Session 9: Solid State Devices

Optical Devices
Outline

- A
  - B
  - C
  - D
  - E
- F
  - G
- H
- I
- J
Outline

- Ref: ?
Solar cell is simply a semiconductor diode that has been carefully designed to efficiently absorb and convert light energy from the sun into electrical energy.

\[ E_\lambda = \frac{hc}{\lambda} \]

\[ T_{SUN} \sim 5760 \, K \quad \text{black body} \]

air mass zero (AM0): Just above the Earth’s atmosphere \( 1.353 \, \text{kW/m}^2 \)

AM1.5 (\( \theta = 48.2^\circ \)) \( 1 \, \text{kW/m}^2 \)

AM1.5g (global)

AM1.5d (direct)
Radiation Spectrum
Light Absorption: Direct

Si—GaAs, GaInP, Cu(InGa)Se2, and CdTe,

Si:
- well developed technology
- absorption characteristics are a fairly good match to the solar spectrum

\[
E_2 - E_C = \frac{p^2}{2m_n^*}
\]

\[
E_V - E_1 = \frac{p^2}{2m_p^*}
\]

\[
h\nu - E_G = \frac{p^2}{2} \left( \frac{1}{m_n^*} + \frac{1}{m_p^*} \right)
\]

\[
\alpha(h\nu) \approx A^*(h\nu - E_G)^{1/2}
\]

\[
\alpha(h\nu) \approx \frac{B^*}{h\nu} (h\nu - E_G)^{3/2}
\]
Light Absorption: Indirect

\[
\alpha_a(h\nu) \approx \frac{A(h\nu - E_G + E_{ph})^2}{e^{E_{ph}/kT} - 1}
\]

\[
\alpha_e(h\nu) \approx \frac{A(h\nu - E_G - E_{ph})^2}{1 - e^{-E_{ph}/kT}}
\]

\[\alpha(h\nu) = \alpha_e(h\nu) + \alpha_a(h\nu)\]
Absorption Coefficient vs. Photon Energy

\[ E_{GSi} = 1.12 \text{ eV} \quad \quad E_{GaAs} = 1.42 \text{ eV} \]

light penetration?
Recombination

SRH recom-gen: 

\[ R_{SRH} = \frac{np - n_i^2}{\tau_p(n + n_i e^{-\beta(E_T - E_i)}) + \tau_n(p + n_i e^{-\beta(E_i - E_T)})} \]

\[ \tau = \frac{1}{\sigma v_{th} N_T} \]

p-type (low injection) \[ R_{SRH} \approx \frac{n - n_0}{\tau_n} \]

High injection \[ R_{SRH} \approx \frac{n \approx p}{\tau_p + \tau_n} \]

Direct \[ R_D = B(np - n_i^2) \]

n-type (low injection) \[ R_D \approx \frac{p - p_0}{\tau_{pD}} \]

Auger \[ R_A = (C_n n + C_p p)(np - n_i^2) \]

n-type low-level \[ R_A \approx \frac{p - p_0}{\tau_{pA}} \]

\[ R = \left[ \sum_{\text{traps } i} R_{SRH,i} \right] + R_D + R_A \]

\[ \frac{1}{\tau} = \left[ \sum_{\text{traps } i} \frac{1}{\tau_{SRH,i}} \right] + \frac{1}{\tau_D} + \frac{1}{\tau_A} \]

(minority-carrier lifetime)

low-level injection

(surface states) ?
Consider carrier-flux into/out-of an infinitesimal volume:

\[ Adx \left( \frac{\partial n}{\partial t} \right) = -\frac{1}{q} [J_n(x)A - J_n(x + dx)A] - \frac{\Delta n}{\tau_n} Adx \]

\[ J_n(x + dx) = J_n(x) + \frac{\partial J_n(x)}{\partial x} dx \]

\[ \frac{\partial n}{\partial t} = \frac{1}{q} \frac{\partial J_n(x)}{\partial x} - \frac{\Delta n}{\tau_n} \]

