1. A hollow spherical shell carries charge density $\rho = \frac{k}{r^2}$ in the region $a \leq r \leq b$. Find the electric field in three regions: $r < a$, $a \leq r \leq b$, $r > b$.

2. Electric charge with surface density $\sigma = \sigma_0 \cos^2 \varphi$ lies on the circular disk $\rho < 1$ in the $xy$ plane. Find the electric field density at an arbitrary point on $z$ axis.

3. A thin conductor ring of radius $R$ which has the uniform line charge of $\lambda$ is located on the $xy$ plane centered at the origin. (it’s axis is along the $z$ axis).
   i) Find the electrical field on the axis of the ring at an arbitrary distance $z$ above its center. Call this point $P_1 = (0, 0, z)$.
   ii) Now suppose that the point of observation is moved to a very small distance $\delta \rho$ from the axis. Call this point $P_2$. As there is no symmetry anymore, the field doesn’t point to the $z$ direction, so it has a radial component too. Find the radial component of the field at $P_2$, in the terms of $z$ and $\delta \rho$. (hint: use Gauss’s law for a cylinder with radius $\delta \rho$ and with its ends on the heights $z$ and $z + \delta z$, note that there is no charge in this cylinder.)

4. Two spheres, each of radius $R$ and carrying uniform charge density $\rho$ and $-\rho$, respectively, are placed so that they partially overlap. Call the vector from positive center to negative center $\vec{d}$. Show that the field in the region of overlap is constant, and find its value.

5. Assume that the following triangle has a surface density of $\sigma_s$, find the electric potential at point O in the terms of $a, \varphi$ and $\sigma_s$.

6. Suppose that the charge distribution is spherically symmetric ($\rho = \rho(r)$), prove that the electric potential is calculated as follows:

$$\phi(\vec{r}) = \frac{1}{\varepsilon_0 r} \int_0^r \rho(r') r'^2 dr' + \frac{1}{\varepsilon_0} \int_r^\infty \rho(r') r' dr'.$$
7. Four charges are situated on the corners of a square with the side length of $a$:

i) find the electric potential at the origin.

ii) suppose that $P(x, y)$ is so close to the origin ($x \ll a, y \ll a$). Find the electric potential at $P$ to the second order of ($\frac{x}{a}$) and ($\frac{y}{a}$), using Taylor extension.

iii) Now find the electric field at $P$ to the first order of ($\frac{x}{a}$) and ($\frac{y}{a}$).