Session 4: Solid State Devices

Heterojunction devices
Ref: Brennan and Brown
Review Homojunction!

Homojunction: the junction is between two regions of the same material.

Heterojunction: the junction is between two different semiconductors.
PN junctions – Before Being Joined

Electrically neutral in every region

Electron affinity: $X_{Si}$

Work function $\Phi$: $\Phi = E_{vac} - E_F$

$\Phi_n \neq \Phi_p$
PN junctions (Qualitative)
PN junctions (Qualitative)

Remember \( \frac{dE_F}{dx} = 0 \) under equilibrium. Band bending occurs around the metallurgical junction!
PN junctions (Qualitative)

depletion region

$J_{n\text{drift}}$ $J_{n\text{diff}}$ $J_{p\text{drift}}$ $J_{p\text{diff}}$

$E_{CP}$ $E_{Fn}$ $E_{CN}$ $E_{FP}$ $E_{VP}$ $E_{VN}$

$qV_{bi}$
PN junctions (Qualitative)
Reverse Biased

$V_R$

$J_{n\text{drift}}$

$J_{n\text{diff}}$

$E_{Fp}$

$qV_R$

$E_{Fn}$

$J_{p\text{drift}}$

$J_{p\text{diff}}$
PN junctions (Qualitative)
Forward Biased

\[ I = \frac{N_{d}}{d_{n}} \exp\left(\frac{qV_{F}}{kT}\right) \]

- \( N_{d} \): Density of donor atoms
- \( d_{n} \): Thickness of the depletion region
- \( V_{F} \): Forward bias voltage
- \( q \): Elementary charge
- \( k \): Boltzmann constant
- \( T \): Temperature

\[ J_{n_{diff}} \] and \[ J_{p_{diff}} \]

\[ J_{n_{drift}} \] and \[ J_{p_{drift}} \]

Depletion region

\[ E_{Fp} \] and \[ E_{Fn} \]

\[ qV_{F} \]

\[ \text{Current - Voltage (I-V) characteristics} \]
PN junctions (Qualitative)

The figure illustrates a PN junction with the depletion region highlighted. The diagram shows the distribution of electric field (E), voltage (V), and charge density (ρ) across the junction. The depletion region is the area where the PN junction is formed, where the charge carriers are depleted.
PN junctions - Assumptions

The Depletion Approximation: Obtaining closed-form solutions for the electrostatic variables

Charge Distribution: \[ \rho = q(p - n + N_D - N_A) \]

Note that:
1. \(-x_p \leq x \leq x_n\): p & n are negligible (\because \mathcal{E} \text{ exist}).
2. \(x \leq -x_p \text{ or } x \geq x_n\): \(\rho = 0\)
How to Find $\rho(x), \mathcal{E}(x), V(x)$

1. Find the built-in potential $V_{bi}$

2. Use the depletion approximation $\rightarrow \rho(x)$ (depletion-layer widths $x_p, x_n$ unknown)

3. Integrate $\rho(x)$ to find $\mathcal{E}(x)$ boundary conditions $\mathcal{E}(-x_p) = 0, \mathcal{E}(x_n) = 0$

4. Integrate $\mathcal{E}(x)$ to obtain $V(x)$ boundary conditions $V(-x_p) = 0, V(x_n) = V_{bi}$

5. For $\mathcal{E}(x)$ to be continuous at $x = 0$, $N_A x_p = N_D x_n \rightarrow$ solve for $x_p, x_n$
Built-In Potential $V_{bi}$

$$qV_{bi} = q\varphi_{sp} + q\varphi_{sn}$$

$$= (E_i - E_F)_p + (E_F - E_i)_n$$

For non-degenerately doped material:

$$\begin{align*}
(E_i - E_F)_p &= kT \ln \left( \frac{p}{n_i} \right) = kT \ln \left( \frac{N_A}{n_i} \right) \\
(E_F - E_i)_n &= kT \ln \left( \frac{n}{n_i} \right) = kT \ln \left( \frac{N_D}{n_i} \right)
\end{align*}$$

$$\left\{ \begin{array}{c}
V_{bi} = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right)
\end{array} \right.$$

What shall we do for $p^+ - n$ (or $n^+ - p$) junction?!?!

$p^+$:

$$(E_i - E_F)_p = \frac{E_G}{2}$$

$n^+$:

$$(E_F - E_i)_n = \frac{E_G}{2}$$
The Depletion Approximation

The electric field is continuous at $x = 0$

$$ x_p N_A = x_n N_D $$

Charge neutrality condition as well!
Electrostatic Potential in the Depletion Layer

\[ dV \over dx = -\varepsilon \]

\(-x_p < x < 0:\)
\[ \varepsilon(x) = -\frac{qN_A}{\varepsilon}(x + x_p) \]
\[ V(x) = \frac{qN_A}{2\varepsilon}(x + x_p)^2 + C = \frac{qN_A}{2\varepsilon}(x + x_p)^2 \]

\(0 < x < x_n:\)
\[ \varepsilon(x) = -\frac{qN_D}{\varepsilon}(x_n - x) \]
\[ V(x) = -\frac{qN_D}{2\varepsilon}(x_n - x)^2 + C' = V_{bi} - \frac{qN_D}{2\varepsilon}(x_n - x)^2 \]
Depletion Layer Width

\(-x_p < x < 0: \quad V(x) = \frac{qN_A}{2\epsilon} (x + x_p)^2\)

\(0 < x < x_n: \quad V(x) = V_{bi} - \frac{qN_D}{2\epsilon} (x_n - x)^2\)

\[ V(0) = \frac{qN_A}{2\epsilon} x_p^2 = V_{bi} - \frac{qN_D}{2\epsilon} x_n^2 \]