Continuity Equation:

\[
\begin{align*}
\frac{\partial n}{\partial t} &= \frac{1}{q} \frac{\partial J_n(x)}{\partial x} - \frac{\Delta n}{\tau_n} + G_L \\
\frac{\partial p}{\partial t} &= -\frac{1}{q} \frac{\partial J_p(x)}{\partial x} - \frac{\Delta p}{\tau_p} + G_L
\end{align*}
\]
Continuity Equation

\[ \nabla J_p = q \left( G - R_p - \frac{\partial p}{\partial t} \right) \]

\[ \nabla J_n = q \left( R_n - G + \frac{\partial n}{\partial t} \right) \]

\[ J_p = q \mu_p \varepsilon - qD_p \nabla p = -q \mu_p \nabla \varphi - qD_p \nabla p = -q \mu_p \nabla (\varphi - \varphi_p) - kT \mu_p \nabla p \]

\[ J_n = qn \mu_n \varepsilon + qD_n \nabla n = -qn \mu_n \nabla \varphi + qD_n \nabla n = -qn \mu_n \nabla (\varphi + \varphi_n) - kT \mu_n \nabla n \]

low-level injection

Thus the minority carrier diffusion equations are

\[ \frac{\partial \Delta n_p}{\partial t} = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L \quad \text{in p-type material} \]

\[ \frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L \quad \text{in n-type material} \]
PN junctions (Qualitative)
**PN junctions - Assumptions**

The Depletion Approximation: Obtaining closed-form solutions for the electrostatic variables

Charge Distribution:\n\[ \nabla^2 \varphi = \frac{q}{\varepsilon} (p - n + N_D - N_A) \]

\[ \rho = q(N_D - N_A) \]

Note that:
1. \(-x_p \leq x \leq x_n\): p & n are negligible (\(\because \mathcal{E}\) exist).
2. \(x \leq -x_p\) or \(x \geq x_n\): \(\rho = 0\)
Built-In Potential $V_{bi}$

$$qV_{bi} = q\varphi_{sp} + q\varphi_{sn}$$

$$= (E_i - E_F)_p + (E_F - E_i)_n$$

For non-degenerately doped material:

$$ (E_i - E_F)_p = kT \ln \left( \frac{p}{n_i} \right) = kT \ln \left( \frac{N_A}{n_i} \right) $$

$$ (E_F - E_i)_n = kT \ln \left( \frac{n}{n_i} \right) = kT \ln \left( \frac{N_D}{n_i} \right) $$

$$ \to V_{bi} = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right) $$

What shall we do for $p^+ - n$ (or $n^+ - p$) junction?!?!

$p^+$:

$$ (E_i - E_F)_p = \frac{E_G}{2} $$

$n^+$:

$$ (E_F - E_i)_n = \frac{E_G}{2} $$
The Depletion Approximation

\[ \frac{d\mathcal{E}}{dx} = \frac{\rho}{\varepsilon} \]

\[ \rho = -qN_A \rightarrow \]

\[ \mathcal{E}(x) = \frac{-qN_A}{\varepsilon} + C = \frac{-qN_A}{\varepsilon} (x + x_p) \]

\[ \rho = qN_D \rightarrow \]

\[ \mathcal{E}(x) = \frac{qN_D}{\varepsilon} + C' = \frac{qN_D}{\varepsilon} (x - x_n) \]

The electric field is continuous at \( x = 0 \)

\[ x_p N_A = x_n N_D \]

Charge neutrality condition as well!
Depletion Layer Width

\(-x_p < x < 0:\) \quad V(x) = \frac{qN_A}{2\varepsilon}(x + x_p)^2

\(0 < x < x_n:\) \quad V(x) = V_{bi} - \frac{qN_D}{2\varepsilon}(x_n - x)^2

V(0) = \frac{qN_A}{2\varepsilon}x_p^2 = V_{bi} - \frac{qN_D}{2\varepsilon}x_n^2 \quad \rightarrow \quad \begin{cases} x_n = \sqrt{\frac{2\varepsilon_s V_{bi}}{q} \left( \frac{N_A}{N_D(N_A + N_D)} \right)} \\ x_p = \sqrt{\frac{2\varepsilon_s V_{bi}}{q} \left( \frac{N_D}{N_A(N_A + N_D)} \right)} \end{cases}

x_p N_A = x_n N_D

Summing, we have:

\[ W = x_p + x_n = \sqrt{\frac{2\varepsilon_s V_{bi}}{q} \left( \frac{1}{N_D} + \frac{1}{N_A} \right)} \]
Now as we assumed all voltage drop is in the depletion region (Note that \( VA \leq V_{bi} \))

