EE101: Sinusoidal steady state analysis

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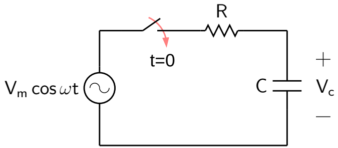
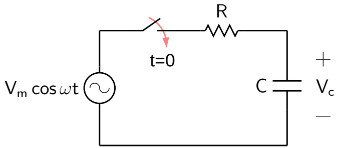
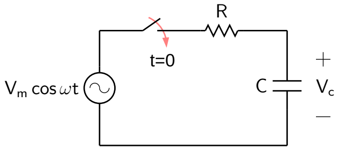

R
t=0
$V_m \cos \omega t$
C
+
V_c
−

Sinusoidal steady state



$$R\,(C\,V_c') + V_c = V_m\,\cos\,\omega t\,, \qquad t > 0\,. \qquad (1)$$



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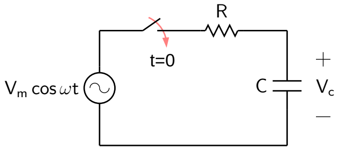

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$V_c^{(h)}(t)$ satisfies the homogeneous differential equation,

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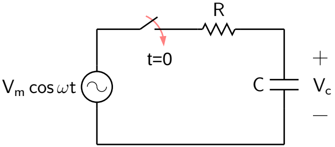
from which, $V_c^{(h)}(t) = A\,\exp(-t/\tau)\,,$ with $\tau = RC\,.$



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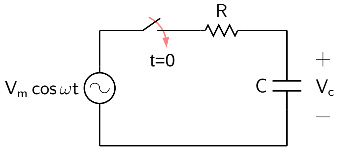

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Substituting in (1), we get,

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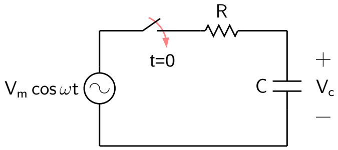

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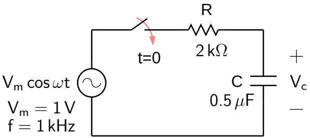
C_1 and C_2 can be found by equating the coefficients of $\sin \omega t$ and $\cos \omega t$ on the left and right sides.

Sinusoidal steady state



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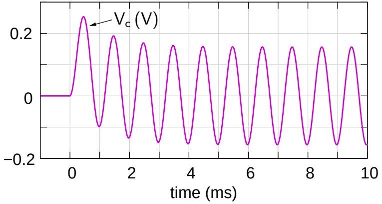

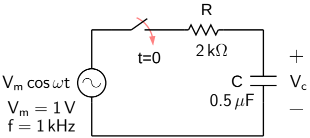

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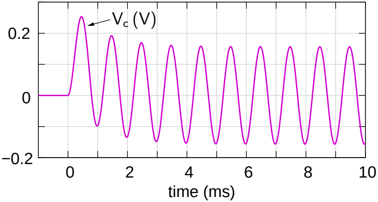

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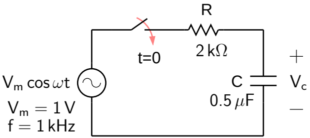

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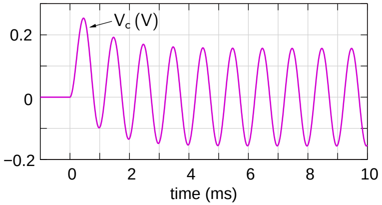

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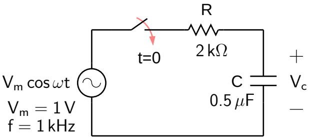

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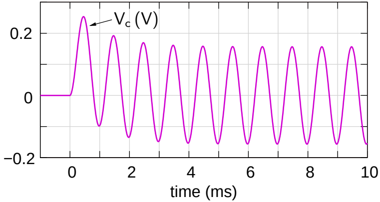

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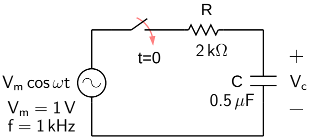

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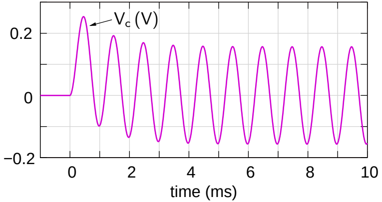

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- This is known as the "sinusoidal steady state" response since all quantities (currents and voltages) in the circuit are sinusoidal in nature.
- Any circuit containing resistors, capacitors, inductors, sinusoidal voltage and current sources (of the same frequency), dependent (linear) sources behaves in a similar manner, viz., each current and voltage in the circuit becomes purely sinusoidal as $t \rightarrow \infty$.

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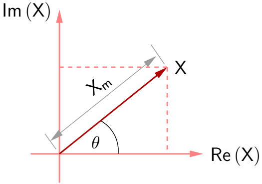
* Use of phasors substantially simplifies analysis of circuits in the sinusoidal steady state.
* Note that a phasor can be written in the polar form or rectangular form,
  $\mathbf{X} = X_m \angle\theta = X_m \exp(j\theta) = X_m \cos\theta + j\, X_m \sin\theta$.

  The term ωt is always *implicit*.



Phasors: examples

Time domain	Frequency domain
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Addition of phasors

Consider addition of two sinusoidal quantities:

$$v(t) = v_1(t) + v_2(t)$$
$$= V_{m1}\cos(\omega t + \theta_1) + V_{m2}\cos(\omega t + \theta_2)$$

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Now consider addition of the phasors corresponding to $v_1(t)$ and $v_2(t)$.

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$$\tilde{v}(t) = Re\left[\mathbf{V} e^{j\omega t}\right]$$
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$$\mathbf{V} = \mathbf{V_1} + \mathbf{V_2}$$
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In the time domain, $\mathbf{V}$ corresponds to $\tilde{v}(t)$, with

$$\tilde{v}(t) = Re\left[\mathbf{V} e^{j\omega t}\right]$$
$$= Re\left[\left(V_{m1} e^{j\theta_1} + V_{m2} e^{j\theta_2}\right) e^{j\omega t}\right]$$
$$= Re\left[V_{m1} e^{j(\omega t + \theta_1)} + V_{m2} e^{(\omega t + j\theta_2)}\right]$$

Addition of phasors

Consider addition of two sinusoidal quantities:

$$v(t) = v_1(t) + v_2(t)$$
$$= V_{m1}\cos(\omega t + \theta_1) + V_{m2}\cos(\omega t + \theta_2)$$

Now consider addition of the phasors corresponding to $v_1(t)$ and $v_2(t)$.

$$\mathbf{V} = \mathbf{V_1} + \mathbf{V_2}$$
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$$= Re\left[V_{m1}e^{j(\omega t + \theta_1)} + V_{m2}e^{(\omega t + j\theta_2)}\right]$$
$$= V_{m1}\,\cos(\omega t + \theta_1) + V_{m2}\,\cos(\omega t + \theta_2)$$

which is the same as $v(t)$.

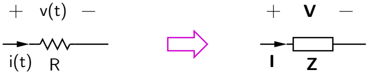
Addition of phasors

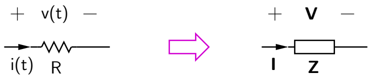
* Addition of sinusoidal quantities in the time domain can be replaced by addition of the corresponding phasors in the sinusoidal steady state.