\(x_p N_A = x_n N_D\)

\[
\begin{align*}
  x_n &= \sqrt{\frac{2\epsilon s V_{bi}}{q} \left( \frac{N_A}{N_D(N_A + N_D)} \right)} \\
  x_p &= \sqrt{\frac{2\epsilon s V_{bi}}{q} \left( \frac{N_D}{N_A(N_A + N_D)} \right)}
\end{align*}
\]

Summing, we have:

\[ W = x_p + x_n = \sqrt{\frac{2\epsilon s V_{bi}}{q} \left( \frac{1}{N_D} + \frac{1}{N_A} \right)} \]
Depletion Layer Width

If $N_A \gg N_D$ as in a $p^+ - n$ junction:

$$W = \sqrt{\frac{2\varepsilon_s V_{bi}}{q} \left( \frac{1}{N_D} + \frac{1}{N_A} \right)} \quad \Rightarrow \quad W = \sqrt{\frac{2\varepsilon_s V_{bi}}{q N_D}} \approx x_n$$

$$x_p N_A = x_n N_D \quad \Rightarrow \quad x_p \ll x_n \quad \Rightarrow \quad x_p \approx 0$$

Note:

$$V_{bi} = \frac{E_G}{2q} + \frac{kT}{q} \ln \frac{N_D}{n_i}$$
A $p^+ - n$ junction has $N_A = 10^{20} \, \text{cm}^{-3}$ and $N_D = 10^{17} \, \text{cm}^{-3}$. What is

a) its built in potential,

$$V_{bi} = \frac{E_G}{2q} + \frac{kT}{q} \ln \frac{N_D}{n_i} \approx 1V$$

b) $W$,

$$W \approx \sqrt{\frac{2\epsilon_s V_{bi}}{q N_D}} = \sqrt{\frac{2 \times 12 \times 8.85 \times 10^{-14} \times 1}{1.6 \times 10^{19} \times 10^{17}}} = 0.12\mu m$$

c) $x_n$, and

$$x_n \approx W = 0.12\mu m$$

d) $x_p$

$$x_p = x_n \frac{N_D}{N_A} = 1.2 \times 10^{-4} \, \mu m = 1.2 \, \text{Å} \sim 0$$
Biases pn Junction (assumptions)

1. Negligible voltage drop (Ohmic contact)
2. $V_A$ dropped here
3. Will apply continuity equation in this region
4. Low level injection
5. Zero voltage drop ($\varepsilon = 0$)
6. Since ($\varepsilon = 0$) may apply minority carrier diffusion equations

Note: $V_A$ should be significantly smaller than $V_{bi}$ (Otherwise, we cannot assume low-level injection)
Effect of Bias on Electrostatics

Energy Band Diagram
1) The Fermi level is omitted from the depletion region because the device is no longer in equilibrium: We need the quasi Fermi energy level.
2) $E_{fp} - E_{fn} = -qV_A$
Now as we assumed all voltage drop is in the depletion region (Note that $V_A \leq V_{bi}$)

\[ x_n + x_p = W = \sqrt{\frac{2\varepsilon_s (V_{bi} - V_A)}{q} \left( \frac{1}{N_D} + \frac{1}{N_A} \right)} \]

\[ x_p N_A = x_n N_D \]
The junction width for one-sided step junctions in silicon as a function of junction voltage with the doping on the lightly doped side as a parameter.

\[ V_j = V_{bi} - V_a(V) \]
Junction width for a one-sided junction is plotted as a function of doping on the lightly doped side for three different operating voltages.

- $V_a = -5V$
- $V_a = 0V$
- $V_a = 0.5V$
Assumption:

1) low-level injection: \( n_p \ll p_p \sim N_A \) (or \( \Delta n \ll p_0 , p \sim p_0 \) in p-type)
\[
p_n \ll n_n \sim N_D \) (or \( \Delta p \ll n_0 , n \sim n_0 \) in n-type)
\]

2) In the bulk, \( n_n \sim n_{n0} = N_D , p_p \sim p_{p0} = N_A \)

3) For minority carriers \( J_{drift} \ll J_{diff} \) in quasi-neutral region

4) Nondegenerately doped step junction

5) Long-base diode in 1-D (both sides of quasi-neutral regions are much longer than their minority carrier diffusion lengths, \( L_n \) or \( L_p \))

6) No Generation/Recombination in depletion region

7) Steady state \( d/dt = 0 \)

8) \( G_{opt} = 0 \)
pn Junction: I-V Characteristic

Game plan:

i) continuity equations for minority carriers

\[
\frac{\partial \Delta n_p}{\partial t} = \frac{1}{q} \frac{\partial J_n}{\partial x} - \frac{\Delta n_p}{\tau_n} + \frac{\Delta p_n}{\tau_p} + \text{sources}
\]

\[
\frac{\partial \Delta p_n}{\partial t} = -\frac{1}{q} \frac{\partial J_p}{\partial x} - \frac{\Delta p_n}{\tau_p} + \text{sources}
\]

ii) minority carrier current densities in the quasi-neutral region

\[
J_p = J_{p\,\text{drift}} + J_{p\,\text{diff}} = qp\mu_p \frac{dp}{dx} - qD_p \frac{dp}{dx}
\]

\[
J_n = J_{n\,\text{drift}} + J_{n\,\text{diff}} = qn\mu_n \frac{dn}{dx} + qD_n \frac{dn}{dx}
\]
pn Junction: I-V Characteristic

Steady-State solution is:

\[
\frac{d^2 \Delta n_p}{dx^2} = \frac{\Delta n_p}{D_n \tau_n} \implies \Delta n_p = Ae^{x/L_n} + Be^{-x/L_n} \quad (L_n = \sqrt{D_n \tau_n})
\]

diode is long enough!

