_2|^2 + V(z_1) + |z_3|^2$.

(ii) ISS (with respect to $d_2(t)$).

(iii) If $d_2(t) = \bar{d}_2$, the equilibrium $z^\star = (q^*, 0, z_3^*)$ is GAS.

Remark Similar result for $(d_1(t), d_2(t))$, with complex control.

2. UGES Output Feedback Tracking

For all twice differentiable, bounded, references $(q_d(t), \mathbf{p}_d(t))$, there exists a dynamic position–feedback IDA–PBC that ensures uniform global exponential stability (UGES) of the closed–loop system. More precisely, there exist two mappings

$$\mathbf{F} : \mathbb{R}^{3n+1} \times \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{3n+1},\ \mathbf{H} : \mathbb{R}^{3n+1} \times \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^n$$

such that the mechanical system in closed–loop with

$$\dot{\chi} = \mathbf{F}(\chi, q, t),\ u = \mathbf{H}(\chi, q, t)$$

is a (perturbed) port–Hamiltonian system that verifies

$$\left\| \begin{bmatrix} q(t) - q_d(t) \\ \mathbf{p}(t) - \mathbf{p}_d(t) \\ \chi(t) \end{bmatrix} \right\| \leq \kappa \exp^{-\alpha(t-t_0)} \left\| \begin{bmatrix} q(t_0) - q_d(t_0) \\ \mathbf{p}(t_0) - \mathbf{p}_d(t_0) \\ \chi(t_0) \end{bmatrix} \right\|, \ \forall t \geq t_0.$$

for all $(q(t_0), \mathbf{p}(t_0), \chi(t_0)) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^{3n} \times \mathbb{R}_{\geq 0}$.

3. Robust Globally Convergent Adaptive Speed Observers

Consider perturbed, mechanical systems

$$\begin{bmatrix} \dot{q} \\ \dot{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ -I_n & -\mathfrak{R} \end{bmatrix} \nabla H(q, \mathbf{p}) + \begin{bmatrix} 0 \\ G(q) \end{bmatrix} u + \begin{bmatrix} 0 \\ d \end{bmatrix}$$

- Unknown constant disturbances $d = \text{col}(d_i) \in \mathbb{R}^n$.
- Coulomb friction captured by

$$\mathfrak{R} = \text{diag}\{r_1, r_2, ., r_n\} \in \mathbb{R}^{n \times n},$$

with unknown $r_i \geq 0, \ i \in \bar{n}$.

Problem Design a globally convergent robust adaptive observer for the momenta $\mathbf{p}$.

Assumptions

Assumption 1 The factor $T(q)$ verifies

$$[(T)_i, (T)_j] = 0, \ i, j \in \bar{n}.$$

Lemma

The following statements are equivalent:

(i) $M(q)$ satisfies Assumption 1.

(ii) The Riemann symbols of $M(q)$ are all zero.

(iii) There exists a mapping $Q : \mathbb{R}^n \to \mathbb{R}^n$ such that

$$\nabla Q(q) = T^{-1}(q).$$

Assumption 2 The rows of the factor $T(q)$ where there are friction terms are independent of q.

Main Result

Let $r \in \mathbb{R}^n$ be the friction coefficients and $r_u = C^\top r \in \mathbb{R}^s$ the unknown ones. The I&I adaptive momenta observer

$$\begin{aligned}
\dot{p}_I &= -T^\top(q)[\nabla V - G(q)u - \hat{\partial}] - (\sum_{i=1}^{n} Y_i \hat{p}_i)\hat{r}_u - \lambda Q(q)\hat{p} \\
\dot{r}_{u_I} &= (\sum_{i=1}^{n} Y_i^\top \hat{p}_i)(\dot{p}_I + \lambda \hat{p}) \\
\dot{d}_I &= T(q)\hat{p},\ \hat{p} = p_I + \lambda Q(q),\ \hat{\mathbf{p}} = T^{-\top}(q)\hat{p} \\
\hat{\partial} &= d_I + q,\ \hat{r}_u = r_{u_I} + \frac{1}{2\lambda}(\sum_{i=1}^{s} \hat{p}^\top L_i \hat{p}) e_i
\end{aligned}$$

with $Q(q)$ given in the Lemma, $\lambda > 0$ and

$$L_i := T^\top(q) e_i e_i^\top T(q),\ Y_j = \sum_{i=1}^{n} L_i e_j e_i^\top C$$

ensures $\lim_{t\to\infty}[\hat{\mathbf{p}}(t) - \mathbf{p}(t)] = 0$ for all $(q(0), \mathbf{p}(0)) \in \mathbb{R}^n \times \mathbb{R}^n$.