Addition of phasors

* Addition of sinusoidal quantities in the time domain can be replaced by addition of the corresponding phasors in the sinusoidal steady state.
* The KCL and KVL equations,
  $\sum i_k(t) = 0$ at a node, and
  $\sum v_k(t) = 0$ in a loop,
  amount to addition of sinusoidal quantities and can therefore be replaced by the corresponding phasor equations,
  $\sum \mathbf{I}_k = \mathbf{0}$ at a node, and
  $\sum \mathbf{V}_k = \mathbf{0}$ in a loop.

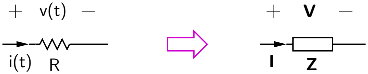
Impedance of a resistor





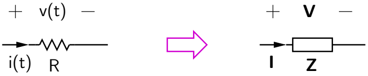
Let $i(t) = I_m \cos(\omega t + \theta)$.

Impedance of a resistor



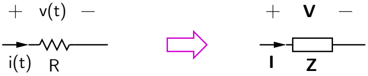
Let $i(t) = I_m \cos(\omega t + \theta)$.

$v(t) = R\,i(t)$



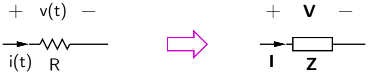
Let $i(t) = I_m \cos(\omega t + \theta)$.

$$v(t) = R\, i(t)$$
$$= R\, I_m \cos(\omega t + \theta)$$



Let $i(t) = I_m \cos(\omega t + \theta)$.

$$v(t) = R\, i(t)$$
$$= R\, I_m \cos(\omega t + \theta)$$
$$\equiv V_m \cos(\omega t + \theta).$$



Let $i(t) = I_m \cos(\omega t + \theta)$.

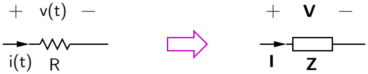
$v(t) = R\, i(t)$

$\quad = R\, I_m \cos(\omega t + \theta)$

$\quad \equiv V_m \cos(\omega t + \theta)$.

The phasors corresponding to $i(t)$ and $v(t)$ are, respectively,

$\mathbf{I} = I_m \angle\theta$, $\mathbf{V} = R \times I_m \angle\theta$.



Let $i(t) = I_m \cos(\omega t + \theta)$.

$$
\begin{aligned}
v(t) &= R\, i(t) \\
&= R\, I_m \cos(\omega t + \theta) \\
&\equiv V_m \cos(\omega t + \theta).
\end{aligned}
$$

The phasors corresponding to $i(t)$ and $v(t)$ are, respectively,

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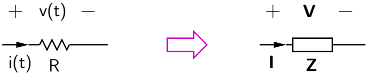
We have therefore the following relationship between $\mathbf{V}$ and $\mathbf{I}$: $\mathbf{V} = R \times \mathbf{I}$.



Let $i(t) = I_m \cos(\omega t + \theta)$.

$$v(t) = R\, i(t)$$
$$= R\, I_m \cos(\omega t + \theta)$$
$$\equiv V_m \cos(\omega t + \theta).$$

The phasors corresponding to $i(t)$ and $v(t)$ are, respectively,

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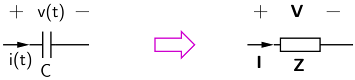
We have therefore the following relationship between $\mathbf{V}$ and $\mathbf{I}$: $\mathbf{V} = R \times \mathbf{I}$.

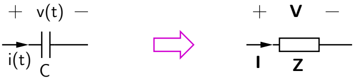
Thus, the *impedance* of a resistor, defined as, $\mathbf{Z} = \mathbf{V}/\mathbf{I}$, is

$$\boxed{\mathbf{Z} = R + j\,0}$$

Impedance of a capacitor

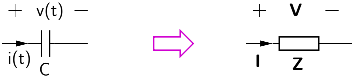


Impedance of a capacitor



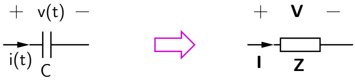
Let $v(t) = V_m \cos(\omega t + \theta)$.

Impedance of a capacitor



Let $v(t) = V_m \cos(\omega t + \theta)$.

$i(t) = C\,\dfrac{dv}{dt} = -C\,\omega\,V_m \sin(\omega t + \theta)$.



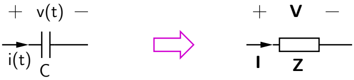
Let $v(t) = V_m \cos(\omega t + \theta)$.

$i(t) = C \dfrac{dv}{dt} = -C\,\omega\,V_m \sin(\omega t + \theta)$.

Using the identity, $\cos(\phi + \pi/2) = -\sin\,\phi$, we get

$i(t) = C\,\omega\,V_m\ \cos(\omega t + \theta + \pi/2)$.

Impedance of a capacitor



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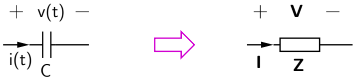
Using the identity, $\cos(\phi + \pi/2) = -\sin\,\phi$, we get

$i(t) = C\,\omega\,V_m\ \cos(\omega t + \theta + \pi/2)$.

In terms of phasors, $\mathbf{V} = V_m \angle\theta$, $\mathbf{I} = \omega C V_m \angle(\theta+\pi/2)$.

Impedance of a capacitor



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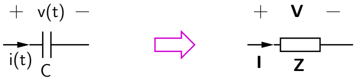
Using the identity, $\cos(\phi + \pi/2) = -\sin\,\phi$, we get

$i(t) = C\,\omega\,V_m\ \cos(\omega t + \theta + \pi/2)$.

In terms of phasors, $\mathbf{V} = V_m \angle\theta$, $\mathbf{I} = \omega C V_m \angle(\theta+\pi/2)$.

$\mathbf{I}$ can be rewritten as,

$\mathbf{I} = \omega C V_m\, e^{j(\theta+\pi/2)} = \omega C V_m\, e^{j\theta}\, e^{j\pi/2} = j\omega C\ \left(V_m\, e^{j\theta}\right) = j\omega C\,\mathbf{V}$



Let $v(t) = V_m \cos(\omega t + \theta)$.

$i(t) = C\,\dfrac{dv}{dt} = -C\,\omega\,V_m \sin(\omega t + \theta)$.

Using the identity, $\cos(\phi + \pi/2) = -\sin\phi$, we get

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In terms of phasors, $\mathbf{V} = V_m \angle\theta$, $\mathbf{I} = \omega C V_m\,\angle(\theta + \pi/2)$.

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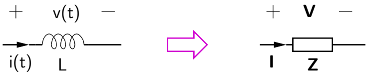
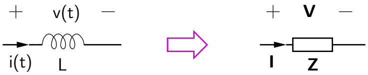
$\mathbf{I} = \omega C V_m\, e^{j(\theta+\pi/2)} = \omega C V_m\, e^{j\theta}\, e^{j\pi/2} = j\omega C\,\left(V_m\, e^{j\theta}\right) = j\omega C\,\mathbf{V}$

Thus, the *impedance* of a capacitor, $\mathbf{Z} = \mathbf{V}/\mathbf{I}$, is $\boxed{\mathbf{Z} = 1/(j\omega C)}$,

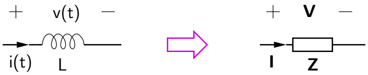
and the *admittance* of a capacitor, $\mathbf{Y} = \mathbf{I}/\mathbf{V}$, is $\boxed{\mathbf{Y} = j\omega C}$.