\[
\Delta n_p(x'') = A''e^{-x''/L_n}
\]

\[
\Delta n_p(x'') = \Delta n_p(-x_p)e^{-x''/L_n}
\]

\[
J_n = qD_n \frac{dn}{dx}
\]

\[
J_n(x'') = \frac{qD_n}{L_n} \Delta n_p(-x_p)e^{-x''/L_n}
\]

\[
J_p = -qD_p \frac{dp}{dx}
\]

\[
J_p(x') = \frac{qD_p}{L_p} \Delta p_n(x_n)e^{-x'/L_p}
\]

\[
J = J_n(0'') + J_p(0')
\]
pn Junction: I-V Characteristic

\[ J = J_n(0'') + J_p(0') \]

\[ J_n(x'') = \frac{qD_n}{L_n} \Delta n_p(-x_p)e^{-x_n/L_n} \]

\[ J_p(x') = \frac{qD_p}{L_p} \Delta p_n(x_n)e^{-x'/L_p} \]

Now! we need to find \( \Delta n_p(-x_p) \) and \( \Delta p_n(x_n) \) vs \( V \)

\[ V_2 - V_1 = \frac{kT}{q} \ln \frac{n_2}{n_1} = \frac{kT}{q} \ln \frac{p_1}{p_2} \]

\[ \rightarrow V_0 - V = \frac{kT}{q} \ln \frac{n(x_n)}{n(-x_p)} = \frac{kT}{q} \ln \frac{p(-x_p)}{p(x_n)} \]

\[ V_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2} \rightarrow n(-x_p) = n_{p0} e^{qV/kT} \]

\[ p(x_n) = p_{n0} e^{qV/kT} \]
pn Junction: I-V Characteristic

Forward:

\[ n(-x_p) = n_{p0} e^{qV/kT} \]

\[ p(x_n) = p_{n0} e^{qV/kT} \]

Reverse:

\[ n(-x_p) = n_{p0} e^{qV/kT} \]

\[ p(x_n) = p_{n0} e^{qV/kT} \]
pn Junction: I-V Characteristic

\[ J = J_n(0') + J_p(0') = \frac{qD_n}{L_n} \Delta n_p(-x_p) + \frac{qD_p}{L_p} \Delta p_n(x_n) \]

\[ n(-x_p) = n_{p0} \ e^{qV/kT} \quad ; \quad \Delta n_p(-x_p) = n - n_{p0} = n_{p0}(e^{qV/kT} - 1) \quad ; \quad n_{p0} = n_i^2/N_A \]

\[ p(x_n) = p_{n0} \ e^{qV/kT} \quad ; \quad \Delta p_n(x_n) = p - p_{n0} = p_{n0}(e^{qV/kT} - 1) \quad ; \quad p_{n0} = n_i^2/N_D \]

\[ J = q \left( \frac{D_n}{L_n} n_{p0} + \frac{D_p}{L_p} p_{n0} \right) (e^{qV/kT} - 1) \quad I = AJ \]

\[ I = qA \left( \frac{D_n n_i^2}{L_n N_A} + \frac{D_p n_i^2}{L_p N_D} \right) (e^{qV/kT} - 1) = I_0 (e^{qV/kT} - 1) \]

\[ I_0 = qA n_i^2 \left( \frac{1}{\sqrt{\tau_n N_A}} + \frac{1}{\sqrt{\tau_p N_D}} \right) \]
I = qA\left( \frac{D_n n_i^2}{L_n N_A} + \frac{D_p n_i^2}{L_p N_D} \right) \left( e^{qV/kT} - 1 \right) = I_0 \left( e^{qV/kT} - 1 \right)

asymmetrically doped junction

If \( p^+ - n \) diode (\( N_A \gg N_D \)), then
\[
I_0 \approx qA \frac{D_p n_i^2}{L_p N_D}
\]

If \( n^+ - p \) diode (\( N_D \gg N_A \)), then
\[
I_0 \approx qA \frac{D_n n_i^2}{L_n N_A}
\]

That is, one has to consider only the lightly doped side of such junction in working out the diode I-V characteristics.
pn Junction: I-V Characteristic

V=0

\[ J_{n_{\text{drift}}} \]

\[ E_{Cp} \]

\[ J_{p_{\text{drift}}} \]

V>0

\[ J_{n_{\text{drift}}} \]

\[ J_{n_{\text{diff}}} \]

\[ qV_{bi} \]

\[ E_{Cn} \]

\[ E_{Fn} \]

\[ E_{Fn} \]

\[ q(V_{bi} - V_F) \]

\[ E_{Fn} \]

\[ qV_F \]

\[ E_{Fn} \]
The minority carrier concentrations on either side of the junction under forward bias

\[ n_p(x) = \Delta n_p(x) + n_{p0} \]

\[ p_n(x) = \Delta p_n(x) + p_{n0} \]
Minority-Carrier Charge Storage

\[ p(x_n) = p_{n0} e^{qV/\kappa T} \]

\[ \Delta n_p(x'') = \Delta n_p(-x_p) e^{-x''/L_n} \]

\[ \Delta p_n(x') = \Delta p_n(x_n) e^{-x'/L_p} \]

\[ Q_N = -qA\Delta n_p(-x_p)L_n \]

\[ Q_p = -qA\Delta p_n(x_n)L_p \]

\[ \frac{\partial \Delta n_p}{\partial t} = \frac{1}{q} \frac{\partial J_n}{\partial x} - \frac{\Delta n_p}{\tau_n} + \nabla \cdot \tau_t \]

\[ \frac{\partial \Delta p_n}{\partial t} = -\frac{1}{q} \frac{\partial J_p}{\partial x} - \frac{\Delta p_n}{\tau_p} + \nabla \cdot \tau_t \]
Charge Control Model

\[ p(x_n) = p_{n0} e^{qV/kT} \]

\[ \Delta p_n(x') = \Delta p_n(x_n) e^{-x'/L_p} \]

\[ J = J_n(0') + J_p(0') \]