\[
x_n + x_p = W = \sqrt{\frac{2\varepsilon_s (V_{bi} - V_A)}{q}} \left( \frac{1}{N_D} + \frac{1}{N_A} \right)
\]

\[
x_p N_A = x_n N_D
\]
Solar Cell Boundary Conditions

\[ \rho = q(N_D - N_A) \]

\[ \rho = 0 \]

ohmic contact? \[ \Delta p(-W_n) = 0 \]

\[ \frac{d\Delta p}{dx} = \frac{S_{surf}}{D_p} \Delta p(-W_n) \]

back contact: ohmic \[ \Delta n(W_p) = 0 \]

back-surface field (BSF),

\[ \frac{d\Delta n}{dx} \bigg|_{x=W_p} = -\frac{S_{BSF}}{D_n} \Delta n(W_p) \]

? BC at \( x_p \) and \(-x_n\)

\[ qV = F_N(-W_n) - F_p(W_p) \]

\[ p_n(-x_n) = \frac{n_i^2}{N_D} e^{qV/kT} \quad n_p(x_p) = \frac{n_i^2}{N_A} e^{qV/kT} \]
Generation Rate

\[ G(x) = (1 - s) \int (1 - r(\lambda)) f(\lambda) \alpha(\lambda) e^{-\alpha(x+W_n)} d\lambda \]

See text for derivations!
Solar Cell Equivalent Circuit

\[ I = I_{o2}(e^{qV/2kT} - 1) \] recombination in the depletion region

\[ I = I_{o1}(e^{qV/kT} - 1) \] recombination current in the quasi-neutral regions

\[ I_{o1} = I_{o1,n} + I_{o1,p} \]

\[ I_{o1,p} = qA \frac{n_i^2 D_p}{N_D L_P} \left[ \frac{D_p}{L_P} \sinh \frac{W_n - x_n}{L_P} + S_F \cosh \frac{W_n - x_n}{L_P} \right] \]

\[ I_{o1,n} = qA \frac{n_i^2 D_n}{N_A L_n} \left[ \frac{D_n}{L_n} \sinh \frac{W_p - x_p}{L_n} + S_{BSF} \cosh \frac{W_p - x_p}{L_n} \right] \]

\[ I_{o2} = qA \frac{W_D n_i}{\tau_D} \]
Solar Cell Equivalent Circuit

\[ I_{sc} = I_{scd} + I_{scn} + I_{scp} \]

\[ I_{scd} = qA(1 - s) \int_{\lambda} (1 - r(\lambda))f(\lambda) \left( e^{-\alpha(W_n-x_n)} - e^{-\alpha(W_n+x_p)} \right) d\lambda \]
# Solar Cell I–V Characteristic

![Solar Cell I–V Characteristic Graph](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{SC}$</td>
<td>3.67 A</td>
</tr>
<tr>
<td>$V_{OC}$</td>
<td>0.604 V</td>
</tr>
<tr>
<td>$I_{MP}$</td>
<td>3.50 A</td>
</tr>
<tr>
<td>$V_{MP}$</td>
<td>0.525 V</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$n$-type Si emitter</th>
<th>$p$-type Si base</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness</td>
<td>$W_N = 0.35 \mu m$</td>
<td>$W_P = 300 \mu m$</td>
</tr>
<tr>
<td>Doping density</td>
<td>$N_D = 1 \times 10^{20} \text{ cm}^{-3}$</td>
<td>$N_A = 1 \times 10^{15} \text{ cm}^{-3}$</td>
</tr>
<tr>
<td>Surface recombination</td>
<td>$D_p = 1.5 \text{ cm}^{-2} / \text{V s}$</td>
<td>$D_n = 35 \text{ cm}^{-2} / \text{V s}$</td>
</tr>
<tr>
<td>Minority-carrier diffusivity</td>
<td>$S_{F, eff} = 3 \times 10^4 \text{ cm/s}$</td>
<td>$S_{BSF} = 100 \text{ cm/s}$</td>
</tr>
<tr>
<td>Minority-carrier lifetime</td>
<td>$\tau_p = 1 \mu s$</td>
<td>$\tau_n = 350 \mu s$</td>
</tr>
<tr>
<td>Minority-carrier diffusion length</td>
<td>$L_p = 12 \mu m$</td>
<td>$L_n = 1100 \mu m$</td>
</tr>
</tbody>
</table>
recombination losses in emitter ↓ → η ↑
Solar Cell Efficiency
LED - Light emitting diode