+
v(t)
−
i(t)
L
+
V
−
I
Z

Impedance of an inductor



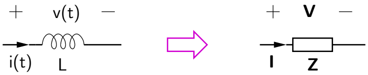
Let $i(t) = I_m \cos(\omega t + \theta)$.



Let $i(t) = I_m \cos(\omega t + \theta)$.

$$v(t) = L \frac{di}{dt} = -L\,\omega\, I_m \sin(\omega t + \theta).$$

Impedance of an inductor

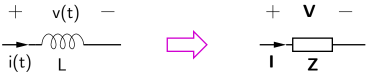


Let $i(t) = I_m \cos(\omega t + \theta)$.

$$v(t) = L\,\frac{di}{dt} = -L\,\omega\,I_m \sin(\omega t + \theta).$$

Using the identity, $\cos(\phi + \pi/2) = -\sin\,\phi$, we get

$v(t) = L\,\omega\,I_m\ \cos(\omega t + \theta + \pi/2)$.



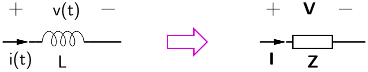
Let $i(t) = I_m \cos(\omega t + \theta)$.

$v(t) = L\,\dfrac{di}{dt} = -L\,\omega\, I_m \sin(\omega t + \theta)$.

Using the identity, $\cos(\phi + \pi/2) = -\sin\phi$, we get

$v(t) = L\,\omega\, I_m \cos(\omega t + \theta + \pi/2)$.

In terms of phasors, $\mathbf{I} = I_m \angle\theta$, $\mathbf{V} = \omega L I_m \angle(\theta + \pi/2)$.



Let $i(t) = I_m \cos(\omega t + \theta)$.

$v(t) = L\,\dfrac{di}{dt} = -L\,\omega\, I_m \sin(\omega t + \theta)$.

Using the identity, $\cos(\phi + \pi/2) = -\sin\phi$, we get

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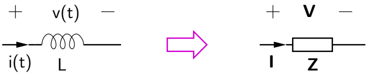
In terms of phasors, $\mathbf{I} = I_m \angle \theta$, $\mathbf{V} = \omega L I_m \angle(\theta + \pi/2)$.

V can be rewritten as,

$\mathbf{V} = \omega L I_m\, e^{j(\theta+\pi/2)} = \omega L I_m\, e^{j\theta}\, e^{j\pi/2} = j\omega L\, \left(I_m\, e^{j\theta}\right) = j\omega L\, \mathbf{I}$

Impedance of an inductor



Let $i(t) = I_m \cos(\omega t + \theta)$.

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Using the identity, $\cos(\phi + \pi/2) = -\sin\phi$, we get

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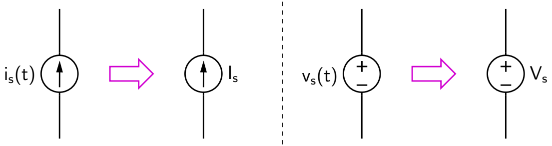
V can be rewritten as,

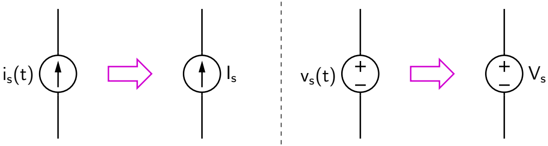
$\mathbf{V} = \omega L I_m\, e^{j(\theta+\pi/2)} = \omega L I_m\, e^{j\theta}\, e^{j\pi/2} = j\omega L\ \left(I_m\, e^{j\theta}\right) = j\omega L\,\mathbf{I}$

Thus, the *impedance* of an indcutor, $\mathbf{Z} = \mathbf{V}/\mathbf{I}$, is $\boxed{\mathbf{Z} = j\omega L}$,

and the *admittance* of an inductor, $\mathbf{Y} = \mathbf{I}/\mathbf{V}$, is $\boxed{\mathbf{Y} = 1/(j\omega L)}$.

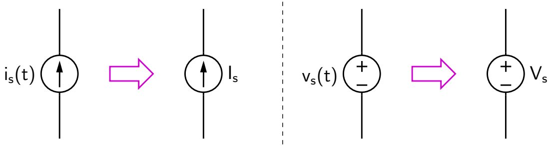

$i_s(t)$
I_s
$v_s(t)$
+
−
+
−
V_s

Sources



* An independent sinusoidal current source, $i_s(t) = I_m \cos(\omega t + \theta)$, can be represented by the phasor $I_m \angle \theta$ (i.e., a *constant* complex number).

Sources



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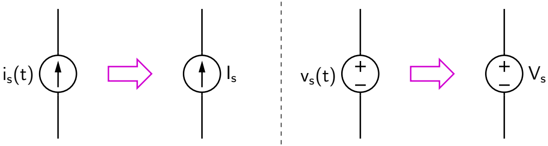

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* An independent sinusoidal voltage source, $v_s(t) = V_m \cos(\omega t + \theta)$, can be represented by the phasor $V_m \angle \theta$ (i.e., a *constant* complex number).
* Dependent (linear) sources can be treated in the sinusoidal steady state in the same manner as a resistor, i.e., by the corresponding phasor relationship.
  For example, for a CCVS, we have,
  $v(t) = r\, i_c(t)$ in the time domain.
  $\mathbf{V} = r\, \mathbf{I_c}$ in the frequency domain.

* The time-domain KCL and KVL equations $\sum i_k(t) = 0$ and $\sum v_k(t) = 0$ can be written as $\sum \mathbf{I}_k = \mathbf{0}$ and $\sum \mathbf{V}_k = \mathbf{0}$ in the frequency domain.

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* Resistors, capacitors, and inductors can be described by $\mathbf{V} = \mathbf{Z}\,\mathbf{I}$ in the frequency domain, which is similar to $V = R\,I$ in DC conditions (except that we are dealing with complex numbers in the frequency domain).

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* An independent sinusoidal source in the frequency domain behaves like a DC source, e.g., $\mathbf{V_s} = constant$ (a complex number).

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* An independent sinusoidal source in the frequency domain behaves like a DC source, e.g., $\mathbf{V_s} = constant$ (a complex number).
* For dependent sources, a time-domain relationship such as $i(t) = \beta\, i_c(t)$ translates to $\mathbf{I} = \beta\, \mathbf{I_c}$ in the frequency domain.

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* Circuit analysis in the sinusoidal steady state using phasors is therefore very similar to DC circuits with independent and dependent sources, and resistors.

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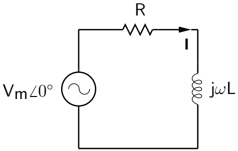
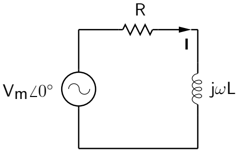
* Circuit analysis in the sinusoidal steady state using phasors is therefore very similar to DC circuits with independent and dependent sources, and resistors.
* Series/parallel formulas for resistors, nodal analysis, mesh analysis, Thevenin's and Norton's theorems can be directly applied to circuits in the sinusoidal steady state.

RL circuit



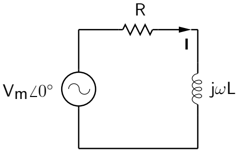
RL circuit



$$\mathbf{I} = \frac{V_m \angle 0}{R + j\omega L} \equiv I_m \angle(-\theta),$$

where $I_m = \dfrac{V_m}{\sqrt{R^2 + \omega^2 L^2}}$, and $\theta = \tan^{-1}(\omega L / R)$.