Steady state: \[ \frac{d}{dt} = 0 \]

\[ I_p(x_n) = \frac{Q_P}{\tau_p} \quad \text{similarly} \quad I_n(-x_p) = \frac{Q_P}{\tau_n} \]

In general: \[ \Delta p_n(x, t) \]

\[ \frac{\partial \Delta p_n}{\partial t} = - \frac{1}{q} \frac{\partial J_p}{\partial x} - \frac{\Delta p_n}{\tau_p} \]

\[ \frac{\partial (qA\Delta p_n)}{\partial t} = -A \frac{\partial J_p}{\partial x} - \frac{qA\Delta p_n}{\tau_p} \]

\[ \frac{d}{dt} Q_P = AJ_p(x_n) - \frac{Q_P}{\tau_p} \]

\[ \frac{d}{dt} Q_P = I_p(x_n) - \frac{Q_P}{\tau_p} \]
PN junctions – Before Being Joined

electrically neutral in every region
electron affinity: \( X_{Si} \)
work function \( \Phi \): \( \Phi = E_{vac} - E_F \)
\( \Phi_n \neq \Phi_p \)
MS junctions – Before Being Joined

1. \( E_0 \)  
2. \( E_{Fm} \)  
3. \( q \varphi_m \)  
4. \( q \chi \)  
5. \( \Phi_{sp} \)  

\( q \varphi_m \)  work function  \( \varphi_{Au} = 4.75\text{eV} \), \( \varphi_{Cu} = 4.5\text{eV} \), \( \varphi_{Al} = 4.28\text{eV} \)  
\( q \chi \)  electron affinity  \( \chi_{Si} = 4.05\text{eV} \), \( \chi_{Ge} = 4\text{eV} \), \( \varphi_{GaAs} = 4.07\text{eV} \)
Reminder

1. I
2. n
3. n
4. p
5. p

$D(E)$

$E_{Fm}$

$E_{Cp}$

$E_{Fn}$

$E_{Cn}$

$E_{Fp}$

$E_{Vp}$

$E_{Vn}$

metal

p-type

n-type
Step by step:

1. Vacuum energy \( (E_0) \) is continuous.

2. \( E_G \) and \( \chi \) are intrinsic properties of materials and should remain constant. (which means \( E_C, E_V, \) and \( E_0 \) are all parallel)

3. At equilibrium \( E_F \) is constant while by applying voltage \( \Delta E_F = -qV \).
MS junctions – Before Being Joined

Metal

\[ E_0 \]

\[ q\phi_m \]

4.75 eV

\[ E_{Fm} \]

n-type

\[ E_0 \]

4.05 eV

\[ q\chi \]

\[ q\phi_s \]

\[ E_C \]

\[ E_{FS} \]

\[ E_V \]
MS junctions – Qualitative

Metal

n-type

$E_0$

$q \varphi_m$

4.75 eV

$E_{Fm}$

$E_C$

$q \chi$

4.05 eV

$q \varphi_s$

$E_{FS}$

$E_V$
MS junctions – Qualitative

\[ q\varphi_B = q(\varphi_m - \chi) = qV_i + (E_C - F_F) \]

\[ qV_i = q(\varphi_m - \varphi_s) \]
MS junctions – Qualitative

\[ q\varphi_b = qV_i + (E_C - F_F) \]

\[ \int \]

\[ \frac{1}{\varepsilon} \int \]

\[ qN_D \]

\[ x_p \]

\[ -x_p \]

\[ V \]

\[ qV_i \]

\[ x \]

\[ qV_i = q(\varphi_m - \varphi_s) \]
MS junctions - Schottky Effect

\[ \mathcal{E}(0) = -qN_D W / \varepsilon \]
\[ V_i = -\frac{1}{2} W \mathcal{E}(0) = qN_D W^2 / 2\varepsilon \]
\[ W = \sqrt{\frac{2\varepsilon}{q N_D}} (V_i - V_a) \]
\[ \mathcal{E}(0) = -\sqrt{\frac{2qN_D}{\varepsilon}} (V_i - V_a) \]

as \( qV_i = q(\varphi_m - \varphi_s) \) seems that \( V_i \) is independent of the applied voltage

But it is not! This is known as “Schottky Effect” This will lower \( V_i (\varphi_b) \) a little bit.

Image method

\[ F(x) = \frac{-q^2}{16\pi \varepsilon x^2} \]
\[ \rightarrow \Phi(x) = -qV(x) = \frac{-q^2}{16\pi \varepsilon x} \]
MS junctions, I-V Curve

\[ V \alpha 1/x \]

\[ q\Delta \varphi_b = qV_i + (E_C - F_F) \]

\[ q\varphi_b = qV_i \]

\[ \varphi'_b = \varphi_b - \Delta \varphi_b \]

\[ \Delta \varphi_b = \sqrt[4]{\frac{q^3 N_D}{8\pi^2 \epsilon^2} (V_i - V_a)} \]

\[ I_{m\rightarrow s} = AR^*T^2 e^{-q(\varphi'_b - V_a)/kT} \]

\[ I_{s\rightarrow m} = -AR^*T^2 e^{-q\varphi'_b/kT} \]

\[ I = I_{m\rightarrow s} - I_{s\rightarrow m} = AR^*T^2 e^{-q\varphi'_b/kT} \left[e^{qV_a/kT} - 1\right] = I_s \left(e^{qV_a/kT} - 1\right) \]

\[ I_{S,\text{Schottky}} \approx 100 - 1000I_{0,\text{pn}} \]
MS junctions – Ohmic Contact