LED: a p-n junction in forward biased

\[ E_{\text{light}} \sim E_G \]

LED for optical communication sources (InP, GaAs)
LED for display (GaN, InGaN, AlGaInP)
p+ - n+ junction under forward bias

- At high injection carrier density in such a junction there is an active region near the depletion layer that contains simultaneously degenerate populations of electrons and holes.
- An LED emits incoherent, non-directional, and unpolarized spontaneous photons that are not amplified by stimulated emission.
- An LED does not have a threshold current. It starts emitting light as soon as an injection current flows across the junction.
Emission Energy

\[
\frac{1}{\tau} = \frac{1}{\tau_r} + \frac{1}{\tau_{nr}}
\]

1. high-quality direct emission
   \[\eta \sim 1\]
2. indirect emission
   \[\eta \sim 10^{-2} \ldots 10^{-3}\]

\[
\tau_r = \frac{1}{R_{ec}N_A}
\]

\[
\tau_{nr} = \frac{1}{\sigma v_{th} N_T}
\]

\[
\eta = \frac{1}{1 + \frac{\tau_r}{\tau_{nr}}}
\]

(i) increasing the direct recombination rate and leading to higher light output,

(ii) having an emission region that is lower in energy than the injection (cladding) regions which allows the generated photons to escape without being re-absorbed in the injection regions,

(iii) minimizing the overflow of electrons into the cladding regions where the injected carriers either recombine non-radiatively or generate light of an undesired wavelength.
recombination coefficients and lifetimes

<table>
<thead>
<tr>
<th></th>
<th>$R_r [cm^{-3}s^{-1}]$</th>
<th>$\tau_r [ns]$</th>
<th>$\tau [ns]$</th>
<th>$\tau [ns]$</th>
<th>$\eta_{int}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>$10^{-15}$</td>
<td>100000000</td>
<td>100</td>
<td>100</td>
<td>$10^{-5}$</td>
</tr>
<tr>
<td>GaAs</td>
<td>$10^{-10}$</td>
<td>100</td>
<td>100</td>
<td>50</td>
<td>0.5</td>
</tr>
</tbody>
</table>

$R_r$  Carrier pair injection rate $[cm^{-3}s^{-1}]$

steady-state excess-carrier concentration $\delta n = R_r \tau [1/cm^3]$

$$
\Phi \left[ \frac{photon}{s} \right] = \eta_{int} R_r V \\
= V \frac{\delta n}{\tau_r} = \frac{\eta_{int} i}{q} \quad \delta n \uparrow \\
\tau_r \downarrow \quad P = h\nu \Phi
$$

Very effective carrier and optical confinement can be simultaneously accomplished with double heterostructures. A basic configuration can be either P-p-N or P-n-N (the capital P, N represents wide-gap materials, p, n represents narrow-gap materials). The middle layer is a narrow-gap material. (e.g. Ga$_{1-y}$Al$_y$As – GaAs - Ga$_{1-x}$Al$_x$As)
Recombination Rate

\[ \hbar \omega - E_g = \frac{\hbar^2 k^2}{2 \left( \frac{1}{m_e^*} + \frac{1}{m_h^*} \right)} \]

\[ W_{em} \sim 1.5 \times 10^9 \hbar \omega \ [eV \ s^{-1}] \]

\[ t_0 \sim \frac{0.67}{\hbar \omega [eV]} [ns] \]

\[ V_{bi} = 3.4V \]

\[ V_{ON} = 2.8V \]
Lightly doped:

\[
R_{spon} = \frac{1}{2t_0} \left( \frac{2\pi \hbar^2 m_r^*}{k_B T \mu_e \mu_h^*} \right)^{3/2} \quad np
\]

\[
\frac{R_{spon}}{n} = \frac{1}{t_r} = \frac{1}{2t_0} \left( \frac{2\pi \hbar^2 m_r^*}{k_B T \mu_e \mu_h^*} \right)^{3/2} \quad p
\]

Heavy doped:

\[
R_{spon} \sim \frac{1}{2t_0} \left( \frac{m_r^*}{m_h^*} \right)^{3/2} \quad n
\]

\[
R_{spon} \sim \frac{1}{2t_0} \left( \frac{m_r^*}{m_h^*} \right)^{3/2} \quad p
\]