RL circuit



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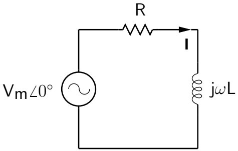

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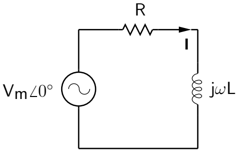
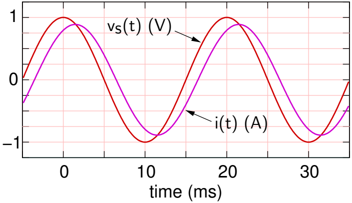
For $R = 1\,\Omega$, $L = 1.6\,\text{mH}$, $f = 50\,\text{Hz}$, $\theta = 26.6^\circ$, $t_{\text{lag}} = 1.48$ ms.

(SEQUEL file: `ee101_rl_ac_1.sqproj`)

RL circuit





R = 1 Ω
L = 1.6 mH

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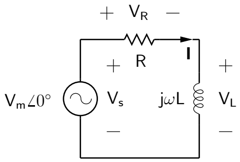
RL circuit



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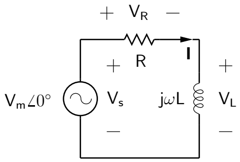

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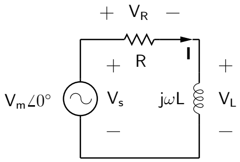
RL circuit



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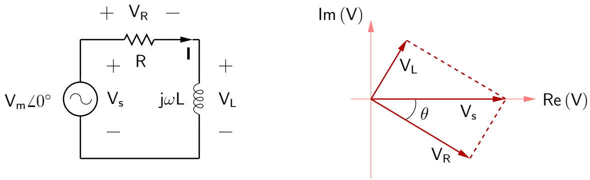
The KVL equation, $\mathbf{V_s} = \mathbf{V_R} + \mathbf{V_L}$, can be represented in the complex plane by a "phasor diagram."



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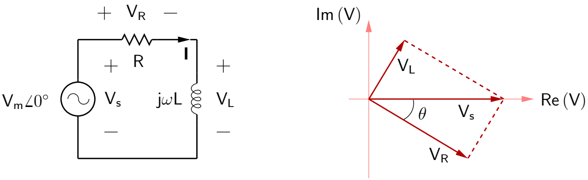

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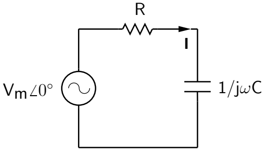
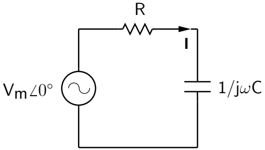
If $R \ll |j\omega L|$, $\theta \to \pi/2$, $|\mathbf{V_L}| \simeq |\mathbf{V_s}| = V_m$.

RC circuit



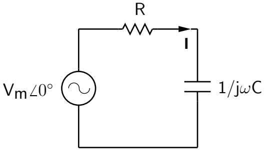
RC circuit



$$\mathbf{I} = \frac{V_m \angle 0}{R + 1/j\omega C} \equiv I_m \angle \theta,$$

where $I_m = \dfrac{\omega C V_m}{\sqrt{1 + (\omega RC)^2}}$, and $\theta = \pi/2 - \tan^{-1}(\omega RC)$.

RC circuit



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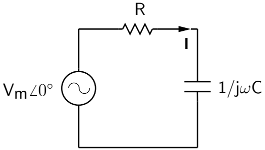

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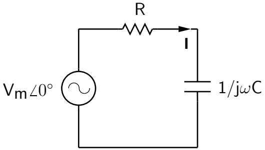
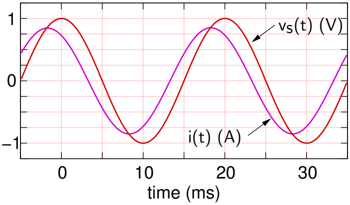
For $R = 1\,\Omega$, $C = 5.3\,\text{mF}$, $f = 50\,\text{Hz}$, $\theta = 31^\circ$, $t_{\text{lead}} = 1.72$ ms.

(SEQUEL file: `ee101_rc_ac_1.sqproj`)

RC circuit





R = 1 Ω
C = 5.3 mF

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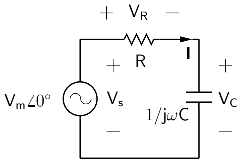

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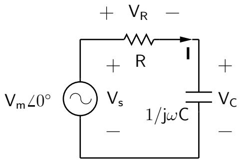

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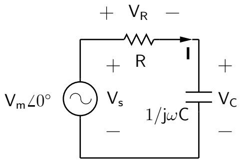

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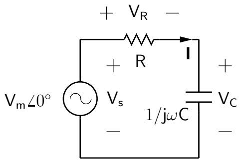

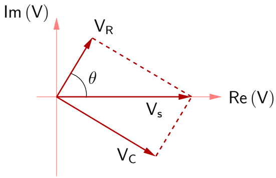

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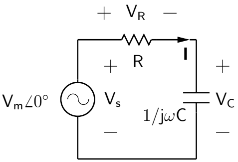

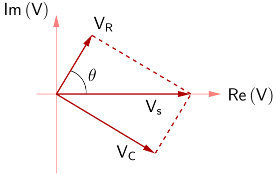

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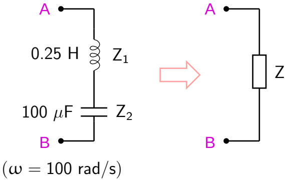
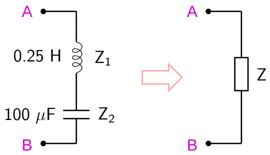
Series/parallel connections



Series/parallel connections

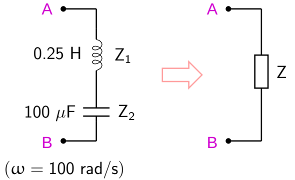


$(\omega = 100 \text{ rad/s})$

$$Z_1 = j \times 100 \times 0.25 = j\,25\,\Omega$$

$$Z_2 = -j/(100 \times 100 \times 10^{-6}) = -j\,100\,\Omega$$

$$Z = Z_1 + Z_2 = -j\,75\,\Omega$$



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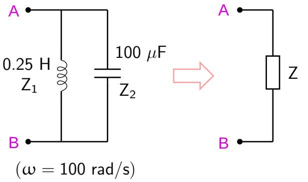

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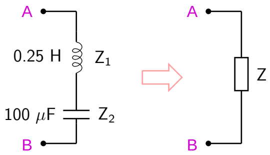

($\omega = 100$ rad/s)

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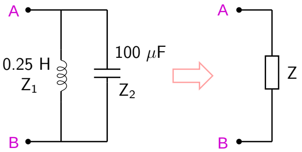


($\omega = 100$ rad/s)

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

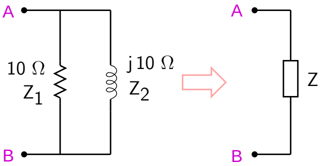
$$= \frac{(j\,25) \times (-j\,100)}{j\,25 - j\,100}$$

$$= \frac{25 \times 100}{-j\,75}$$

$$= j\,33.3\,\Omega$$

Impedance example

Obtain Z in polar form.



