Remark. starred problems are obligatory.

1. A uniform line charge of $\lambda$ is placed on an infinite straight wire, a distance $d$ above a grounded conducting plane.(the wire is parallel to the $x$ axis and the conducting plane is $xy$ plane)
   i) find the potential in the region above the plane.
   ii) find the charge density $\sigma$ induced on the conducting plane.

2*. The applied force from $\vec{p}_1$ dipole to $\vec{p}_2$ dipole can be found from the following relation:

$$ F_{12} = \frac{1}{4\pi \varepsilon_0} \left[ \frac{3(\vec{p}_1 \cdot \vec{r})\vec{p}_2 + 3(\vec{p}_2 \cdot \vec{r})\vec{p}_1 + 3(\vec{p}_1 \cdot \vec{p}_2)\vec{r}}{r^5} - \frac{15(\vec{p}_1 \cdot \vec{r})(\vec{p}_2 \cdot \vec{r})\vec{r}}{r^7} \right] $$

which $\vec{r}$ is the vector from $\vec{p}_1$ to $\vec{p}_2$. Now consider there’s a dipole $\vec{p} = p_0 \sin \theta \hat{y} + p_0 \cos \theta \hat{z}$ on the $z$ axis, a distance $d$ above the grounded conducting plane(xy plane). Find the force which is applied from the conducting plane to the dipole.

3. In this problem we want to find the mutual capacity of two cylinders centered at $x = x_{01}$ and $x = x_{02}$ and straighten along $z$ axis, with radius $R_1$ and $R_2$. So solve the following questions hierarchically:
   a) Consider two line charges($\lambda, -\lambda$) in a distance $2a$ apart. Locate the origin on the $z$ axis midway between the line charges. Find the potential due to a superposition of the two line charges.(assume both lines must have the same zero potential, which then must lie on the plane $x = 0$.)
   b) Show the equipotential surfaces are generated from $\frac{1}{\rho_-} + \frac{1}{\rho_+} = m.$ (which $\rho_-$ and $\rho_+$ are the distances from $\lambda$ and $-\lambda$ respectively)
   c) Rewrite the preceding equation in the Cartesian coordinates. Show that the equipotential surfaces are circular cylinders whose centers lie on the $x$ axis. With $x_0$ indicating the center of the circle and $R$ its radius. Prove that

$$ x_0 = \frac{m^2 + 1}{m^2 - 1}a $$  \hspace{1cm} (1) \\
$$ R^2 = x_0^2 - a^2 = \left( \frac{2ma}{m^2 - 1} \right)^2 $$  \hspace{1cm} (2)

Now we wish to find the potential due to a line charge $\lambda$ and a conducting circular cylinder of radius $R$ whose center is a distance $\delta$ from the charge.(Consider this cylinder as one of the equipotential surfaces made by two line charges,i.e $\lambda, -\lambda$)

   d) Find $x_0$ and $a$ in the figure below. Apply relations 1 and 2 .
e) Prove that for this circular cylinder \( m = \frac{R}{2} \), and find the induced potential on the conductor. Finally, we seek to solve the main question (mutual capacity of two cylinders).

f) Consider the problem of two cylinders of radius \( R_1 \) and \( R_2 \) whose axes are separated by a distance \( \Delta \). Each cylinder is an equipotential surface caused by two virtual line charges. Use equation 2 to find \( a \) in the figure below.

g) From part e find \( m_1 \) and \( m_2 \), then find the mutual capacity in the terms of \( m_1 \) and \( m_2 \).

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4*. Consider a point charge \( q_0 \), external to a grounded conducting sphere of radius \( R \). Can you find a convenient set of imaginary point charges, located inside the sphere, that will force the potential of the sphere to be zero when the potential of \( q_0 \) (located at \( z_0 \) from the center of the sphere) is added? answer the preceding question when the potential of conducting sphere is set to \( V = V_0 \).

5*. A conducting plate has a semispherical boss of radius \( R \) with a center on the plate as shown below. The plate is grounded and a point charge \( q \) is brought next to it at a distance \( D > R \). The charge is on the normal to the plate passing through the center of the boss.
a) Determine the image charge needed to replace the plate.
b) Determine the potential on the side of the charge.
c) Determine the charge induced on the whole conducting plate, including the boss.
d) Determine the force between the charge and the plate.

6. A conducting sphere of radius $a$ is placed at the origin, a uniform field $\vec{E} = E_0 \hat{z}$ is applied:
   a) Find the electrical potential outside the sphere.
   b) Find the surface charge density of the conducting sphere.
   c) Find the total charge which is induced on the sphere.
   hint: A uniform field can be thought of as being produced by appropriate positive and negative charges at infinity. For example, if there are two charges $Q, -Q$, located at positions $z = -R$ and $z = R$, then in a region near the origin whose dimensions are very small compared to $R$ there is an approximately constant electric field $E_0 = \frac{2Q}{4\pi\varepsilon_0 R^2}$. In the limit as $Q, R \to \infty$, with $Q/R^2$ constant, this approximation becomes exact. Now use the method of images and observe the following figure.