$q\varphi_m < q\varphi_s$

$E_0$

$q\varphi_m$

$E_{Fm}$

$E_{F0}$

$q\varphi_s$

$q\chi$

$E_C$

$E_{FS}$

$E_V$
MS junctions – Ohmic Contact, Tunneling

\[ q \phi_m = qV_i + (E_C - E_F) \]

\[ qV_i = q(\phi_m - \phi_s) \]

\[ W \propto 1/\sqrt{N_D} \]
End of Review!
We expect discontinuities in the energy bands
1. Vacuum energy \((E_0)\) is continuous.
2. \(E_G\) and \(\chi\) are intrinsic properties of materials and should remain constant.
   (which means \(E_C, E_V,\) and \(E_0\) are all parallel)
3. At equilibrium \(E_F\) is constant while by applying voltage \(\Delta E_F = -qV\).

1. Align the Fermi level with 2 semiconductors separated (leave enough room for transition region).
2. Indicate \(\Delta E_C, \Delta E_V\) at the metallurgical junction.
3. Connect conduction and valance band regions, keeping the band gap constant in each region.
Plotting!

$E_{C1}$

$E_{F1}$

$E_{V1}$

$E_{C2}$

$E_{F2}$

$E_{V2}$

$\Delta E_C$

$\Delta E_V$
Plotting!
Plotting!
Interface structure

I: $\text{Ga}_x\text{In}_{1-x}\text{As}_y\text{P}_{1-y}$
II: $\text{Ga}_x\text{In}_{1-x}\text{As}_y\text{Sb}_{1-y}$
III: $\text{Al}_x\text{Ga}_{1-x}\text{As}_y\text{Sb}_{1-y}$

Lattice matched to

$F_{st}$ (eV) vs $a$ (Å)
Interface structure
For heterojunctions in the GaAs–AlGaAs system, the direct (Γ) band gap difference $\Delta E_G$ is accommodated approximately $\frac{2}{3}$ in the conduction band and $\frac{1}{3}$ in the valence band. For an Al composition of 0.3, the AlGaAs is direct (see Fig. 3–6) with $\Delta E_G^\Gamma = 1.85$ eV. Sketch the band diagrams for two heterojunction cases: $N^+-$Al$_{0.3}$Ga$_{0.7}$As on n-type GaAs, and $N^+-$Al$_{0.3}$Ga$_{0.7}$As on p$^+-$GaAs.$^{18}$

$$\Delta E_G = 1.85 - 1.43 = 0.42 eV \quad \Delta E_C = 0.28 eV \quad \Delta E_V = 0.14 eV$$

**AlGaAs**

**GaAs**
Streetman’s Example

$n^+ - AlGaAs$

$n - GaAs$

1.85

1.85

0.14

0.14

0.28

0.28

1.43

1.43
Streetman’s Example

\[ n^+ - AlGaAs \quad n - GaAs \]

\[ 0.28 \quad 1.43 \]

\[ 1.85 \quad 0.14 \]

\[ n^+ - AlGaAs \quad p^+ - GaAs \]

\[ 0.28 \quad 1.43 \]

\[ 1.85 \quad 0.14 \]
Why it is important?

MATHEMATICAL FORMULAS AND EQUATIONS

![Diagram of heterostructure with labels and notations]

Figure 5-46
A heterojunction between N⁺-AlGaAs and lightly doped GaAs, illustrating the potential well for electrons formed in the GaAs conduction band. If this well is sufficiently thin, discrete states (such as $E_1$ and $E_2$) are formed, as discussed in Section 2.4.3.

carrier transport MODFET: along the heterostructure HBT: perpendicular to the heterojunction
3 types of Heterostructures

Type I

\[ Al_xGa_{1-x}As \quad GaAs \]

Type II

\[ Al_{0.45}In_{0.52}As \quad InP \]

Type III

\[ GaSb \quad InAs \]
Carrier Flow Parallel to the Heterointerface
Carrier flow perpendicular to the heterointerface

Carrier flow in a heterostructure from the narrow gap semiconductor towards the wider gap semiconductor. In this case, the carriers encounter a potential barrier at the interface that arises from the band edge discontinuity. The electrons can overcome the barrier and enter the wide gap material provided they have sufficient kinetic energy. (b) Carrier flow in a heterostructure from the wide gap semiconductor towards the narrow gap semiconductor. In this case, the carriers gain energy from crossing the potential step.
modulation doping provides a means of increasing the free carrier concentration without introducing donor atoms into the channel.

MODFETs, HEMTs (high electron mobility transistors)

80s, GaAs (channel) and AlGaAs (n-doped layer) are lattice matched
Ultra-High-Speed Digital Circuit Performance in 0.2-μm Gate-Length AlInAs/GaInAs HEMT Technology


Fig. 1. AlInAs/GaInAs HEMT device structure.