4. Energy Shaping without Solving PDE's

Partition $q = \text{col}(q_a, q_u)$, with $q_a \in \mathbb{R}^m$ and $q_u \in \mathbb{R}^{n-m}$ and

$$M(q) = \begin{bmatrix} m_{aa}(q) & m_{au}(q) \\ m_{au}^\top(q) & m_{uu}(q) \end{bmatrix}, \; G = \begin{bmatrix} I_m \\ 0 \end{bmatrix}.$$

Assumptions

A1. The inertia matrix depends only on the unactuated variables q_u, *i.e.*, $M(q) = M(q_u)$.

A2. The sub-block matrix m_{aa} of the inertia matrix is constant.

A3. The potential energy can be written as

$$V(q) = V_a(q_a) + V_u(q_u).$$

A4. The rows of the matrix $m_{au}(q_u)$ satisfy

$$\frac{\partial (m_{au})_k}{\partial q_{uj}} = \frac{\partial (m_{au})_j}{\partial q_{uk}}, \; \forall j \neq k, \; j, k \in \overline{n-m}.$$

cont'd

A5. The columns of $m_{au}(q_u)$ are gradient vector fields, that is,

$$\nabla(m_{au})^i = [\nabla(m_{au})^i]^\top, \ \forall i \in \bar{m}.$$

Equivalently, there exists a function $V_N : \mathbb{R}^{n-m} \to \mathbb{R}^m$ such that

$$\dot{V}_N = -m_{au}(q_u)\dot{q}_u.$$

A6. There exist $k_e, k_a, k_u \in \mathbb{R}, K_k, K_I \in \mathbb{R}^{m\times m}, \ K_k, K_I \geq 0$, such that

(i) $\det[K(q_u)] \neq 0$, where $K : \mathbb{R}^{n-m} \to \mathbb{R}^{m\times m}$ is defined as

$$K(q_u) := k_e I_m + k_a K_k + k_u K_k m_{au}(q_u) m_{uu}^{-1}(q_u) m_{au}^\top(q_u).$$

cont'd

(ii) The matrix

$$M_d(q_u) := \begin{bmatrix} k_e k_a I_m + k_a^2 K_k & -k_a k_u K_k m_{au}(q_u) \\ -k_a k_u m_{au}^\top(q_u) K_k^\top & M_d^{22}(q_u) \end{bmatrix} > 0$$

with

$$M_d^{22}(q_u) := k_e k_u m_{uu}(q_u) + k_u^2 m_{au}^\top(q_u) K_k m_{au}(q_u),$$

and the function

$$V_d(q) := k_e k_u V_u(q_u) + \frac{1}{2}\|k_a q_a + k_u V_N(q_u)\|_{K_I}^2,$$

has a minimum in q_*.

Main Result

There exists a static state–feedback control law such that the closed–loop has a globally stable equilibrium at the desired point $(q, \dot{q}) = (q_\star, 0)$ with Lyapunov function

$$H_d(q, \dot{q}) = \frac{1}{2}\dot{q}^\top M_d(q)\dot{q} + V_d(q).$$

Moreover, if $K_k = 0$ the control law is the simple PI

$$u = -\frac{1}{k_e}\left(K_p + \frac{1}{p}K_I\right)(k_a y_a + k_u y_u),$$

with $p := \frac{d}{dt}$ and

$$y_a := \dot{q}_a, \quad y_u := -m_{au}(q_u)\dot{q}_u.$$

Cart-pendulum on inclined plane ♡

Current Challenges

- Walking robots:
  - passive robots with natural gait,
  - effect of impacts,
  - multi–legged,
  - energy–efficient.
- Dextrous robots:
  - juggling,
  - gymnastics,
  - swimming...
- Transparent teleoperation.
- Coordination of mobile robots.
- Human–robot interaction: cyberphysical systems.
- Visual servoing.
- Humanoid robots.