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Impedance example

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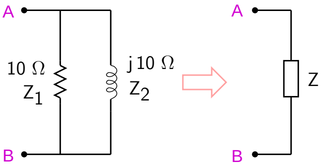


($\omega = 100$ rad/s)

Method 1:

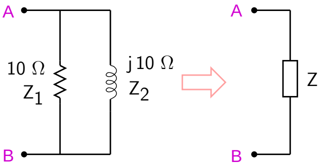
$$Z = \frac{10 \times j10}{10 + j10} = \frac{j10}{1 + j}$$

$$= \frac{j10}{1 + j} \times \frac{1 - j}{1 - j}$$

$$= \frac{10 + j10}{2} = 5 + j5\,\Omega$$

Convert to polar form $\rightarrow Z = 7.07\,\angle\,45^\circ\,\Omega$

Obtain Z in polar form.



($\omega = 100$ rad/s)

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$$Z = \frac{10 \times j10}{10 + j10} = \frac{j10}{1 + j}$$

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$$= \frac{10 + j10}{2} = 5 + j5\,\Omega$$

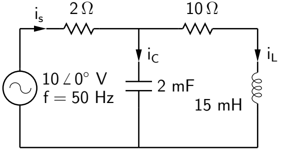
Convert to polar form $\rightarrow Z = 7.07 \angle 45^\circ\,\Omega$

Method 2:

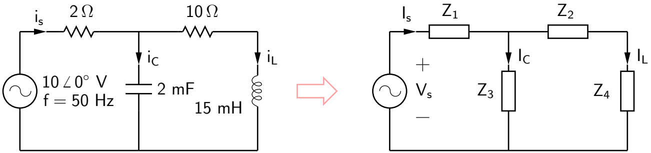
$$Z = \frac{10 \times j10}{10 + j10} = \frac{100 \angle \pi/2}{10\sqrt{2} \angle \pi/4}$$

$$= 5\sqrt{2} \angle (\pi/2 - \pi/4) = 7.07 \angle 45^\circ\,\Omega$$

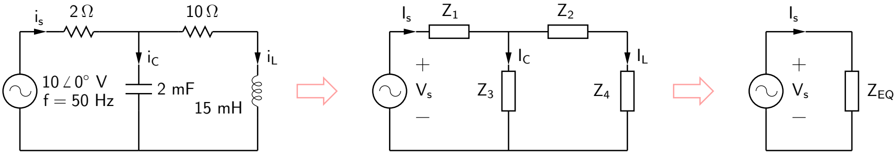
Circuit example



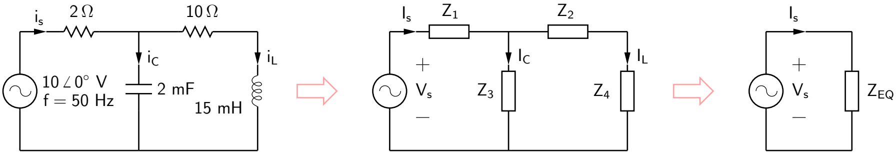
Circuit example



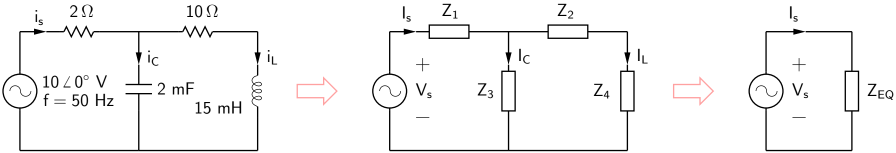
Circuit example



Circuit example



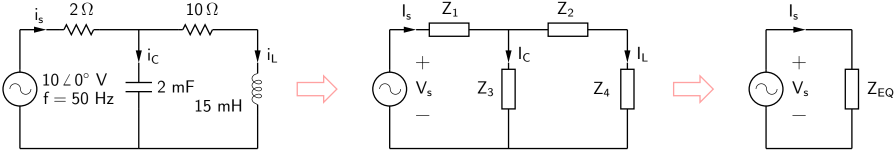
$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j\,1.6\ \Omega$$



$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j\,1.6\ \Omega$$

$$\mathbf{Z}_4 = 2\pi \times 50 \times 15 \times 10^{-3} = j\,4.7\ \Omega$$

Circuit example

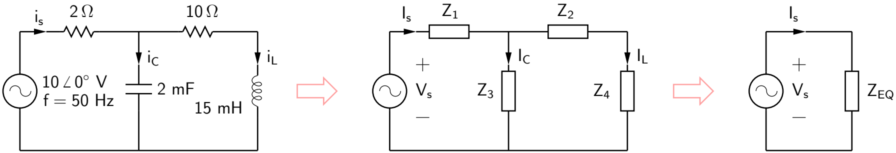


$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j\,1.6\ \Omega$$

$$\mathbf{Z}_4 = 2\pi \times 50 \times 15 \times 10^{-3} = j\,4.7\ \Omega$$

$$\mathbf{Z}_{EQ} = \mathbf{Z}_1 + \mathbf{Z}_3 \parallel (\mathbf{Z}_2 + \mathbf{Z}_4)$$

Circuit example



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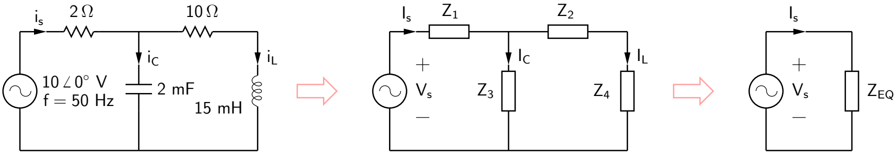
$$= 2 + (-j\,1.6) \parallel (10 + j\,4.7) = 2 + \frac{(-j\,1.6) \times (10 + j\,4.7)}{-j\,1.6 + 10 + j\,4.7}$$

Circuit example



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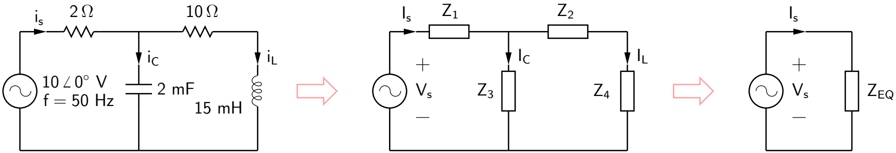
$$= 2 + \frac{1.6\angle(-90^\circ) \times 11.05\angle(25.2^\circ)}{10.47\angle(17.2^\circ)} = 2 + \frac{17.7\angle(-64.8^\circ)}{10.47\angle(17.2^\circ)}$$



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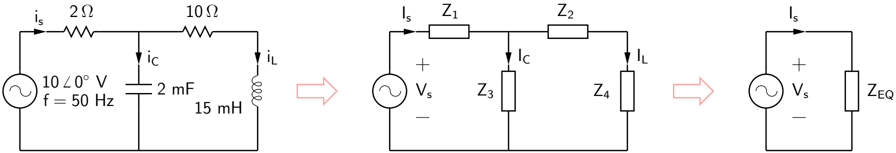
$$= 2 + 1.69\angle(-82°) = 2 + (0.235 - j\,1.67)$$