Fig. 5. The measured $f_T$ of a 0.2-μm AlInAs/GaInAs HEMT extrapolated from the current gain measurement.
AlSb/InAs HEMT’s for Low-Voltage, High-Speed Applications

J. Brad Boos, Member, IEEE, Walter Kruppa, Member, IEEE, Brian R. Bennett, Doewon Park, Steven W. Kirchoefer, Member, IEEE, Robert Bass, and Harry B. Dietrich, Member, IEEE

TABLE I
FET CHANNEL MATERIAL PROPERTIES

<table>
<thead>
<tr>
<th>InAs</th>
<th>In\textsubscript{0.53}Ga\textsubscript{0.47}As</th>
<th>GaAs</th>
<th>InP</th>
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<tbody>
<tr>
<td>0.023</td>
<td>0.041</td>
<td>0.067</td>
<td>0.077</td>
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</table>

<table>
<thead>
<tr>
<th>Electron Effective Mass (m\textsubscript{\Gamma}/m\textsubscript{0})</th>
<th>16000</th>
<th>7800</th>
<th>4600</th>
<th>2800</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron Mobility (cm\textsuperscript{2}/V-sec @ 300K, N\textsubscript{d}=10\textsuperscript{17} cm\textsuperscript{-3})</td>
<td>0.9</td>
<td>0.55</td>
<td>0.31</td>
<td>0.53</td>
</tr>
<tr>
<td>\Gamma-L Valley Separation (eV)</td>
<td>4.0</td>
<td>2.7</td>
<td>2.2</td>
<td>2.5</td>
</tr>
<tr>
<td>Energy Bandgap (eV @ 300K)</td>
<td>0.36</td>
<td>0.72</td>
<td>1.42</td>
<td>1.35</td>
</tr>
</tbody>
</table>

Fig. 1. HEMT starting material.
Performance Metric

\[ f_T = \frac{g_m}{2\pi C_{GS}} = \frac{1}{2\pi \tau_r} \]

\( \tau_r \) transit time of electrons in the device depends on the length of the channel

Long Channel: \( \tau_r = \frac{L^2}{\mu V_{DS}} \)

Short Channel: \( \tau_r = \frac{L}{V_{sat}} \)

\[ L = 1 \mu m \quad (1450) \]

\[ \mu_{Si} = 300 \frac{cm^2}{Vs} \rightarrow f_T = 4GHz \]

\[ \mu_{2D} = 4000 \frac{cm^2}{Vs} \rightarrow f_T = 62GHz \quad (8500) \]

tradeoff between \( f_T \) and Power dissipation
InGaAs Channel (PHEMT)

channel layer is formed with In$_{0.15}$Ga$_{0.85}$As and the doped layer is GaAs. (Rosenberg et al. 1985)

The InGaAs layer is pseudomorphic.

critical thickness of about 20.0 nm

Though the GaAs and InGaAs are not lattice matched, if the InGaAs layer is grown sufficiently thin it will adopt the lattice constant of the underlying GaAs layer.

\[
\mu_{Si} = 1450 \, \frac{cm^2}{Vs} \\
\mu_{GaAs} = 8500 \, \frac{cm^2}{Vs} \\
\mu_{InAs} = 33000 \, \frac{cm^2}{Vs}
\]

\[
E_{G_{GaAs}} = 1.42eV \\
E_{G_{InAs}} = 0.35eV
\]
HEMT

(Brown et al., 1989).

deep well!

\[ \mu_{2D} = 12000 \frac{cm^2}{Vs} \]

\[ v_{sat} = 3 \times 10^7 \text{ cm/s} \]

\[ f_T = 170 \text{ GHz} \]
Bulk Semiconductor Potential, $\varphi_F$

**Definition:**

$$q \varphi_F \equiv E_i - E_F = E_{i(bulk)} - E_F$$

$p$-type

$$\varphi_F = \frac{kT}{q} \ln \left( \frac{N_A}{n_i} \right) > 0$$

$n$-type

$$\varphi_F = -\frac{kT}{q} \ln \left( \frac{N_D}{n_i} \right) < 0$$

$$V_{bi} = \varphi_{FP} - \varphi_{FN} = \frac{kT}{q} \ln \left( \frac{N_D N_A}{n_i^2} \right) = \frac{kT}{q} \ln \left( \frac{n_n}{n_p} \right) = \frac{kT}{q} \ln \left( \frac{p_p}{p_n} \right)$$
Built in Voltage

\[ V_{bi} = \varphi_1 - \varphi_2 \]
\[ = \frac{\Delta E_C}{q} + \frac{(E_{C1} - E_f) - (E_{C2} - E_F)}{q} \]
\[ n = N_C \, e^{-(E_C - E_f)/kT} \]
\[ p = N_V \, e^{-(E_F - E_V)/kT} \]

Effective density of states

\[ n = n_i \, e^{(E_F - E_i)/kT} \]
\[ p = n_i \, e^{(E_i - E_F)/kT} \]

\[ V_{bi} = \frac{\Delta E_C}{q} + \frac{kT}{q} \ln \left( \frac{n_{10} N_{C2}}{n_{20} N_{C1}} \right) \]
Modulation Doping

Modulation doping: free carrier concentration (within semiconductor layer) can be increased significantly without the introduction of dopant impurities.

\[ N_D \uparrow \implies \text{ionized impurity scattering} \uparrow \implies \text{carrier mobility} \downarrow \]

Typically, an undoped AlGaAs spacer layer is formed between the doped AlGaAs and undoped GaAs layers to increase the spatial separation of the electrons from the ionized donors, further reducing the ionized impurity scattering.
Free Electron

\[-\frac{\hbar^2}{2m_0} \frac{d^2\psi}{dx^2} + V(r)\psi = E\psi\]  

(time-independent Schrödinger equation)

Free electron \( V(x) = 0 \), with energy \( E \)

\[\psi = A_+ e^{-ikx} + A_- e^{ikx}\]

\[E = \frac{\hbar^2 k^2}{2m_0}\]

\[m^* = \frac{\hbar^2}{d^2E/dk^2}\]
1-D Quantum Well (Box)

Consider a particle with mass $m$ under potential as:

Outside the box: $V = \infty \rightarrow \psi = 0$

Inside the box: $\frac{d^2 \psi}{dx^2} + \frac{2mE}{\hbar^2} \psi = 0$

$\sin kx, \cos kx$ as $k = \sqrt{2mE}/\hbar$

continuity of $\psi$ at 0 and $L$:

