1. Prove the following relations:
   i) \[ \int_V \nabla f \, dv = \oint_S f \, d\mathbf{s} \] (hint: let \( \mathbf{A} = \mathbf{c} f \) in divergence theorem, \( \mathbf{c} \) is constant)
   ii) \[ \int_V (\nabla \times \mathbf{v}) \, dv = -\oint_S \mathbf{v} \times d\mathbf{s} \] (hint: replace \( \mathbf{v} \) by \( \mathbf{v} \times \mathbf{c} \) in the divergence theorem)
   iii) \[ \int_S \nabla f \, d\mathbf{s} = \oint_C f \, d\mathbf{l} \] (hint: let \( \mathbf{A} = \mathbf{c} f \) in Stokes theorem).

2. Show that the vector \( \mathbf{A} = (6xy + z^3) \mathbf{x} + (3x^2z - y) \mathbf{y} + (3xyz^2) \mathbf{z} \) is conservative. Find the potential function, \( \phi \), such that \( \mathbf{A} = \nabla \phi \).

3. Compute the line integral of \( \oint \mathbf{v} \cdot d\mathbf{l} \) along the triangular path below. Check the answer using the Stokes theorem.
   \[ \mathbf{v} = 6\mathbf{x} + yz^2 \mathbf{y} + (3y + z) \mathbf{z} \]

4. Show that the Stokes theorem holds for the field vector \( \mathbf{A} = r \phi \) when the closed path is the circle \( \rho = a, z = 0 \), and open surface is:
   i) The disk \( 0 < \rho < a, z = 0 \).
   ii) The lower hemisphere \( r = a, \pi/2 < \theta < \pi \).

5. The integral \( \mathbf{A} = \int_S d\mathbf{s} \) is called the vector area of surface \( S \):
   i) find the vector area of a hemispherical bowl of radius \( R \).
   ii) show that \( \mathbf{A} = 0 \), for any closed surface.
   iii) show that \( \mathbf{A} \) is same for all surfaces sharing the same boundary.

6. Assume that \( \mathbf{A} = \frac{\cos^2 \varphi}{r^2} \mathbf{r} \)
   i) check the divergence theorem for the region, \( 1 < r < 2 \).
   ii) check the divergence theorem for a hemisphere with \( R = 2 \). Explain that why it doesn’t hold here.

7. Test the Stokes theorem for the function \( \mathbf{v}(\rho, \varphi, z) \) on the shaded area shown on the next page.
   \[ \mathbf{v} = (\rho^2 \cos^2 \varphi \sin \varphi + 2\rho z \sin^2 \varphi) \hat{\rho} + (\rho z \sin 2\varphi - \rho^2 \cos \varphi \sin^2 \varphi) \hat{\varphi} + 3\rho z \cos \varphi \hat{z} \]
8. prove the following equations in the Cartesian coordinates:

i) $\nabla.(f \vec{A}) = f\nabla.\vec{A} + \vec{A}.\nabla f$

ii) $\nabla \times (f \vec{A}) = f\nabla \times \vec{A} + \nabla f \times \vec{A}$

iii) $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla.\vec{A}) - \nabla^2 \vec{A}$