Circuit example



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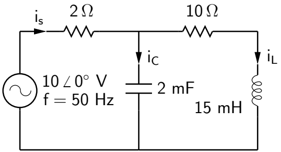
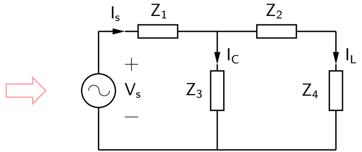
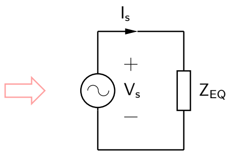
$$= 2 + 1.69\angle(-82^\circ) = 2 + (0.235 - j\,1.67)$$

$$= 2.235 - j\,1.67 = 2.79\angle(-36.8^\circ)\ \Omega$$

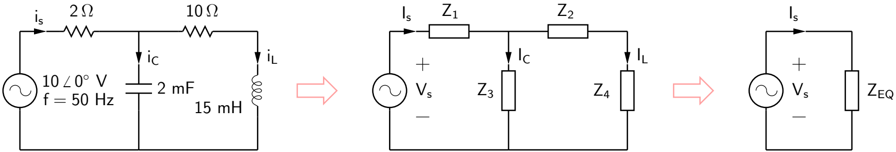
Circuit example (continued)





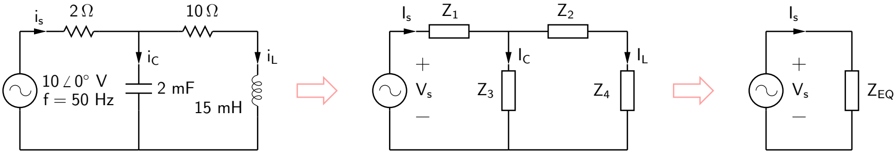


Circuit example (continued)



$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_{EQ}} = \frac{10\angle(0^\circ)}{2.79\angle(-36.8^\circ)} = 3.58\angle(36.8^\circ)\ A$$

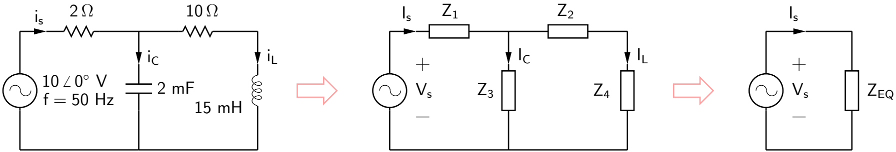
Circuit example (continued)



$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_{EQ}} = \frac{10\angle(0^\circ)}{2.79\angle(-36.8^\circ)} = 3.58\angle(36.8^\circ)\ A$$

$$\mathbf{I}_C = \frac{(\mathbf{Z}_2 + \mathbf{Z}_4)}{\mathbf{Z}_3 + (\mathbf{Z}_2 + \mathbf{Z}_4)} \times \mathbf{I}_s = 3.79\angle(44.6^\circ)\ A$$

Circuit example (continued)

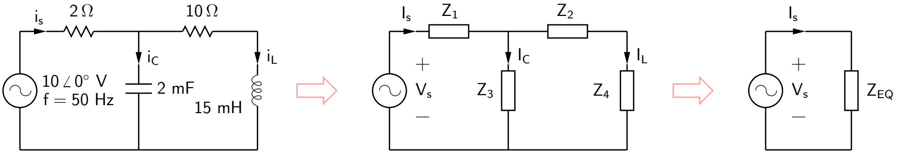


$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_{EQ}} = \frac{10\angle(0^\circ)}{2.79\angle(-36.8^\circ)} = 3.58\angle(36.8^\circ)\ A$$

$$\mathbf{I}_C = \frac{(\mathbf{Z}_2 + \mathbf{Z}_4)}{\mathbf{Z}_3 + (\mathbf{Z}_2 + \mathbf{Z}_4)} \times \mathbf{I}_s = 3.79\angle(44.6^\circ)\ A$$

$$\mathbf{I}_L = \frac{\mathbf{Z}_3}{\mathbf{Z}_3 + (\mathbf{Z}_2 + \mathbf{Z}_4)} \times \mathbf{I}_s = 0.546\angle(-70.6^\circ)\ A$$

Circuit example (continued)



$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_{EQ}} = \frac{10\angle(0^\circ)}{2.79\angle(-36.8^\circ)} = 3.58\angle(36.8^\circ)\ A$$

$$\mathbf{I}_C = \frac{(\mathbf{Z}_2 + \mathbf{Z}_4)}{\mathbf{Z}_3 + (\mathbf{Z}_2 + \mathbf{Z}_4)} \times \mathbf{I}_s = 3.79\angle(44.6^\circ)\ A$$

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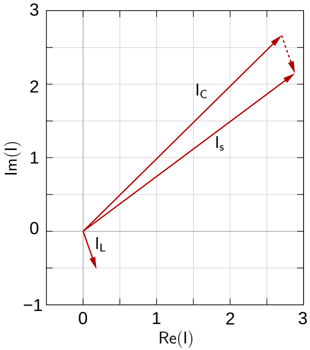

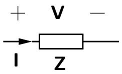
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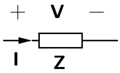
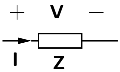


Let $v(t) = V_m \cos(\omega t + \theta)$, i.e., $\mathbf{V} = V_m \angle\theta$,
$i(t) = I_m \cos(\omega t + \phi)$, i.e., $\mathbf{I} = I_m \angle\phi$.



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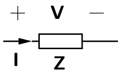
The *instantaneous* power absorbed by **Z** is,

$$
\begin{aligned}
P(t) &= v(t)\, i(t) \\
&= V_m I_m \cos(\omega t + \theta) \cos(\omega t + \phi) \\
&= \frac{1}{2} V_m I_m \left[\cos(2\omega t + \theta + \phi) + \cos(\theta - \phi)\right] \qquad (1)
\end{aligned}
$$



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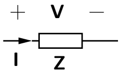
The *average* power absorbed by **Z** is

$$P = \frac{1}{T} \int_0^T P(t)\, dt, \text{ where } T = 2\pi/\omega.$$



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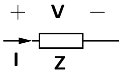
The first term in Eq. (1) has an average value of zero and does not contribute to P. Therefore,



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\end{aligned}
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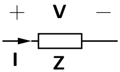
The first term in Eq. (1) has an average value of zero and does not contribute to P. Therefore,

$$\boxed{P = \frac{1}{2} V_m I_m \cos(\theta - \phi)}$$ gives the average power absorbed by **Z**.