$\psi(0) = \psi(L) = 0 \rightarrow \psi = A \sin kx$ ; $k = \frac{n\pi}{L}$, $n = 1, 2, 3, \ldots$

normalization:

$$\int_{-\infty}^{+\infty} \psi^* \psi \, dx = \int_0^L A^2 (\sin \frac{n\pi x}{L})^2 \, dx = 1 \rightarrow A = \sqrt{2/L}$$

$$\psi_n = \sqrt{\frac{2}{L}} \sin \left( \frac{n\pi x}{L} \right)$$

$n$ is the quantum number
1-D Quantum Well (Box)

\[ E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \]

\[ \psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \]

\( L \uparrow \infty \rightarrow \text{free electron} \)

\( L \sim 0.5 \text{ nm (atom)} \)

\( E_1 = 1.5 \text{ eV} \)

\( E_2 - E_1 = 4.5 \text{ eV} \)
1-D Finite Well

We first need to find the values of the energy for which there are solutions to the Schrödinger equation, then deduce the corresponding wavefunctions. Boundary conditions are given by continuity of the wavefunction and its first derivative.

assume $E < U$

Region $\Box II$

$$\frac{d^2 \psi}{dx^2} = -\frac{2mE}{\hbar^2} \psi = -k^2 \psi$$

$$k = \frac{\sqrt{2mE}}{\hbar}$$

$$\psi_{II} = F \sin kx + G \cos kx$$

Region $\Box I$ and $\Box III$

$$\frac{d^2 \psi}{dx^2} = \frac{2m(U - E)}{\hbar^2} \psi = \alpha^2 \psi$$

$$\alpha = \frac{\sqrt{2m(U - E)}}{\hbar}$$

$$\Rightarrow \psi = Ae^{\alpha x} + Be^{-\alpha x}, \quad x < 0 \text{ and } x > L$$

Finite $\psi$: $\Rightarrow \begin{cases} \psi_I = Ae^{\alpha x}, & x < 0 \\ \psi_{III} = Be^{-\alpha x}, & x < L \end{cases}$
Finite vs. Infinite Well

**Finite Well**
- n=1: 0.39 nm
- n=2: 2.47 eV
- n=3: 9.87 eV
- n=4: 22.2 eV
- n=5: 39.5 eV
- n=6: 61.7 eV

**Infinite Well**
- n=1: 1.95 eV
- n=2: 7.76 eV
- n=3: 17.4 eV
- n=4: 30.5 eV
- n=5: 46.7 eV
- n=6: 63.6 eV

**Energies:**
- 0.39 nm
- 64 eV


Bulk Semiconductor Potential, $\varphi_F$

Infinite triangular potential well

The Schrödinger and Poisson equations

$E = \frac{\hbar^2 k_x^2}{2m_0} + \frac{\hbar^2 k_y^2}{2m_0} + E_i$

self-consistent solution of the Schrödinger and Poisson equations

$E_i = \left( \frac{\hbar^2}{2m_0} \right)^{\frac{1}{3}} \left( \frac{3}{2} \pi q \varepsilon \right)^{\frac{2}{3}} \left( i + \frac{3}{4} \right)^{\frac{2}{3}}$

$\varepsilon$: electric field strength corresponding to the slope of the energy band

$\Rightarrow$ electric field strength corresponding to the slope of the energy band
Electron Concentration

\[ E_F = \Delta E_C - E_d - qV_{dep} \]

\[ V_{dep} = - \int_0^W \varepsilon dz = \frac{qN_D W^2}{2\varepsilon_0\varepsilon_{AlGaAs}} \]

\[ E_1 + \frac{\pi \hbar^2 N_S}{m^*} = \Delta E_C - E_d - qV_{dep} \]

\[ D_{1D}(E) \propto E^{-1/2} \]

\[ D_{2D}(E) \propto cte = m^*/\pi\hbar^2 \]

\[ D_{3D}(E) \propto E^{1/2} \]

\[ N_S = \int_{E_1}^{E_1+E_f} f(E)D(E)dE \]

\[ E_F = \frac{\pi \hbar^2 N_S}{m^*} \]

\[ E_F = E_1 + \frac{\pi \hbar^2 N_S}{m^*} \]
EXAMPLE 2.2.1: Determination of the Total Carrier Density in a Two-Dimensional System

Consider a two-dimensional system formed in the GaAs–AlGaAs materials system. With the assumption that the energy levels can be determined from the infinite triangular well approximation, determine the total carrier density in the system at $T = 0$ K if only one subband is occupied. Assume the following information: $m^* = 0.067m_e$, $\varepsilon_{AlGaAs} = 13.18 - 3.12x$ (where $x$ is the Al concentration); the donor concentration within the AlGaAs layer, $N_D$, is $3.0 \times 10^{17}$ cm$^{-3}$; and the effective field $F$ in the triangular well is $1.5 \times 10^5$ V/cm. The conduction band edge discontinuity in the GaAs–AlGaAs system is usually estimated as 62% of the difference in the energy band gaps. Assume that the Al concentration within the AlGaAs is 40%. The donor energy in AlGaAs is assumed to be 6 meV. The width of the depletion region in the AlGaAs is given as 18.2 nm.

