Linear Algebra
Homework #3

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Problem 1. Suppose that $A$ and $B$ are $m \times n$ matrices. Show that $A = B$, if and only if $Ax = Bx$ for all $n \times 1$ columns $x$.

Problem 2. Let $A = \begin{pmatrix} 1 & 5 & 0 & -3 \\ 2 & 10 & 1 & -4 \\ -1 & -5 & 1 & 5 \end{pmatrix}$.

(a) Determine a spanning set for the column space of $A$.
(b) Determine a spanning set for the row space of $A$.
(c) Determine a spanning set for the null space of $A$.
(d) Find a complete solution of the equation $Ax = b$ where $b = (6 \ 4 \ -14)^T$.

Problem 3. Given the matrix

$$A = \begin{pmatrix} 0 & a & 0 & 0 \\ b & 0 & c & 0 \\ 0 & d & 0 & e \\ 0 & 0 & f & g \\ 0 & 0 & 0 & h \end{pmatrix},$$

find the inverse, $A^{-1}$, if $A$ is invertible. If it is not, state why it’s the case.

Problem 4. Suppose that $A$ is a square matrix and $A^2 + I = 0$. Compute $e^{At}$. Your answer should be in a closed form. (Hint: $e^{At} = \sum_{n=0}^{\infty} \frac{t^n A^n}{n!}$)

Problem 5. For each matrix $A_{n \times n}$, prove that why it is impossible to find a solution for $X_{n \times n}$ in the equation $AX - XA = I$.

Problem 6. Matrix $A$ of order $n$ is such that for any traceless matrix $X$ of order $n$, the product $AX$ is also traceless. Show that $A$ is a multiple of the identity matrix, i.e., $A = \lambda I$.

Problem 7. Let $A$ be a square matrix with real positive entries such that every row adds up to 1. Show that for any positive integer $k$, the maximum entry of the matrix $I + A + A^2 + \cdots + A^k$ lies on the principal diagonal.

Question 1. What does the Latin phrase “reductio ad absurdum” mean?

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A square matrix $M$ is said to be traceless if $\text{tr}(M) = 0$. 