Average power for R, L, C

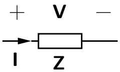


General formula:

$$\mathsf{V} = \mathsf{V_m} \angle \theta \,, \mathsf{I} = \mathsf{I_m} \angle \phi$$

$$\mathsf{P} = \frac{1}{2} \mathsf{V_m} \, \mathsf{I_m} \cos\left(\theta - \phi\right)$$

Average power for R, L, C



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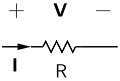
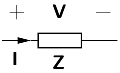


$$V = R\, I$$

For $I = I_m \angle \alpha, \;\; V = R\, I_m \angle \alpha \, ,$

$$P = \frac{1}{2} \left(R\, I_m\right) I_m \cos\left(\alpha - \alpha\right) = \frac{1}{2} I_m^2 \, R = \frac{1}{2} V_m^2 / R$$

Average power for R, L, C



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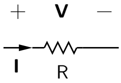

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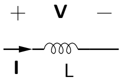
$$P = \frac{1}{2} (R\, I_m)\, I_m \cos(\alpha - \alpha) = \frac{1}{2} I_m^2 R = \frac{1}{2} V_m^2 / R$$

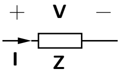


$$V = j\omega L\, I$$

For $I = I_m \angle \alpha, \;\; V = \omega L\, I_m \angle (\alpha + \pi/2) \,,$

$$P = \frac{1}{2} V_m I_m \cos\,[(\alpha + \pi/2) - \alpha] = 0$$

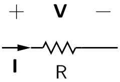
Average power for R, L, C



General formula:

$$V = V_m \angle \theta \, , I = I_m \angle \phi$$

$$P = \frac{1}{2} V_m \, I_m \cos(\theta - \phi)$$



$$V = R \, I$$

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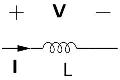
$$P = \frac{1}{2} (R \, I_m) \, I_m \cos(\alpha - \alpha) = \frac{1}{2} I_m^2 \, R = \frac{1}{2} V_m^2 / R$$



$$V = j\omega L \, I$$

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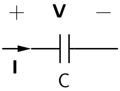
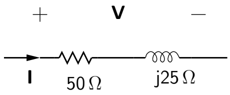
$$P = \frac{1}{2} V_m \, I_m \cos\left[(\alpha + \pi/2) - \alpha\right] = 0$$



$$I = j\omega C \, V$$

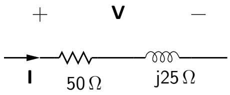
For $V = V_m \angle \alpha$, $I = \omega C \, V_m \angle (\alpha + \pi/2)$,

$$P = \frac{1}{2} V_m \, I_m \cos\left[\alpha - (\alpha + \pi/2)\right] = 0$$



Given: $\mathrm{I} = 2\angle 45^\circ$ A

Find the average power absorbed.



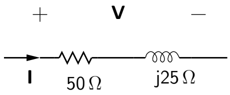
Given: $I = 2\angle 45^\circ$ A

Find the average power absorbed.

Method 1:

$$\begin{aligned} \mathbf{V} &= (50 + j25) \times 2\angle 45^\circ \\ &= 55.9\angle 26.6^\circ \times 2\angle 45^\circ \\ &= 111.8\angle (45^\circ + 26.6^\circ) \end{aligned}$$

Average power: example

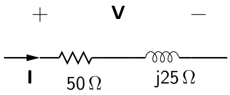


Given: $I = 2\angle 45^\circ$ A

Find the average power absorbed.

Method 1:

$$\mathbf{V} = (50 + j25) \times 2\angle 45^\circ$$
$$= 55.9\angle 26.6^\circ \times 2\angle 45^\circ$$
$$= 111.8\angle (45^\circ + 26.6^\circ)$$
$$P = \frac{1}{2} \times 111.8 \times 2 \times \cos(26.6^\circ) = 100\ W.$$



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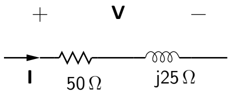
Method 2:

No average power is absorbed by the inductor.
$\Rightarrow P = P_R$ (average power absorbed by R)

Average power: example



Given: $I = 2\angle 45^\circ$ A

Find the average power absorbed.

Method 1:

$$\mathbf{V} = (50 + j25) \times 2\angle 45^\circ$$
$$= 55.9\angle 26.6^\circ \times 2\angle 45^\circ$$
$$= 111.8\angle (45^\circ + 26.6^\circ)$$
$$P = \frac{1}{2} \times 111.8 \times 2 \times \cos(26.6^\circ) = 100\ W.$$

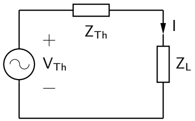
Method 2:

No average power is absorbed by the inductor.

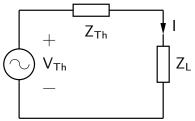
$\Rightarrow P = P_R$ (average power absorbed by R)

$$= \frac{1}{2} I_m^2 R = \frac{1}{2} \times 2^2 \times 50$$
$$= 100\ W.$$

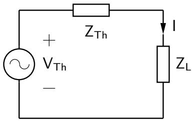
Maximum power transfer



Let $\mathbf{Z}_L = R_L + jX_L$, $\mathbf{Z}_{Th} = R_{Th} + jX_{Th}$, and $\mathbf{I} = I_m \angle \phi$.



Maximum power transfer



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The power absorbed by $\mathbf{Z}_L$ is,

$$P = \frac{1}{2}\, I_m^2 R_L$$

$$= \frac{1}{2} \left| \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + \mathbf{Z}_L} \right|^2 R_L$$

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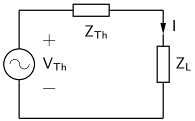

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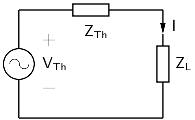
For P to be maximum, $(X_{Th} + X_L)$ must be zero. $\Rightarrow X_L = -X_{Th}$.



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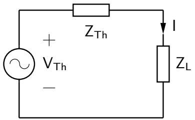

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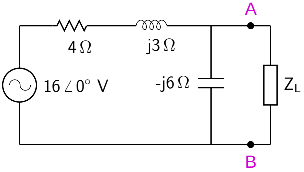
Therefore, for maximum power transfer to the load $\mathbf{Z}_L$, we need,

$R_L = R_{Th}$, $X_L = -X_{Th}$, i.e., $\boxed{\mathbf{Z}_L = \mathbf{Z}^*_{Th}.}$

Maximum power transfer: example



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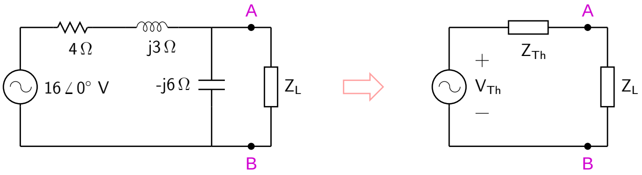

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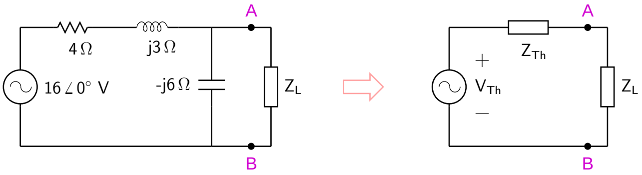

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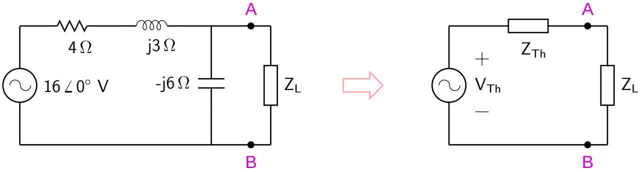

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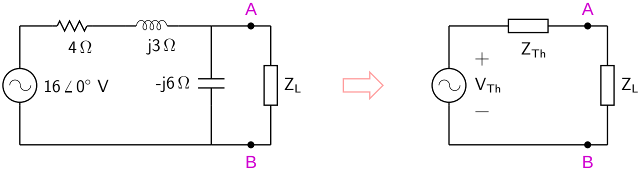