We start with Eq. 2.2.9,

$$E_1 + \frac{N_D \pi^2}{m^*} = \Delta E_c - V_{dep} - E_d$$

The first subband energy, $E_1$, can be calculated using the infinite triangular well approximation with the field $F$ of $3.0 \times 10^5$ V/cm:

$$E_i = \left( \frac{\pi^2}{2m^*} \right)^{1/3} \left( \frac{3}{2} \pi qF \right)^{2/3} \left( i + \frac{3}{4} \right)^{2/3}$$

Substituting in for $i$, 1, and $E_1$ is equal to 0.205 eV. The bandgap discontinuity $\Delta E_g$ is found using Eq. 2.1.2 as

$$\Delta E_g = 1.247x = (1.247)(0.40) = 0.50$$

The conduction band edge discontinuity is then

$$\Delta E_c = (0.62)(0.5) = 0.31 \text{ eV}$$

$V_{dep}$ can be calculated from

$$V_{dep} = \frac{qN_D W^2}{2\varepsilon_{0} \varepsilon_{AlGaAs}}$$

Substituting in the relevant values, $V_{dep}$ is computed to be 0.075 V. $N_x$, the two-dimensional electron concentration, can now be determined as

$$\frac{N_x \pi^2}{m^*} = \Delta E_c - V_{dep} - E_d - E_1 = 0.31 - 0.075 - 0.006 - 0.205 = 0.024 \text{ eV}$$

Solving for $N_x$ yields

$$N_x = \frac{m^*}{\pi \pi^2} (0.024) = 6.7 \times 10^{11} \text{ cm}^{-2}$$
Carrier flow in a heterostructure from the narrow gap semiconductor towards the wider gap semiconductor. In this case, the carriers encounter a potential barrier at the interface that arises from the band edge discontinuity. The electrons can overcome the barrier and enter the wide gap material provided they have sufficient kinetic energy. (b) Carrier flow in a heterostructure from the wide gap semiconductor towards the narrow gap semiconductor. In this case, the carriers gain energy from crossing the potential step.
Potential Well

Region I

\[-\hbar^2 \frac{d^2 \psi_I}{2m \, dx^2} = E \psi_I\]

\[
\frac{d^2 \psi_I}{dx^2} + k^2 \psi_I = 0 \quad \text{where} \quad k = \sqrt{2mE}/\hbar
\]

Region II

\[-\hbar^2 \frac{d^2 \psi_{II}}{2m \, dx^2} + V_0 \psi_{II} = E \psi_{II}\]

\[
\frac{d^2 \psi_{II}}{dx^2} - \alpha^2 \psi_{II} = 0 \quad \text{where} \quad \alpha = \sqrt{2m(E - V_0)}/\hbar
\]

\[
\psi_I = Ae^{ikx} + Be^{-ikx} \quad (x < 0)
\]

\[
\psi_{II} = Ce^{\alpha x} + De^{-\alpha x} \quad (x > 0)
\]

B.C. \(\rightarrow A, B, D\)

\[
\Psi_I(x, t) = Ae^{i(kx - Et/\hbar)} + Be^{i(kx + Et/\hbar)}
\]

\[
\Psi_{II}(x, t) = De^{-\alpha x - iEt/\hbar}
\]

\[
\frac{D}{A} = 2 \frac{E - i\sqrt{(V_0 - E)E}}{V_0}
\]

\[
\frac{B}{A} = \frac{2E - V_0 - 2i\sqrt{(V_0 - E)E}}{V_0}
\]

\[
E = 1eV, V_0 = 2 \, eV
\]

\[
\frac{1}{\alpha} = 0.2 \, nm
\]

penetration depth
Potential Well

Energy = 0.15 eV

Energy = 0.15 eV
Bulk Semiconductor Potential, $\varphi_F$

For simplicity, let us assume that the electron is confined completely to the x–z plane.

\[
\frac{\hbar^2 k_2^2}{2m_2} = \frac{\hbar^2 k_1^2}{2m_1} + \Delta E_C
\]

\[
k_{1x} = k_{2x}
\]

\[
k_{1y} = k_{2y}
\]

Solving for $k_{2z}$
Tunneling

\[-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + U\psi = E\psi\]

\[
\begin{cases}
\psi_I = e^{ikx} + re^{-ikx} & (x < 0) \\
\psi_{II} = Ae^{\alpha x} + Be^{-\alpha x} & (0 < x < L) \\
\psi_{III} = te^{ikx} & (x > L)
\end{cases}
\]

\[k = \sqrt{2mE/\hbar} \quad \alpha = \sqrt{2m(U - E)/\hbar}\]

\[
\begin{cases}
\psi_I = \psi_{II} @ x = 0 \\
\frac{d\psi_I}{dx} = \frac{d\psi_{II}}{dx} @ x = 0 \\
\psi_{II} = \psi_{III} @ x = L \\
\frac{d\psi_{II}}{dx} = \frac{d\psi_{III}}{dx} @ x = L
\end{cases}
\Rightarrow \begin{pmatrix} A \\ B \end{pmatrix} \Rightarrow \begin{pmatrix} t & t \rightarrow T = |t|^2 \\ r & r \rightarrow R = |r|^2 \end{pmatrix}
\]

- Transmission coefficient (T): The probability that the particle penetrates the barrier.

- Reflection coefficient (R): The probability that the particle is reflected by the barrier.

- \(T + R = 1\)
Tunneling

\[ T = \left[ 1 + \frac{U^2}{4E(E - U)} \sin^2 \alpha L \right]^{-1} \]
\[ \alpha = \sqrt{2m(U - E)/\hbar} \]

For \( U = E \)
\[ T = \left[ 1 + \frac{E mL^2}{2\hbar^2} \right]^{-1} \]

For \( U \gg E \)
\[ T \sim \exp\left[ -\frac{2L}{\hbar} \sqrt{2m(U - E)} \right] \]
Material Properties
Bulk Semiconductor Potential, $\varphi_F$
<table>
<thead>
<tr>
<th>1.</th>
<th>Bulk Semiconductor Potential, $\phi_F$</th>
</tr>
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<tbody>
<tr>
<td>2.</td>
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<tr>
<td>3.</td>
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Bulk Semiconductor Potential, $\phi_F$