High injection

\[
R_{spon} \sim \frac{n}{t_0} \sim \frac{p}{t_0}
\]
Radiative Lifetime

Semiconductor GaAs
Temperature is 300 K

Typical carrier densities for laser operation

Carrier occupation is degenerate
$f_e = f_h = 1$

$N_d$ (for holes injected into an $n$-type semiconductor)

$n = p$ (for excess electron-hole pairs injected into a region)
Direction of Emitted Light

surface emitter

edge emitter
Luminous Efficiency

LED luminous efficiency with time
3 Optical Processes

**BEFORE**

Absorption

\[ E_C \uparrow \]

Spontaneous emission

\[ E_C \rightarrow E_V \]

Stimulated emission

\[ E_C \rightarrow E_V \]

**AFTER**

Absorption

\[ E_C \rightarrow E_V \]

Spontaneous emission

\[ E_C \rightarrow E_V \]

Stimulated emission

\[ E_C \rightarrow E_V \]
Laser: "light amplification by stimulated emission of radiation"

Spatial coherence:
- focused to a tight spot
- narrow over long distances (collimation)
- narrow spectrum (high temporal coherence) (pulses of light—as short as a femtosecond)

Components of a typical laser:
1. Gain medium
2. Laser pumping energy
3. High reflector
4. Output coupler
5. Laser beam

Watch movie
1. Capable of emitting high powers (e.g. continuous wave ~ W).

2. A relatively directional output beam (compared with LEDs) permits high coupling efficiency (~ 50 %) into single-mode fibers.

3. A relatively narrow spectral width of the emitted light allows operation at high bit rates (~ 10 Gb/s), as fiber dispersion becomes less critical for such an optical source.

laser diode:
semiconductor optical amplifier (SOA) that has an optical feedback.

SOA: Forward -biased p+-n+ junction from a direct-bandgap material

The sharp refractive index difference between the crystal (~3.5) and the surrounding air causes the cleaved surfaces to act as reflectors.
Laser Diodes

Fabry-Perot optical resonator.

Gain coefficient is sufficiently large:
Amplifier + optical feedback $\rightarrow$ oscillator

When stimulated emission is more likely than absorption
$\Rightarrow$ net optical gain (a net increase in photon flux)
$\Rightarrow$ material can serve as a coherent optical amplifier
Population inversion by carrier injection
Population inversion
3 Optical Processes

Absorption

Before

$E_C$

After

$E_C \rightarrow E_V$

$R_{abs} \propto \rho(h\nu)P_V(E_1)[1 - P_C(E_2)]$

Spontaneous emission

Before

$E_C$

After

$E_C \rightarrow E_V$

$R_{spon} \propto P_C(E_2)[1 - P_V(E_1)]$

Stimulated emission

Before

$E_C$

After

$E_C \rightarrow E_V$

$R_{stim} \propto \rho(h\nu)P_C(E_2)[1 - P_V(E_1)]$
R_{stim} \propto \rho(h\nu)P_C(E_2)[1 - P_V(E_1)] \geq R_{abs} \propto \rho(h\nu)P_V(E_1)[1 - P_C(E_2)]

P_C(E_2)[1 - P_V(E_1)] > P_V(E_1)[1 - P_C(E_2)]

P_C(E_2) > P_V(E_1)

This defines the population inversion in a semiconductor
Optical Gain

![Diagram of optical gain](image)

**Optical gain (broadband)**

- FWHM = gain bandwidth
- $E_{FC} - E_{FV}$
- Frequency

1. I
2. II
3. III
4. IV
5. V
LED and Laser Diode

LED

Light Output vs. Current

Laser

Light Output vs. Wavelength

1. I
2. 
3. 
4. 
5. 

Mirror Facet
Single frequency lasers is desirable in the optical fiber communication system to increase the bandwidth of an optical signal. This is because light pulses of different frequencies travel through optical fiber at different speeds thus causing pulse spread. Dispersion mechanisms for a step-index fiber:
(1) intermodal dispersion
(2) waveguide dispersion
(3) material dispersion
Dispersion effects can be minimized by using long wavelength sources of narrow spectral width (a single frequency laser) in conjunction with single mode fibers.
Methods to achieve the single frequency lasers:
(1) Frequency Selective Feedback
   External Grating, Distributed-Feedback (DFB), Distributed Bragg Reflector (DBR)
(2) Coupled Cavity
   Cleaved Coupled Cavity (C3) laser