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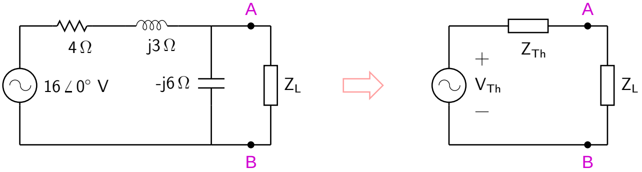
$$\mathbf{V}_{Th} = 16\,\angle 0^\circ \times \frac{-j\,6}{(4 + j\,3) + (-j\,6)} = 19.2\,\angle(-53.13^\circ)\,.$$



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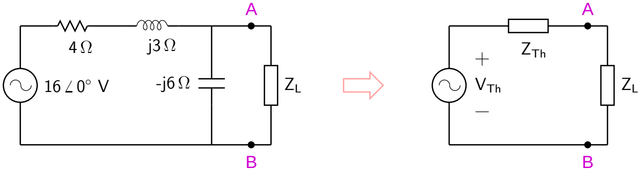

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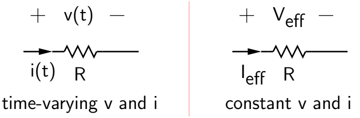
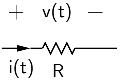
$$P = \frac{1}{2}\,I_m^2 R_L = \frac{1}{2}\left(\frac{19.2}{2\,R_L}\right)^2 \times R_L = \frac{1}{2}\,\frac{(19.2)^2}{4\,R_L} = 8\ W.$$

Effective (rms) values of voltage/current

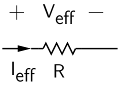


Consider a periodic current $i(t)$ passing through R.

Effective (rms) values of voltage/current



time-varying v and i



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where t_0 is some reference time (we will take t_0 to be 0).

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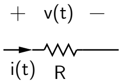

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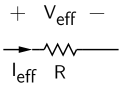

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The *average* power absorbed by R in the DC case is,

$P_2 = I_{eff}^2 \, R$.

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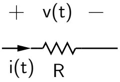

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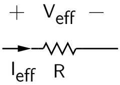

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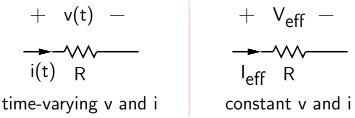
I_{eff}, the *effective* value of $i(t)$, is defined such that $P_1 = P_2$, i.e.,

$$I_{eff}^2 \, R = \frac{1}{T} \int_0^T [i(t)]^2 \, R \, dt \, ,$$

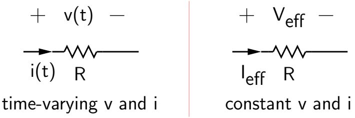
$$I_{eff} = \sqrt{\frac{1}{T} \int_0^T [i(t)]^2 \, dt} \, .$$

Effective (rms) values of voltage/current



$$I_{eff} = \sqrt{\frac{1}{T} \int_0^T [i(t)]^2 \, dt} \, .$$

I_{eff} is called the root-mean-square (rms) value of $i(t)$ because of the operations (root, mean, and square) involved in its computation.



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If $i(t)$ is sinusoidal, i.e., $i(t) = I_m \cos(\omega t + \phi)$,

$$I_{eff} = \sqrt{\frac{1}{T} \int_0^T I_m^2 \cos^2(\omega t + \phi) \, dt} = I_m \sqrt{\frac{1}{T} \int_0^T \frac{1}{2} \left[1 + \cos(2\omega t + 2\phi)\right] \, dt}$$

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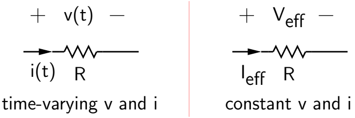

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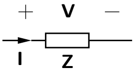

$V = V_m \angle \theta$

$I = I_m \angle \phi$

The average ("real") power absorbed by **Z** is,

$$P = \frac{1}{2} V_m\, I_m\, \cos(\theta - \phi) = \frac{V_m}{\sqrt{2}}\, \frac{I_m}{\sqrt{2}}\, \cos(\theta - \phi)$$

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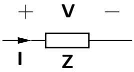
Apparent power, real power, and power factor



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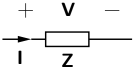

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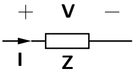

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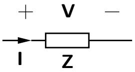
(inductive impedance)



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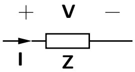
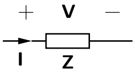


1. Given: $\mathbf{V} = 120\angle 0^\circ$ V (rms), $\mathbf{I} = 2\angle(-36.9^\circ)$ A (rms).
   Find P_{app}, P.F., and P.



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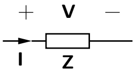
   $P_{app} = 120 \times 2 = 240\ V\text{-}A.$

Power factor: examples



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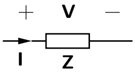
Power factor: examples



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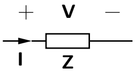
   $P = P_{app} \times$ P.F. $= 192$ W.



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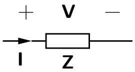
2. Given: $P = 50$ kW, P.F. $= 0.95$ (lagging), $\mathbf{V} = 480\,\angle 0^{\circ}$ V (rms).
   Find **I**.

Power factor: examples



1. Given: $\mathbf{V} = 120\angle 0^\circ$ V (rms), $\mathbf{I} = 2\angle(-36.9^\circ)$ A (rms).
   Find P_{app}, P.F., and P.

   $P_{app} = 120 \times 2 = 240\ V\text{-}A.$

   P.F. $= \cos(0^\circ - (-36.9^\circ)) = 0.8$ lagging (since $\mathbf{I}$ lags $\mathbf{V}$).

   $P = P_{app} \times$ P.F. $= 192\ W.$

2. Given: $P = 50$ kW, P.F. $= 0.95$ (lagging), $\mathbf{V} = 480\angle 0^\circ$ V (rms).
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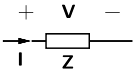
   $V_{eff} \times I_{eff} \times$ P.F $= 50 \times 10^3$



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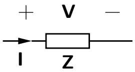

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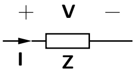

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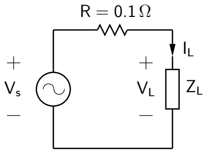
   $\Rightarrow \mathbf{I} = 109.6\angle(-18.2^\circ)$ A (rms).

Why is power factor important?



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Note: lagging power factors are typical of industrial loads (motors).

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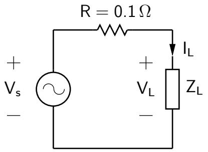

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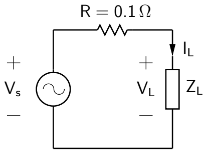

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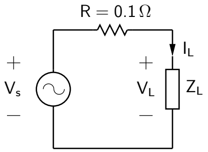

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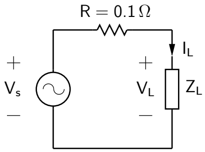

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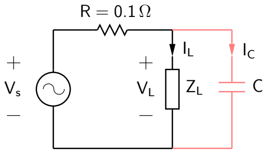

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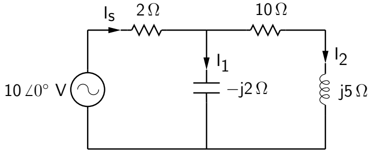
The effective power factor of an inductive load can be improved by connecting a suitable capacitance in parallel.



* Find $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_s$.
* Compute the average power absorbed by each element.
* Verify power balance.

(SEQUEL file: `ee101_phasors_2.sqproj`)